

**Economics 335**  
**Problem Set 2**

NAME: \_\_\_\_\_

1. A single Iowa hog grower and a small packing facility are considering a joint venture in which they plan to produce and market high quality pork. They plan to market the pork under the brand name *IQ Pork*, which is short for *Iowa Quality Pork*.

The grower and packer know that consumers will pay a higher price for their product if the grower takes steps to improve quality and the packer undertakes marketing efforts to make consumers aware that *IQ Pork* is indeed a superior product.

Let  $z$  denote an index of the quality of the pork produced, with higher values of  $z$  indicating higher quality. The grower's *unit cost* (cost per hog) is  $C^G = 20 + .5z^2$ , where the \$20 is the cost of growing a single hog when no efforts are made to improve quality.

There is a traditional market for hogs. That is, if the joint venture breaks down for some reason, the farmer can deliver the hogs to other packers who are not concerned about quality. These packers pay \$20 per animal.

Let  $m$  denote an index of the marketing efforts undertaken by the packing firm. The packer's cost of processing and marketing is  $C^P = 18 + m^2$  per animal. If the marketing is not undertaken the packer can sell the processed pork into an alternative consumer market where quality is not important. In this case the price received for the processed product is \$40 per animal. The price that consumers are willing to pay for pork depends on  $z$  and  $m$ ;

$$P(z, m) = \begin{cases} 40 + 2z + m & \text{if } z > 0, m > 0 \\ 40 & \text{otherwise} \end{cases}$$

The above expression implies that the price will rise above \$40 only if the grower improves quality,  $z > 0$  and the packer makes efforts to market the product,  $m > 0$ .

To determine whether this venture can be profitable we must find out how much money can be made if the grower provides the optimal amount of quality and the packer undertakes the optimal amount of marketing. We must then coordinate the growers and packers efforts to achieve the best outcome.

(1.a) Write down an expression for the total profits *per animal* from producing *IQ Pork*, as a function of  $z$  and  $m$ .

(1.b) Use calculus to determine the optimal level of  $z$  and  $m$ .

(1.c) What is the consumer price when  $z$  and  $m$  take their optimal values?

(1.d) How much profit can be made under the joint venture?

(1.e) How much profit can be made in the traditional hog market (when no efforts are made to improve and market quality)?

Two problems that the grower and packer must resolve are (1) how to split the proceeds from the joint venture, and (2) how to ensure that the grower and packer both carry out their end of the agreement, i.e., choose the optimal  $z$  and  $m$ .

(1.f) Suppose that the grower and the packer decide to seal their partnership with a handshake. The deal is structured so that the grower agrees to sell the high quality pork to the packer for a price of \$24 per animal. Carefully explain the “hold up” problem facing the grower.

Suppose that rather than just a handshake, the grower and packer sign a contract in which they share the **revenues** that are generated under the joint venture. For example, under a revenue sharing arrangement, the unit profit for the grower is  $0.5 \cdot P(z, m) - C^G$ .

(1.g) Will a revenue sharing agreement induce the grower and packer to choose the total profit maximizing amount of  $z$  and  $m$ ? Explain.

(1.h) Suppose now that the grower and the packer decide to split the proceeds from their venture by dividing the total profits equally among themselves. How much quality,  $z$ , and marketing,  $m$  will be selected?

(1.i) Does your answer to the above question change if the packing firm collects 55% of the profits and the grower collects 45% of the profits? Explain.

(1.j) Suppose that the grower is considering going it alone, i.e., forward vertical integration. List and explain some advantages and disadvantages of vertical integration.

2. Suppose that an Agribusiness firm must decide how to pay its sales representatives. It is considering two options. The first method pays the workers a fixed annual salary. The second pays the workers a commission that is based solely on their individual sales revenues.

The actual sales of each worker will depend on the level of effort that is exerted by the worker. Suppose that the utility of a representative worker can be represented as  $U = w - e$ , where  $w$  denotes the salary (dollars) that they receive and  $e$  denotes the amount of effort that they exert. Let  $s(e) = 2.5e^{1/2}$  denote the total sales revenues for the worker if they exert effort level  $e$ .

(2.a) How are sales affected by changes in effort?

(2.b) Explain in words which payment option, fixed annual salary or commission, provides the greatest incentive to exert effort?

Now use a mathematical model to demonstrate your answer to part (2.b).

(2.c) Write down an expression for worker utility under the two payment alternatives. Let  $\alpha \in [0,1]$  denote the proportion of sales revenue that the worker receives as commission.

(2.d) How many units of effort will the worker exert under the fixed salary option?

(2.e) How much effort will the worker exert under the sales commission option.

(2.f) How does the value of  $\alpha$  affect the amount of effort exerted by the salesperson? Derive an expression to show your answer and explain your answer intuitively.