1. Mr. Smith owns a shoe repair shop. He approaches his local bank for a loan of $150,000. This money will be spent on investments in his store and on advertising. Without the loan, his profits will be only $50,000.

- For the loan to be worthwhile, the bank must receive a payment of $170,000 (including interest). The contract requires Mr. Smith to pay back $175,000. If he fails to pay his loan in full, the bank may foreclose on his store to obtain as much money as possible. The bank may also choose to go easy on Mr. Smith, and allow him to refinance his loan. This is sometimes better for the bank, as they can obtain more money in the long run if they allow him more time to pay.
- With this loan, if he works hard, he can increase his profits by $180,000 over the next few years. (So total profits will be $230,000.)
- However, he might also decide to goof off. In that case, his profits will go up by only $25,000 (for a total of $75,000), but he obtains a personal gain he values at $60,000. (For example, he can use the money to buy himself a few vacations in Jamaica.)
- The bank can claim only against his profits and the liquidation value of his store. The liquidation value of his store is $50,000 (irrespective of whether an investment is made). Mr. Smith values his store at $100,000, so liquidation is inefficient.
- If Mr. Smith is unable to pay back his loan, the bank has two choices: (1) Foreclose and get as much money as possible by liquidating his assets. (2) Refinance the loan to give him more time to pay. If they refinance the loan, then he will be able to earn additional profits totalling $60,000, and these can be taken by the bank.

The following game tree represents this situation. The first payoff listed is Mr. Smith’s. The second is the Bank’s.
a. Consider the subgame that begins after the Bank lends money (node B).
   i. In the table below, write out this subgame in Normal form and circle the Nash equilibria.

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<th>Bank</th>
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   Mr. Smith
   |      |
   |      |

   ii. Using backwards induction, find the subgame-perfect equilibrium of the game that begins at node B. Use arrows to indicate the players’ strategies. Place a star by the Nash equilibrium that is subgame-perfect.

   iii. Explain intuitively why the subgame-perfect equilibrium is more plausible.

b. Using your results in part (a), determine the subgame-perfect equilibrium of the game as a whole.
   i. Indicate strategies using arrows on the tree above.

   ii. What actually happens in the subgame-perfect equilibrium?

   iii. What precisely are the players’ strategies (including behavior off the equilibrium path)?
2. Consider two firms in a Cournot duopoly. Each has a constant marginal cost of .2 per unit. Firm 1 produces $q_1$ units and firm 2 produces $q_2$ units. The industry demand curve is

$$Q = 1050 - 300p$$

where $Q = q_1 + q_2$.

a. Calculate the Cournot Nash equilibrium.

i. What is the price as a function of $q_1$ and $q_2$?

ii. What is firm 1’s profit as a function of $q_1$ and $q_2$?

iii. Calculate firm 1’s best response function.

iv. What is firm 2’s best response function?

v. Combine these to get the Nash equilibrium.

vi. What is the equilibrium price?

vii. What is each firm’s profit level?
b. Now suppose that firm 1 acts as the Stackelberg leader.
   i. Suppose that firm 1 assumes that firm 2 will play its best response level of \( q_2 \), taking \( q_1 \) as fixed. What is firm 1’s profit as a function of \( q_1 \)?

   ii. Calculate firm 1’s optimal level of \( q_1 \).

   iii. What are the levels of \( q_2 \) and \( p \) at the equilibrium?

   iv. What is each firm’s profit level?

   v. What do you notice?

   vi. Explain why this outcome is a subgame-perfect equilibrium. What is the game being played here?