Homework Assignment 5 solution.

1. (6 points) Wayne is maximizing his utility by choosing how many hours to work a week. His preferences for leisure (L) and consumption of all other goods (C) are given by \( U = C^{1/4}L^{3/4} \). Wayne’s labor supply (l) can be calculated by deducting his leisure consumption from total number of hours available to him in a week: \( l = 7 \times 24 - L = 168 - L \). The price of leisure (=wage rate, \( w \)) is equal to $5/hour. The price of consumption of other goods \( P_C \) is $1. Wayne’s optimal consumption of leisure can be computed according to the following demand function \( L^* = \frac{3}{4} \left( \frac{I}{w} \right) \), where \( I \) is his income, which is equal to \( I = 168 \times w = 168 \times 5 = $840 \). Wayne’s consumption \( C \) is equal to \( C^* = \frac{1}{4} \left( \frac{I}{P_C} \right) \). (Notice that Wayne won’t actually make $I in cash, he spends part of his income by not working and consuming leisure, which has the same price as his labor.)
a. Carefully draw Wayne’s budget constraint. Calculate his optimal consumption of leisure $L^*$ and all other goods $C^*$. Label this consumption bundle as point A on your graph. What is the level of Utility that Wayne derives from it?

Please refer to the graph above. The optimal consumption bundle is given by:
$L^*=(3/4)(I/w)=(3/4)(840/5)=126$;
$C^*=(1/4)(I/P_c)=(1/4)(840/1)=210$;
$U=C^{1/4}L^{3/4}=143.16$.

b. Now suppose that government introduces a welfare program, which has the following benefit level $B=G-t*w*l$: where $B$ is the benefit level, $G$ is the basic grant equal to $100$, $t$ is the benefit reduction rate equal to $0.25$, and $w$ is the wage rate. How many hours Wayne has to work to reduce his benefit to zero and how much money would he earn in this case? On the same graph draw a new budget constraint (hint: the kink will happen where Wayne’s benefit is zero.) Based on your graph determine whether Wayne will decide to enroll in this welfare program at all? (hint: if Wayne’s new optimal allocation is likely to lie on the new segment of the budget line, he’s going to enroll.)

Again refer to the graph above. The new segment of the budget line is given by line ‘bc’. The location of point c is easy to determine – it is associated with zero labor supply ($L=168$) and consumption is given by the amount of basic grant $G$, $C=G=100$. The location of point b is a bit more difficult to determine. This point is associated with working enough hours to reduce the benefit to zero, i.e. $0=B=G-t*w*l=100-1.25*l$. The labor supply level necessary to reduce the benefit to zero is equal to $l=80$. This corresponds to the consumption level $C=80*5=$$400 and leisure $L=168-80=88$. It should be clear that Wayne will decide to enroll because this new segment of the budget line will allow him to move further away from the origin (direction of utility increase).

c. Now let’s determine analytically whether Wayne will enroll in this program. To do so, we have to calculate his optimal consumption bundle assuming that he enrolls: in this case his income is going to be determined by $I=100+(1-t)*w*l=100+630=730$, the wage is going to be equal to $(1-t)w=$$3.75; plug these new income and wage numbers to determine new optimal consumption of leisure $L$ and other goods $C$. Label this allocation as point B on your graph. What is the level of utility associated with this bundle? Compare this utility level to the utility you obtained in (a). Is Wayne going to enroll in this program? If so, is his labor supply going to be lower than in (a)?

The optimal consumption bundle is given by:
\[ L^* = \left(\frac{3}{4}\right) \left(\frac{I}{w}\right) = \left(\frac{3}{4}\right)(730/3.75) = 146; \]
\[ C^* = \left(\frac{1}{4}\right) \left(\frac{I}{P_c}\right) = \left(\frac{1}{4}\right)(730/1) = 182.5; \]
\[ U = C^{1/4} L^{3/4} = 154.38 > 143.16. \]
Wayne’s utility from enrolling is higher than his utility from not enrolling. So he will choose to enroll. Without the welfare program Wayne was supplying 42 (=168-126) hours of labor a week. With the program in place Wayne chooses to supply only 22 hours a week (=168-146). Intuitively this makes sense because we expect the welfare programs to discourage labor supply. There is a trade-off between efficiency (which requires as little incentive distortion as possible) and equity/fairness (which requires support of needy citizens).

d. Now suppose that the government offers the following welfare program instead: the basic grant \( G \) is equal to $300, the benefit reduction rate \( t \) is equal to 1. Sketch the new budget line (you don’t have to draw it exactly this time). Intuitively, do you expect Wayne to work at all with this program present? Why or why not?

The new budget will have a familiar kink. In contrast to the previous parts of the problem the new segment of the budget line will be absolutely flat (because the benefit reduction rate of 1 means that every dollar made on the job reduces the benefit by exactly the same amount). Wayne’s new
optimal allocation is going to be point B, where he doesn’t work at all. We have extreme case of work disincentives here, which produce this extreme result.

e. Suppose that Medicaid eligibility is tied to the welfare eligibility. Does this have any effect on the incentives to work?

This will have a very large effect on incentives to work. In particular, Wayne will have a very large disincentive to make enough on the job to lose his welfare eligibility. Supposing that he is considering whether to work the last hour which would make his welfare benefits zero (he’ll lose eligibility if he works for one more hour). In addition to losing some of the earnings due to the benefit reduction tax, he’ll lose health insurance potentially valued in thousands of dollars a year. That is, his effective wage associated with this last hour is negative and very large.

2. (2 points) Suppose that the state of Iowa is considering an increase in the funding of primary schools in the state. Explain using graphs how this can lead to a decrease in the overall level of educational spending in the state?
On the graph above, there are three types of families X, Y and Z. In absence of the government provided education each family chooses its optimal allocation according to their preferences (given by the points where indifferences curves are tangent to the budget line). If government introduces publicly provided education at the level $E_1$, families Y and Z stay put (still choose to send their children to private schools), the family X will choose to enroll into public school, which provides more education than they would consume on their own. Now suppose that government decides to increase the amount of education provided in public schools to $E_2 > E_1$. The family X will now consume even more education ($E_2 > E_1$). The family Z will stay put again. The family Y, however, may choose to switch to public schooling even though it provides a bit less education than they would prefer (they may switch because public education is free and therefore frees up a lot of resources that could be spent on other things). Depending on how many families of type X and type Y we have we may potentially have lower overall spending on education in the state. The public educational spending will go up for sure (more spending on families of type X and a lot of additional families of type Y). The private schools will lose families of type Y. The net effect may be negative.

3. (2 points) Read the Policy Debate on School Vouchers at:
http://www.swlearning.com/economics/policy_debates/vouchers.html (read only the main part; do not follow links unless you want to). Briefly summarize the arguments for and against school vouchers based on what you read.