Homework Assignment 10 solution.

1. (1 point) Assume that the demand equation is given by $P = 99 - 3Q$. If the original price of the good was $9 and a $2 tax was imposed, how much marginal excess burden will be created?

Refer to the figure below. The marginal excess burden is equal to the area ‘abc’ (it is the same as excess burden because there was no tax to start with) = $0.5 * (11-9) * (30-29.3) = 0.7$.

![Graph showing demand and supply curves]

2. (1 point) The equation of MC of unreported income is $MC = 4 + 2X$, and the MB of unreported income is $MB = 44 - 3X$, what will be the amount of unreported income? How much of a change will there be if enforcement becomes more strict, resulting in $MC = 9 + 2X$?

We set $MC = MB = 4 + 2X = 44 - 3X$, leading to $X = 8$.
After the change in enforcement we have:
$MC = MB = 9 + 2X = 44 - 3X$, leading to $X = 7$.
So the amount of unreported income goes down because of the increased penalties.

3. (1 point) Suppose that mayor of Ames wants to build a stadium to attract a professional baseball team to the town. To finance this project two alternatives are considered – impose a 2-year sales tax of 10% or impose a 20-year sales tax of
1%. Assuming that the interest rate is zero, which approach will yield more efficient outcome?

Recall that the marginal excess burden from tax is rising with the tax squared. The excess burden from 10% tax will therefore be higher than 10 times excess burden from 1% tax. Hence, it is more efficient to have 1% tax for 20 years instead of 10% tax for 2 years. In addition, there is a time inconsistency problem – once government introduces a 10% tax, they will have no incentive to roll it back in 2 years – politicians are believed to be maximizing tax revenues instead of public welfare.

4. (1 point) Luxury goods usually have very high demand elasticities. On what grounds could we justify the relatively heavy taxation of such goods?

Society can have preferences for vertical equity, which requires that rich people pay relatively more than poor people (rich people have higher average tax rates than poor people). Efficiency would require us to impose relatively low tax rates on luxury goods. However, since luxury goods are disproportionately consumed by rich people, efficient taxation would lead to vertical inequity.

5. (1 points) Suppose the demand for good X can be represented by the following equation: \( Q_d = 22 - \frac{1}{4}P \). Furthermore, suppose that the demand for good Y can be represented by \( Q_d = 50 - P \). The prices of both goods are equal to $10. Suppose that an ad valorem tax is placed on both goods. Good Y is taxed at a rate of 5%. To ensure that the inverse elasticity rule holds, what must be the rate at which good X is taxed? (Hint: Elasticity at a given price is found using the formula \( \varepsilon = \frac{-1}{S} \frac{P}{Q} \), where S is the slope of the demand curve, Q is the quantity demanded, and P is the price.)

We have to determine elasticities of the two goods:

Elasticity of X: \( \varepsilon = 0.25 \frac{10}{19.5} = 0.128 \);
Elasticity of Y: \( \varepsilon = 1 \frac{10}{40} = 0.25 \);

The efficiency requires \( (\text{elasticity of X} \times \text{tax on X}) = (\text{elasticity of Y} \times \text{tax on Y}) \), which translates into

\( 0.128 \times \text{tax on X} = 0.25 \times 5 = 1.25 \), which means that X should be taxed at 9.7%.

6. (2 points) Consider a country with the following progressive tax system – low income is taxed at 0%, moderate income is taxed at 15%, high income is taxed at 28%. The individuals in this country are considering whether to invest in municipal bonds. The market rate of return is 20%.

a. What rates of return on municipal bonds would induce low-income people to invest in these bonds? What about moderate and high income people?

The after-tax return on municipal bonds should be the same as the after-tax return on other assets. The after-tax rate of return depends on the marginal tax rate of each individual. For low income individuals it is 0%,
for medium income it is 15% and for high income it is 28%. Denoting by $t$ the marginal tax rate for each particular person, the after tax market rate of return becomes $20(1-t)$. The after-tax rate of return on municipal bonds is equal to pre-tax rate of return because the return on these bonds is not taxed. So, to induce each income group to buy municipal bonds, the rate of return on these bonds has to be:

- **Low income**: $20(1-0.0)=20\%$
- **Medium income**: $20(1-0.15)=17\%$
- **High Income**: $20(1-0.28)=14.4\%$

b. Now suppose that each income group has exactly $100 in total to invest (or not invest) in municipal bonds. What rate of return would municipality offer if it needs to raise exactly $100? How much money it would save (compared to market rate of return)? How much money would the federal government lose?

If local government needs to raise exactly $100, it only needs to target the high income group (which demands the lowest rate of return because of its high marginal tax rate) by offering 14.4% rate of return. High income people will take this offer up and provide $100. Low and moderate income people will not want to invest. By being able to offer rate of return which is lower than market municipality saves some money: it would have to pay $20 in return if it were offering market interest rate ($100*0.2); instead it pays only $14.4 ($100*0.144), saving $5.6. The federal government, on the other hand, loses money by not collecting taxes from the returns. If these bonds were not tax exempt, they would offer 20% return ($20) and high income people would pay $5.6 in taxes ($20*0.28). The amount that municipal government saves is equal to the amount that federal government loses.

c. What rate of return would municipality offer if it needs to raise exactly $200? How much money it would save (compared to market rate of return)? How much money would the federal government lose?

If local government needs to raise exactly $200, it needs to target both the high income group and moderate income group by offering 17% rate of return. High income people and moderate income people will take this offer up and provide $200. Low income people will not want to invest. By being able to offer rate of return which is lower than market municipality saves some money: it would have to pay $40 in return if it were offering market interest rate ($200*0.2); instead it pays only $34 ($200*0.17), saving $6. The federal government, on the other hand, loses money by not collecting taxes from the returns. If these bonds were not tax exempt, they would offer 20% return ($40). High income people would pay $5.6 in taxes ($20*0.28), moderate income people would pay $3 in taxes ($20*0.15). The amount that municipal government saves ($6) is less than to the amount that federal government loses ($8.6). It happens because
high income group gets some economic rent – this group would have invested in municipal bonds at 14.4%, the offered interest rate is 17% instead. So high income group gets 2.6% extra ($100 \times 0.026 = $2.6), which is exactly the difference between what federal government loses and municipal government gains ($8.6 - $6).

d. Based on your answers in (b) and (c) comment on whether preferential tax treatment is the best way to support local governments? Why does such system exists in U.S.?

Clearly, preferential tax treatment is not the best way to support local governments. Federal government will generally subsidize both local government and rich people. If federal government would just pay local governments, it would have been cheaper. The fundamental reason why it exists is that it is politically much more difficult to get some spending through than it is to reduce taxes.

7. (2 points) Consider the following investment opportunity. The asset is worth $100 now and it offers the rate of return $r=10\%$ each year for the next 20 years. Suppose that the asset value gain is not realized until it expires in 20 years.

a. Assume that only realized gains are taxed at 15%. What is the value of this asset to the investor in 20 years?

If we do not sell the asset until it expires in 20 years and let it grow at 10% a year, we will only have to pay taxes once – after the asset is sold in 20 years. The pre-tax value of this asset is going to be $[100 \times (1+0.1)^{20}] = $672.75. We will pay a tax on gains equal to $672.75 - 100 = $572.75. The after-tax value of the asset is going to be $572.75 \times (1-0.15) + 100 = $586.

b. Now suppose that any gains (realized and not realized) are taxed at the same rate of 15%. What is the value of the asset to the investor in 20 years?

If unrealized gains are taxed as well, the effective after-tax yearly rate of return becomes $10\% \times (1-0.15) = 8.5\%$. The tax is paid every year as opposed to paying it once at the end of the asset’s life. The after-tax value of the asset will be $100 \times (1+0.085)^{20} = $511.

c. Is there any lock-in effect resulting from preferential tax treatment of unrealized capital gains?

There is certainly going to be a lock-in effect resulting from this preferential tax treatment of unrealized capital gains. In reality a lot of assets do not have an expiration date – think of stocks as an example. The essence of the lock-in effect is that if you allow your asset to grow for a
few years, the gains are not going to be taxed. Now, if you decide to change your portfolio, you’d have to sell the asset and buy other assets. The fact that you’d have to pay taxes on these accumulated large gains adds additional costs to changing your portfolio. The longer you allow your asset to grow the more difficult it would be to switch, you’re going to be “locked-in” in your current portfolio. This creates inefficiencies because people will hold some assets despite their inferiority.

8. (1 point) What are the advantages and disadvantages of a flat income tax?

There are two main advantages to a flat income tax:

- It eliminates the possibility to use different tax shelter schemes and loopholes. This greatly reduces tax avoidance by (mostly rich) people. In addition, wherever the tax evasion is a problem, fairly low flat tax rate may reduce the tax evasion because the benefit of not reporting your tax becomes low.
- It is simple, so people spend a lot less time filing their tax returns.

The main disadvantage is that the flat tax is widely perceived to be unfair is the sense that rich people will in general benefit more from it (they will pay less in taxes).