1. Two firms share a market with demand curve
   \[ Q = 3800 - 20p \]
   Each has cost function
   \[ C(y) = 10000 + \frac{1}{20}y^2 \]
   a. What is the outcome in this market if these firms act as price-takers?
      (This is the efficient outcome)
      • How much does each firm produce?
      • What is the market price?
      • What is each firm’s profit?
      • What is consumer surplus?
b. Now suppose that the firms are not price-takers, but that each maximizes its profit taking the other’s production choice as given. Firm 1 produces \( y = q_1 \) and firm 2 produces \( y = q_2 \).

i. Suppose that firm 2 produces 500 units of output, so that \( q_2 = 500 \).
   What is the residual demand curve facing firm 1?

ii. Set firm 1’s marginal revenue equal to marginal cost to obtain the optimal choice of \( q_1 \).

iii. Now we will calculate how each firm’s output depends on the other’s. Calculate the market price as a function of \( q_1 \) and \( q_2 \).

iv. What is firm 1’s profit-maximizing choice of \( q_1 \) when firm 2 produces \( q_2 \) units of output?
v. What is firm 2’s profit-maximizing choice of \( q_2 \) when firm 1 produces \( q_1 \)?
(There is no math to do.)

vi. The following is a graph of the firms’ best response functions:

Derive the Cournot - Nash equilibrium using algebra. What are \( q_1, q_2, \) and \( p \)?
vii. How do the duopolistic quantities and price compare to the competitive outcome? Can you explain why this is the case?

viii. What is the resulting profit for each firm, and how does it compare to the competitive outcome?

ix. What is the resulting consumer surplus, and how does it compare to the competitive outcome?

x. How does the duopolistic outcome compare to the competitive outcome in terms of efficiency?
c. Now suppose that the two firms decide to collude in order to obtain even higher profits. Thus, they agree on a price that each will charge, and they split the market equally between them.
   i. What is each firm’s profit if they each produce $q$ units of output?

ii. What are their profit maximizing outputs and price?

iii. How much profit does each firm earn? Compare this profit to the Cournot outcome.

d. We will now see that even though collusion is beneficial to these firms, it is not strategically stable. One or the other firm will have an incentive to raise output above the cartel level.
   i. Suppose that firm 1 chooses to adhere to the cartel agreement. What level of output is best for firm 2, and how does it compare to their agreement?
ii. How much does firm 2 gain, in terms of increased profits, by deviating from the agreement?

iii. What can you therefore conclude about the cartel, from a practical standpoint?
2. In this problem, you will see that when the market contains more firms, the oligopolistic solution becomes more similar to the competitive one. Suppose now that there are six firms, each of whom has cost function

\[ C(y) = 10000 + \frac{1}{20}y^2 \]

The market demand is

\[ Q = 3 \times (3800 - 20p) = 11400 - 60p \]

Thus, the market is three times as large as the market in problem 1. There are three times as many firms and three times as many consumers.

a. Calculate the outcome in this market when firms are price-takers.
   - How much does each firm produce?
   - What is the market price?
   - What is each firm’s profit?
   - What is consumer surplus?

b. Now suppose that the firms are not price-takers, but maximize their individual profits, taking the outputs of the others as given.
   i. Suppose that all of the other (five) firms each produce 500 units of output. What is the residual demand curve of firm 1?
ii. Use this demand curve to calculate the optimal output of firm 1.

How does this compare to the duopoly result? Can you explain why?

iii. Now we will calculate how each firm’s output depends on the other’s. Suppose that firm 1 produces \( q_1 \) units and all of the other firms produce \( q \) units. (We assume that they all produce the same amount, to make things simple.) Calculate the market price as a function of \( q_1 \) and \( q \).

iv. What is firm 1’s profit-maximizing choice of \( q_1 \) when all of the other firms produce \( q \) units of output?
v. What is firm 2’s profit-maximizing choice of \( q_2 \) when all of the other firms (except firm 2) produce \( q \)? (There is no math to do.)

vi. Since each firm is the same, it is reasonable to suppose that each produces the same amount in the equilibrium. Assuming this, derive the Cournot - Nash equilibrium using algebra. What are \( q \) and \( p \)?

vii. How do the quantities produced compare to the duopoly outcome and the competitive outcome? Can you explain why this is the case?

viii. What is the resulting profit for each firm? How does it compare to the duopoly profit?

c. Now suppose that the two firms decide to collude in order to obtain even higher profits. Thus, they agree on a price that each will charge, and they split the market equally between them.
i. What are their profit maximizing outputs and price?

ii. How much profit does each firm earn? (There is no math to do.)

d. Now we will see that the gains of deviating from a cartel are higher than in the duopoly case.
   i. Suppose that all firms except firm 1 chooses to adhere to the cartel agreement. What level of output is best for firm 1, and how does it compare to their agreement?

   ii. How much additional profit can firm 1 obtain by deviating? (What is its gain from deviating?)

   iii. What can you therefore conclude about the cartel, from a practical standpoint?
3. In this problem, you will see what happens in a duopoly model when the firms have different cost functions. Suppose that two firms exist in an industry with inverse demand curve

\[ P = 120 - 2Q \]

Firm 1 has marginal cost 40 and firm 2 has marginal cost 20.

a. What is the efficient outcome in this market?
   How much does each firm produce and what is the market price?
   (Think of what would happen if the firms were price-takers.)

b. Now we will calculate the Cournot Nash equilibrium.
   i. What is firm 1’s best response function?

   ii. What is firm 2’s best response function? (Careful! This problem is not symmetric.)
iii. Calculate the Cournot Nash equilibrium.

iv. How much profit does each firm earn?

v. How does the Cournot outcome compare to the efficient outcome?
   What are two inefficiencies that you see, and how can you explain them?

c. Now suppose that the two firms decide to collude and form a cartel.
i. What is their joint profit as a function of $q_1$ and $q_2$?

ii. The firms agree on a plan that maximizes this joint profit. One firm will not produce at all at their optimal plan? Which firm is this?

iii. What production plan maximizes their joint profits?

iv. What profit does each firm earn?

v. How much does firm 2 have to pay firm 1 to agree not to produce? Verify that firm 2 is willing to pay this.

d. What would you predict would actually happen in an industry like this?