Notes 9: Profit Today Versus Profit Tomorrow: Firm Decision Making over Time

I. Variation in profits over time and the decision between alternative profit streams

Firms often must choose between taking an action that yields profit immediately versus taking an action that yields perhaps greater profit but not until many periods later. In such a setting, the meaning of maximizing profit is not always clear.

A. Opportunity cost and decision making over time

A decision that leads to increased profit in the current period may have an opportunity cost in terms of profit that is given up in a future period or periods. For example, a firm may choose to forego some production in the current period in order to concentrate resources on the construction of a new plant. The costs of this plan in terms of current profits may be very high, but the firm may realize larger profits in future periods than it would have had it not built the plant. The opportunity cost of investing in a project that has returns in the distant future is the return that would be available from a different project that has returns in the current or a less distant period.

B. Time preferences

Individuals have preferences over the timing of cost and return flows. Economic theory usually assumes that an individual has a positive rate of time preference, meaning that one dollar today is preferred over one dollar one year from now. This is usually attributed to impatience or quasi concavity of the utility function. Exceptions to this positive rate can occur if relative income and wealth levels differ across time periods, if financial markets are not complete, or if there are significant costs for carrying goods between periods. The rate of time preference for an individual commodity is the implicit relative price that would induce an individual to consume or hold equal amounts in adjacent periods and is implied by the shape of indifference curves. When applied to an individual commodity, the rate of time preference is called the own rate of interest; when applied to a numeraire commodity such as money, it is called the discount rate or the rate of interest. Just as the interaction of individual preferences for commodities and the production technology determine the relative prices of goods, the interaction of individuals' time preference, commodity preference, and the technology determine a market rate of discount or interest rate. There are clearly different discount rates for time periods of different lengths. These rates reflect the market's evaluation of the relative worth of the same income flows (or money) occurring in different time periods.

An individual's rate of time preference is determined independently of the market rate of interest, but is a factor in determining the market rate. In an economic equilibrium, where individuals can trade freely on commodity and financial markets, they will make production and consumption decisions such that (at the margin) their individual rate of discount between income in different periods is equal to the market rate of interest. The Fisher separation theorem implies that production decisions can be made independently of consumption decisions when markets are complete. This theorem further implies that individuals will make production decisions based on this market rate of interest, and partially justifies the common practice of using the market rate of interest (discount) for evaluating the relative contributions of returns and costs to an individual's welfare at different periods in time. When markets are not complete or fully functioning, a rate of discount other than the market rate may be applicable.
C. Discounting

The practice of adjusting all cost and return streams to a common point in time to account for time preferences is usually called discounting or present value analysis. The idea is that with properly functioning markets, funds received in one period can be invested at the market discount rate and earn that rate of return over the period. Thus one dollar received today is worth more than one received tomorrow because it can be invested at this usually positive market rate. If the desire is to reflect all future monetary flows on an equivalent current period basis, present value formulas are used. When income streams are adjusted to a future point in time, the practice is sometimes called compounding or future value analysis to contrast it with discounting income flows back to the current period. In order to make the analysis clear, consider a number line taking values from $-\infty$ to $\infty$ as below. Time 0 is considered to be the present time, time 1 is one period in the future, -2 is two periods in the past, and so forth.

\[
\begin{array}{cccccccc}
-4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\end{array}
\]

Of course, the line can be renumbered so that any point on it is time 0. Consider now an income (cost) stream that begins at the present time 0 (or the beginning of the first period) and ends at time \( n \). The value of this stream at time 0 is given by

\[
V_0 = \sum_{t=0}^{n} \frac{Z_t}{(1 + r)^t}
\]

where \( V_0 \) is the present value of the payment stream (of income or costs) on the right-hand side of the equal sign. The notation \( Z_t \) represents the net return or cost at the end of period \( t \), where \( t \) denotes the time period 0, 1, 2, 3, ..., \( n \). The discount rate, which is constant over time, is given by \( r \). If the initial period of the income stream is considered to be the base, as in this example, the discounted value is called the present value of the future income stream. For example, consider an income stream with values (100, 200, 500) at the points 0, 1, and 2. This is represented on the number line by placing the returns above the line as follows.

\[
\begin{array}{cccccccc}
-4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\end{array}
\]

The present value at point 0 of the above stream is given by

\[
V_0 = 100 + \frac{200}{1 + r} + \frac{500}{(1 + r)^2}
\]

If the interest rate is 5\%, this will give

\[
V_0 = 100 + \frac{200}{1.05} + \frac{500}{(1.05)^2} = 743.991.
\]
In many instances it is useful to adjust cost and return streams to points in time other than the present. This can be accomplished using the above formula and allowing the index $t$ to take on both positive and negative values in relation to the point of time considered to be the present or the base ($0$) for the analysis. For example, consider adjusting all income flows to the end of the last period (the $n^\text{th}$ period) as is done in future value analysis. The value at the end of the $n^\text{th}$ period ($V_0$) of a CAR stream occurring over the $n$ periods is given by

$$V_0 = \sum_{t=-n}^{0} \frac{Z_t}{(1+r)^t}$$

where $V_0$ represents the value of the payment stream at the end of the period (time 0). If one prefers to use positive values for the index $t$ and treat the $n^\text{th}$ period as the base, as in standard future value calculations, the above formula would read

$$V_n = \sum_{t=0}^{n} \frac{Z_t}{(1+r)^{t-n}} = \sum_{t=0}^{n} Z_t(1+r)^{n-t}.$$
where \( j \) is the first period considered and \( n \) is the last. When \( k \) is greater than \( t \), flows are adjusted forward to period \( k \); when \( k \) is less than \( t \), flows are adjusted back to \( k \); and when \( k \) is equal to \( t \), the flow is not adjusted. Consider the value at the end of the first period (or time 1 on the number line) for the above payment stream. The formula will give

\[
V_1 = \sum_{t=0}^{2} \frac{Z_t}{(1+r)^{t+1}}
\]

\[
= \frac{100}{(1+r)^{0+1}} + \frac{200}{(1+r)^{1+1}} + \frac{500}{(1+r)^{2+1}}
\]

\[
= 100(1+r) + 200 + \frac{500}{(1+r)}
\]

where \( V_1 \) represents the value at the end of the first period.

To clarify the discussion, consider a stream of CAR flows occurring at the end of each period. Let the flow at the end of period 1 be -10 with a further return at the end of period 2 of -20. Let the returns at the end of periods 3 through 5 be -5, 10, and 50. The number line is as follows:

-10 -20 -5 10 50

-3 -2 -1 0 1 2 3 4 5

Assume a discount rate of 10%. The adjusted (discounted) values of each flow and the total for the entire stream at the end of each period are given in the Table below. The columns give the cash flow adjusted to the end of the period in the column title. For example, consider the first line of the table which reflects cash flow of -10 at the end of the first period. This cash flow has value -10 at the time 1, but declines in value (grows in absolute value) to -11 (11) by the end of period 2. The value at the beginning of period 1 (end of period 0) is -9.091. The adjusted value of this flow at the end of the fifth period is -14.641. Similarly, the value at the end of period 0 of the 50 dollar return occurring is 31.046 and the value of the 50 dollar return at the end of the fifth period valued at the end of the fifth period is 50. The Total row at the bottom of the table gives the total of the cash flows for all periods adjusted to the end of the period in the column title. Thus, for example, the total value of all five cash flows at time 0 is $8.499, while at the end of the first period (time 1) it is 9.35 and at the end of the fifth period it is 13.689. The diagonal elements of the table are the same as the actual cash flows, because the diagonal represents adjustments to that period as the base. Furthermore, the amounts in the Total line can be adjusted to any other period using similar procedures. For example, the value of the entire stream at the end of the fourth period ($12.445) is properly discounted to the end of the first period using the relation \( V_0 = 12.445/(1.1)^3 = 9.35 \).
TABLE  Discounted Values of a Cost and Return Stream

<table>
<thead>
<tr>
<th>Period</th>
<th>Cash Flow at End of Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10.000</td>
</tr>
<tr>
<td>1</td>
<td>-10.000</td>
</tr>
<tr>
<td>2</td>
<td>-20.000</td>
</tr>
<tr>
<td>3</td>
<td>-5.000</td>
</tr>
<tr>
<td>4</td>
<td>10.000</td>
</tr>
<tr>
<td>5</td>
<td>50.000</td>
</tr>
<tr>
<td>Total</td>
<td>$8.499</td>
</tr>
</tbody>
</table>

D. Common discounting formulas

1. The discount factor

Let the real interest rate be given by \( r \). We then define the discount factor by \( R = \frac{1}{1 + r} \). If we multiply a payment received one period from now by \( R \), we obtain the value of this payment at the current time. For example if \( r = .05 \) and we receive a payment of $210.00 at the end of the year, the present value is given by

\[
V_0 = \left( \frac{1}{1.05} \right) (210) = 200.
\]

2. Relationships between \( r \) and \( R \)

The following relationships between \( r \) and \( R \) then hold

\[
R = \frac{1}{1 + r}
\]

\[
1 + r = \frac{1}{R}
\]

\[
r = \frac{1}{R} - 1 = \frac{1 - R}{R}
\]

\[
\frac{1}{r} = \frac{R}{1 - R}
\]

\[
\frac{1 + r}{r} = \frac{1}{1 - R}
\]
3. Common formulas using the discount factor in finite time

We can consider the value of a stream of payments using the discount rate. Let the return or expenditure at the end of the $t^{th}$ period be given by $Z_t$. Then the value of this stream at the end of the $0^{th}$ period (which is also the beginning of the first period) is given by

$$V_0 = Z_0 + \frac{Z_1}{1+r} + \frac{Z_2}{(1+r)^2} + \frac{Z_3}{(1+r)^3} + \ldots + \frac{Z_T}{(1+r)^T}$$

$$= Z_0 + Z_1 R + Z_2 R^2 + Z_3 R^3 + \ldots + Z_{T-1} R^{T-1} + Z_T R^T$$

$$= \sum_{t=0}^{T} \frac{Z_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{R^t Z_t}{(1+r)^t}$$

If the payment stream starts at the end of period 1, we can write this as $PV$ so that

$$PV = \frac{Z_1}{1+r} + \frac{Z_2}{(1+r)^2} + \frac{Z_3}{(1+r)^3} + \ldots + \frac{Z_T}{(1+r)^T}$$

$$= Z_1 R + Z_2 R^2 + Z_3 R^3 + \ldots + Z_{T-1} R^{T-1} + Z_T R^T$$

$$= \sum_{t=1}^{T} \frac{Z_t}{(1+r)^t} = \sum_{t=1}^{T} \frac{R^t Z_t}{(1+r)^t}$$

If $Z_t$ is a constant and there is no return or expenditure until the end of the first period we can write

$$PV = \bar{Z} R + \bar{Z} R^2 + \bar{Z} R^3 + \ldots + \bar{Z} R^{T-1} + \bar{Z} R^T$$

$$= \frac{\bar{Z}}{1+r} + \frac{\bar{Z}}{(1+r)^2} + \frac{\bar{Z}}{(1+r)^3} + \ldots + \frac{\bar{Z}}{(1+r)^T}$$

$$= \sum_{t=1}^{T} \frac{\bar{Z}}{(1+r)^t} = \sum_{t=1}^{T} \frac{R^t \bar{Z}}{(1+r)^t}$$

To find the value of this series it is useful to define $s_T$ which is the value of the constant payment series PV if the payment stream started today instead of at the end of the first period. Specifically,

$$s_T = R^{-1} PV = R^{-1} \left[ \bar{Z} R + \bar{Z} R^2 + \bar{Z} R^3 + \ldots + \bar{Z} R^{T-1} + \bar{Z} R^T \right]$$

$$= \bar{Z} + \bar{Z} R + \bar{Z} R^2 + \ldots + \bar{Z} R^{T-1}$$

$$= \sum_{t=1}^{T} \bar{Z} R^{t-1}$$

This is just the value of a constant payment stream where the first payment occurs at the beginning of the first period. Now compute $Rs_T$ and then subtract from $s_T$ to obtain
\[
s_T = \sum_{t=1}^{T} \bar{Z} R^{t-1} = \bar{Z} + \bar{Z} R + \bar{Z} R^2 + \ldots + \bar{Z} R^{T-1}
\]

\[
R s_T = R \sum_{t=1}^{T} \bar{Z} R^{t-1} = \sum_{t=1}^{n} \bar{Z} R^{t} = \bar{Z} R + \bar{Z} R^2 + \bar{Z} R^3 + \ldots + \bar{Z} R^{T-1} + \bar{Z} R^T
\]

\[
\Rightarrow s_T - R s_T = \bar{Z} - \bar{Z} R^T
\]

\[
\Rightarrow s_T(1 - R) = \bar{Z}(1 - R^T)
\]

\[
\Rightarrow s_T = \frac{\bar{Z}(1 - R^T)}{1 - R}, \quad R \neq 1
\]

Since \( s_T = R^{-1} PV \), then \( PV = R s_T \). This then implies that

\[
PV = R s_T = R \left( \frac{\bar{Z}(1 - R^T)}{1 - R} \right)
\]

\[
= \frac{\bar{Z}(R - R^{T+1})}{1 - R}
\]

3. Common formulas using the discount factor when the time horizon is infinite

If \( T \to \infty \) and \( |R| < 1 \) then \( R^T \to 0 \). This then implies

\[
s_T = \sum_{t=1}^{T} \bar{Z} R^{t-1} = \bar{Z} + \bar{Z} R + \bar{Z} R^2 + \ldots + \bar{Z} R^{T-1} = \frac{\bar{Z}(1 - R^T)}{1 - R}
\]

\[
\Rightarrow \lim_{T \to \infty} s_T = \frac{\bar{Z}}{1 - R} = \frac{\bar{Z}(1 + r)}{r}
\]

Similarly,

\[
PV = R s_T = \sum_{t=1}^{T} \bar{Z} R^{t} = \bar{Z} R + \bar{Z} R^2 + \ldots + \bar{Z} R^{T-1} + \bar{Z} R^T
\]

\[
\lim_{T \to \infty} PV = \lim_{T \to \infty} R s_T = \frac{R \bar{Z}}{1 - R} = \bar{Z} \left( \frac{R}{1 - R} \right) = \frac{\bar{Z}}{r}
\]

In the case where \( T \to \infty \), the payment stream is known as a perpetuity that pays a constant amount forever.
II. Example problems

A. Driving out a competitor

Suite Enterprises is a large restaurant supply firm that dominates the local market. It does, however, have one rival, Loew Supplies. Because of this competition, Suite earns a profit of $100,000 per year. It could, however, cut its prices to cost and drive out Loew. To do this, Suite would have to forgo all profit for one year during which it earned zero. After that year, Loew would be gone forever and Suite could earn $110,000 per year. The interest rate Suite confronts is 12 percent. Hence, the discount factor \( R = 0.8929 \).

1. Is driving Loew out of the market a good “investment” for Suite?

   The discount factor is given by

   \[
   R = \frac{1}{1 + r} = \frac{1}{1.12} = 0.892857 = .89
   \]

   The firm foregoes a benefit of $100,000. It then has a net benefit of $10,000 forever. The value of $10,000 forever received today is given by the value of a perpetuity of $10,000 that starts at the end of 1 year. We multiply this by \( r = \frac{R}{1 - R} \) to get its present value. The present value driving out Loew is

   \[
   PV(\text{driving out Loew}) = -100,000 + \frac{1}{r} (10,000)
   \]

   \[
   = -100,000 + \frac{10,000}{0.12}
   \]

   \[
   = -100,000 + 83,333.33 = -16,666.66
   \]

2. Consider the alternative strategy in which Suite buys Loew for $80,000 today and then operates the new combined firm, Suite & Loew, as a monopoly earning $110,000 in all subsequent periods. Is this a good investment?

   We can simply repeat the analysis from part 1 using a cost of $80,000 at the beginning of the current period and no returns until the end of the current period. This yields

   \[
   PV(\text{driving out Loew}) = -80,000 + \frac{10,000}{0.12}
   \]

   \[
   = -80,000 + 83,333.33 = 3,333.33
   \]

   This is a good investment.
B. Choosing between alternative strategies

Lindon Enterprises is considering a takeover offer by Pope, Chambers and Just. Lindon Enterprises has an expected profit of $20,000 per year for an indefinite period in the future. This is received at the end of each year. The rate of interest is 10%. Pope, Chambers and Just are offering a cash payment as of right now of $180,000. Alternatively they would be willing to pay $50,000 per year for 5 years with the first payment at the end of 1 year. A competing firm, Lopez and Chavas, is offering a cash payment buy partial interest in Lindon Enterprises. Lindon Enterprises’ share of the profits of the new joint firm are expected to be $14,000 per year. Lopez and Chavas are offering $70,000 in cash to gain partial ownership.

1. What is the present value of the firm if it keeps operating

The present value of a perpetuity of $20,000 can be obtained by dividing $20,000 by the interest rate or by multiplying it by \( \frac{1}{r} \). Carrying out the computations we obtain

$$\lim_{T \to \infty} PV = \frac{Z}{r} = \frac{20,000}{0.1} = (20,000)(10) = 200,000$$

2. What is the present value of the payment of $50,000 per year for 5 years.

$$PV = \sum_{t=1}^{5} \frac{Z_t}{(1+r)^t} = \sum_{t=1}^{5} \frac{500}{(1.1)^t}$$

$$= \frac{500}{1.1} + \frac{500}{(1.1)^2} + \frac{500}{(1.1)^3} + \frac{500}{(1.1)^4} + \frac{500}{(1.1)^5}$$

$$= \frac{500}{1.1} + \frac{500}{1.21} + \frac{500}{1.331} + \frac{500}{1.4641} + \frac{500}{1.61051}$$

$$= 45,454.545 + 41,322.314 + 37,565.740 + 34,150.673 + 31,046.066$$

$$= 189,539.338$$

3. What is the value of the buyout?

We need to add the value of a perpetuity of $14,000 to the cash payment of $70,000. The present value is then

$$V_0 = 70,000 + \frac{Z}{r} = 70,000 + 14,000(10)$$

$$= 70,000 + 140,000 = 210,000$$

4. What should Lindon do?

Sell a portion of the firm to Lopez and Chavas.