

Homework Assignment 4 solution.

1. (2 points) Consider the following net benefits (measured in billions of dollars) that will result from the passage of two legislative bills, X and Y:

Voter	Issue X	Issue Y
A	6	-3
B	-1	5
C	-2	-3

Is there a logrolling opportunity present in this situation? What are the potential gains to voters? Explain why logrolling may enhance the wellbeing of the society.

Notice that neither of the bills would pass if no vote trading is allowed. There are two voters whose net benefits from a bill is negative, so they would vote against it (B and C are going to vote against bill X, and A and C are going to vote against Y). There is a potential for log-rolling. Suppose voter A meets voter B before voting takes place and offers him to vote for bill Y in return for B's vote for X. He himself would certainly prefer that because the benefit from both X and Y ($=6-3=3$) is greater than otherwise (0 because none of the bills passes). B should actually agree to this vote trade because the net benefit of these two bills to him ($=-1+5=4$) is greater than no bill (0). Assuming that society cares equally about all its citizens, it is actually better off with both bills passed than with neither passed. Society has net benefit of zero if neither bill passes and has a net benefit of 2 ($=6-3-1+5-2-3$) if both of them pass.

2. (2 points) Suppose that Jim's utility depends on his income (I_J) only and is given by the following function $U_J(I_J) = 3I_J - 0.25I_J^2$. The marginal utility is then given by $MU_J = 3 - 0.5I_J$. Helen's utility also depends on her income (I_H) only in the following way $U_H(I_H) = I_H$. The corresponding marginal utility is given by $MU_H = 1$. What is the socially optimal income distribution in this two-person economy under the utilitarian social welfare function $W_1 = U_J + U_H$ and with total income being equal to \$15. What if social welfare function is given by $W_2 = \min(U_J, U_H)$?

The key fact to know here is that if the social welfare function is given by $W_1 = U_J + U_H$, then the optimal income distribution should satisfy the following condition

$MU_J = MU_H$. Intuitively, suppose that this is not the case and at the current income distribution $MU_J = 5 \neq MU_H = 10$. This means that the last dollar consumed by Helen is worth 10 utils and last dollar consumed by Jim is worth 5 utils. Since society treats Helen's and Jim's utils in the exactly the same manner (because $W_1 = U_J + U_H$), we can take last dollar consumed by Jim and give it to Helen. Jim (and society) will lose 5 utils, but Helen (and society) will gain 10 utils. It is easy to see that such an improvement (from a society standpoint) is possible as long as

$MU_J \neq MU_H$. Therefore, we set the marginal utility functions equal to each other to get $MU_J = 3 - 0.5I_J = MU_H = 1$, which gives us $I_J^* = \$4$. Given that there are 15 dollars in total, Helen gets the rest $I_H^* = \$11$.

The case with $W_2 = \min(U_J, U_H)$ requires a bit more thought. The thing to realize here is that society only cares about the person with the lowest utility. The changes in the utility of the society can only be brought about by changing the welfare of this least happy person. This means that at the optimal income distribution we must have $U_J = U_H$. Suppose that, on the contrary, $5 = U_J \neq U_H = 10$. This means that the society's welfare is equal to 5 (Jim's utility). We can take \$1 from Helen and give it to Jim. Helen will lose some amount of utils, which depends on the functional form of her utility (Society's welfare isn't going to change). Jim will gain some amount of utils, which depends on his utility function (Society will gain the same amount). It is also easy to see that as long as $U_J \neq U_H$ such an improvement in society's welfare is possible. In order to obtain the optimal income distribution we need to set the utilities of Jim and Helen equal to each other: $U_J(I_J) = 3I_J - 0.25I_J^2 = U_H(I_H) = I_H$. We have to add the constraint on the total amount of money available $I_J + I_H = 15$, leading to $I_H = 15 - I_J$. Plugging it into the equation above we get $3I_J - 0.25I_J^2 = 15 - I_J$. Rearranging terms we get $-0.25I_J^2 + 4I_J - 15 = 0$. This quadratic equation has two roots $I_{J1} = 6$, $I_{J2} = 10$. The respective incomes for Helen are $I_{H1} = 9$, $I_{H2} = 5$. The associated utilities are given by $U_J = U_H = 9$, $U_J = U_H = 5$. Obviously, the first allocation is optimal even though there are two allocations satisfying the equality of utilities requirement.

3. (3 points) Al consumes two goods – Housing (H) and ‘all Other goods’ (O). His utility function is given by $U = H^{1/3}O^{2/3}$ (so that Al's optimal consumption of housing can be computed according to the following demand function $H^* = 1/3(I/P_H)$, and Al's consumption of all other goods is derived according to $O^* = 2/3(I/P_O)$). The price of housing is $P_H = \$2$, the price of all other goods is $P_O = \$1$. Al's current income is $I = \$240$.
- What is the optimal consumption of housing and all other goods? What is the resulting utility?

We have to plug the values of income and prices into the demand (optimal consumption) equations given above:

$$H^* = 1/3(I/P_H) = 1/3 * (240/2) = 240/6 = 40.$$

$$O^* = 2/3(I/P_O) = 2/3 * (240/1) = 160.$$

Plugging back into the utility function we get:

$$U^* = (H^*)^{1/3} (O^*)^{2/3} = 32.89.$$

- Now suppose that in addition to his earned income, Al is also receiving support from the government, which amounts to \$120 in cash. How much housing and all other goods is Al going to consume? How much utility does Al get from it?

We have to plug the values of new income (=old income+\$120=\$360) and prices into the demand (optimal consumption) equations given above:

$$H^* = 1/3(I/P_H) = 1/3 * (360/2) = 360/6 = 60.$$

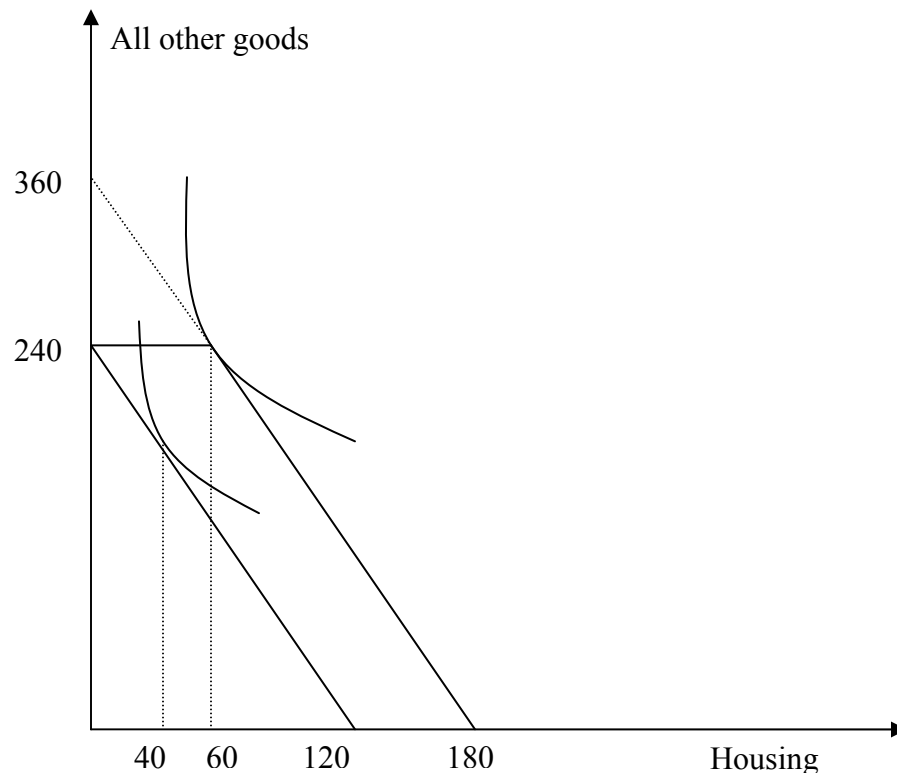
$$O^* = 2/3(I/P_O) = 2/3 * (360/1) = 240.$$

Plugging back into the utility function we get:

$$U^* = (H^*)^{1/3} (O^*)^{2/3} = 42.53.$$

- c. Now suppose instead of giving Al cash, government is giving him a housing subsidy worth \$120. This housing subsidy cannot be resold and used in any way other than acquiring housing services. How much housing and all other goods is Al going to consume? What is Al's utility in this case?

We already know that if Al's income is increased to \$360, he's going to demand 60 units of housing. If instead of cash he were given \$120 worth of housing subsidy (which is exactly 60 units), he'd also want 60 units of housing. So in this case, he's going to spend all of this cash income (\$240) on other goods and let the subsidy take care of his housing needs. See the graph illustrating his decisions below:



$$H^* = 60.$$

$$O^* = 240/1 = 240.$$

Plugging back into the utility function we get:

$$U^* = (H^*)^{1/3} (O^*)^{2/3} = 42.53.$$

- d. Which type of support (cash or subsidy) would Al prefer to have? Who can gain from the government giving out support to needy citizens in the form of subsidy as opposed to cash? Explain how this special interest group can persuade politicians to distribute aid in the form of subsidies?

It is obvious from the graph that Al is indifferent between two policies. His demand for housing after the increase in his income is not lower than what the subsidy gives him (in this case it is actually exactly equal to subsidy) so he's not forced into an inferior allocation because he has to spend his housing subsidy on housing only. In general, subsidy recipients with different utility functions may in fact be worse off with housing subsidy than with cash (because their demand with cash may be lower than what housing subsidy forces them to consume). Despite this possible inferiority of subsidy, real estate developers/landlords can gain from it. The reason is that some people may be forced to consume more housing than they would want increasing the demand for housing. If real estate developers/landlords could form an association able to support a lobby, they could persuade politicians to use housing subsidy instead of cash to support poor people by donating money for political campaigns.

4. (1 point) How would you feel about a policy that would raise someone else's income without lowering yours? Are you any worse off?

To feel nothing about it is perfectly fine. It is also absolutely normal to feel happy about other people who get more money. It is also normal to feel envious sometimes (just imagine that you don't get any tax cut while all of your friends get sizeable tax cuts). The point is that if you have any emotions at all, be it a positive or negative reaction, it must mean that other people wellbeing enters your utility function (either in a positive or negative way).

5. (2 points) List all the requirements of the Arrow's Impossibility Theorem and briefly explain what each means.

Arrow's Impossibility Theorem requires that:

- **preferences (both individual and social) are rational: if A is preferred to B, and B is preferred to C, then A must be preferred to C.**
- **Social Aggregation Rule should be unanimous: if everyone in a society prefers A to B, then society must prefer A to B.**
- **Social Aggregation Rule should satisfy Independence of Irrelevant Alternatives: the society's choice between A and B depends on each individual's preference between A and B ONLY. That is, how any person values A vs C, or B vs D should be irrelevant for society's choice between A and B.**

The only Social Aggregation Rule that satisfies all three is Dictatorial.