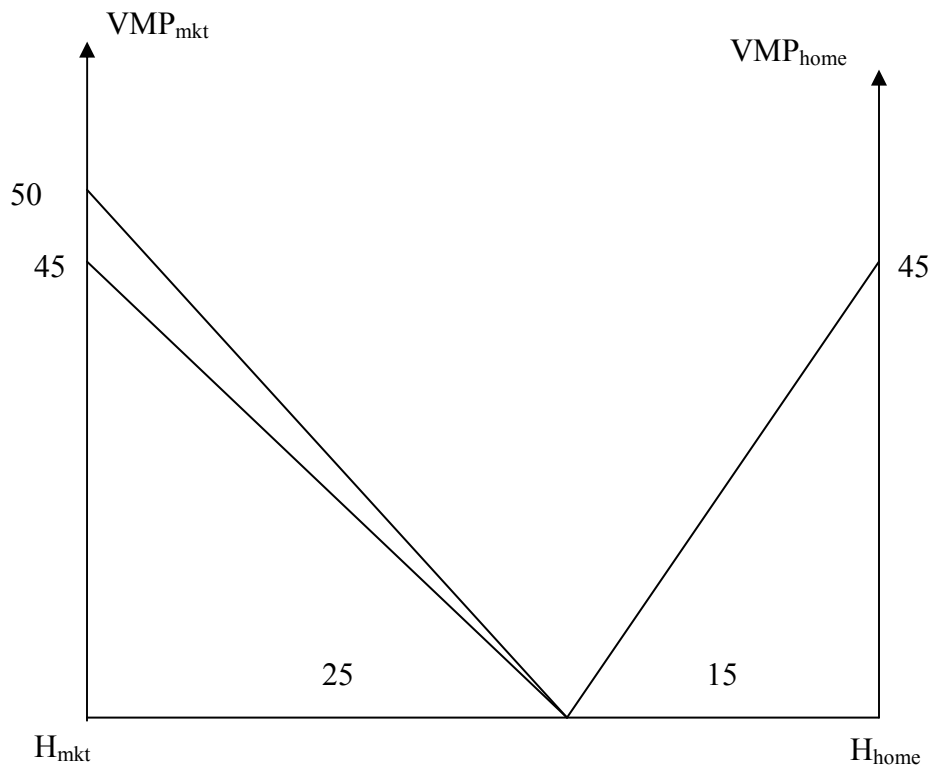


Homework Assignment 9 solution.

1. (3 points) Ashley divides her working time between home production and market production. The Value of marginal Product (=productivity in dollar terms) of market production is given by $VMP_{mkt} = 50 - 2H_{mkt}$, where H_{mkt} is the number of hours spend on market production, and the Value of Marginal Product of home production is given by $VMP_{home} = 45 - 3H_{home}$, where H_{home} is the number of hours spend on home production. What is the optimal division of her time between two types of production if there are a total of 40 hours to be worked between work and home? Suppose that government taxes Ashley market income by 10%. What is the new optimal division of time? What is the excess burden resulting from this tax?



To determine how many hours Ashley will work at home and on the market we set $VMP_{mkt} = 50 - 2H_{mkt} = VMP_{home} = 45 - 3H_{home}$. Keeping in mind that $H_{mkt} + H_{home} = 40$, we obtain $H_{mkt} = 40 - H_{home}$. Plugging this expression into the equality above we get

$VMP_{mkt} = 50 - 2(40 - H_{home}) = VMP_{home} = 45 - 3H_{home}$,
 which results in
 $H_{home} = 15$, $H_{mkt} = 40 - H_{home} = 40 - 15 = 25$.

If government taxes Ashley's market income by 10%, her Value of Marginal Product of market production becomes $VMP'_{mkt} = (1-0.1) * VMP_{mkt} = (1-0.1) * (50 - 2H_{mkt}) = 45 - 1.8H_{mkt}$.

Once again, we express $H_{home} = 40 - H_{mkt}$.

We have $VMP_{mkt} = 45 - 1.8H_{mkt} = VMP_{home} = 45 - 3(40 - H_{mkt})$,
 which results in $H_{mkt} = 25$, $H_{home} = 40 - H_{mkt} = 40 - 25 = 15$.

In other words, the optimal allocation of time between the two activities will not change as a result of this tax. The excess burden is zero.

2. (7 points) Jim is maximizing his utility by consuming two goods – color markers and all other goods. The price of one color marker is $P_{CM} = \$0.5$, the price of all other goods $P_{OG} = \$1$. Jim's income is $I = \$50$, his utility is given by $U = (CM)^{0.5} (OG)^{0.5}$. (The respective optimal consumption levels are given by $(CM)^* = 0.5 * (I/P_{CM})$, $(OG)^* = 0.5 * (I/P_{OG})$. Now assume that government imposes a \$.50 tax on color markers. (The color markers are produced competitively so that market supply is perfectly elastic – a tax is born completely by consumers.)
- What is Jim's optimal consumption of color markers and all other goods before tax?

We just plug the prices and income into the optimal consumption formulas given above (we call this allocation E1):

$(CM)^* = 0.5 * (50/0.5) = 50$, $(OG)^* = 0.5 * (50/1) = 25$.

The associated level of utility is $U(E1) = (50)^{0.5} (25)^{0.5} = 25(2)^{0.5} = 35.35$.

- What is Jim's optimal consumption of color markers and all other goods after tax?

Tax changes the price of color markers from \$0.5 to \$1. (We call this new after-tax allocation E2):

$(CM)^* = 0.5 * (50/1) = 25$, $(OG)^* = 0.5 * (50/1) = 25$.

The associated level of utility is $U(E2) = (25)^{0.5} (25)^{0.5} = 25$.

- What are the tax revenues?

Refer to the graph below. The tax revenues are given by the distance between points M and E2. Intuitively, the vertical coordinate of each of these two points represents the amount of other goods that Jim will consume before and after tax, respectively, if he consumes exactly 25 color markers (i.e., the height of M tells us the amount of other goods that Jim can consume if he consumes 25 markers and the height of E2 represents the amount of other goods that Jim can consume if he consumes 25 markers). The difference therefore is the reduction in his

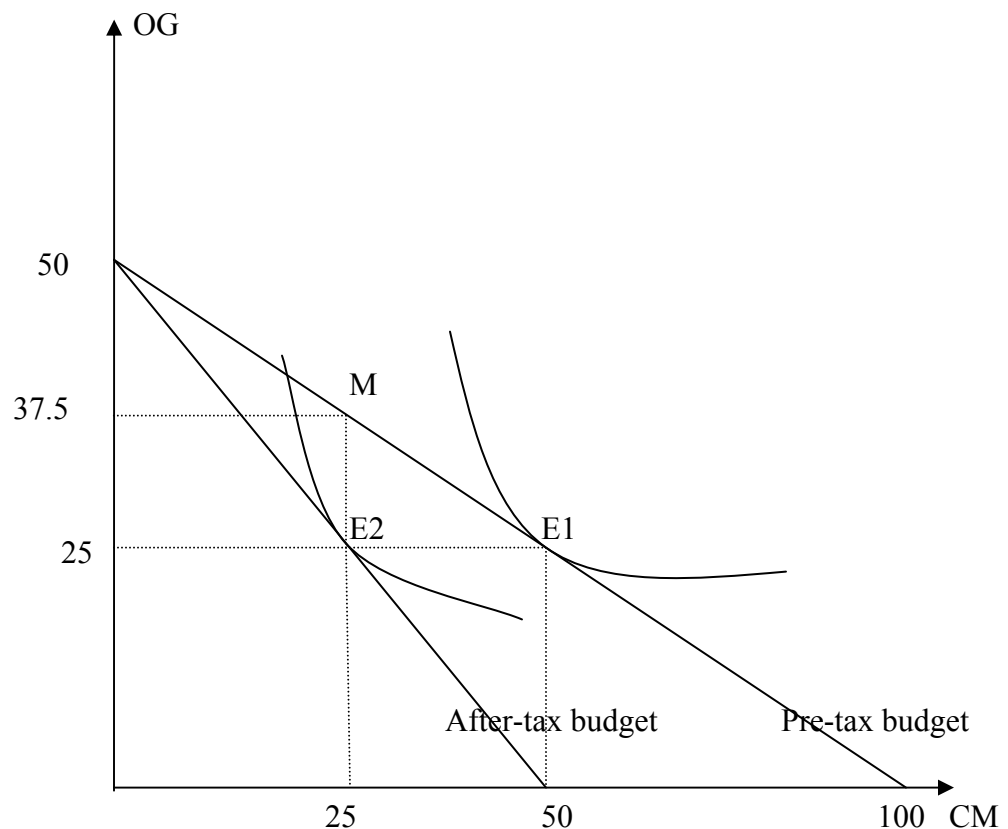
purchasing power expressed in units of other goods. Since the price of other goods is \$1 this reduction represents the drop in his income, which goes to the government in the form of tax revenues. We do this calculation at the level of consumption of color markers of 25 because it is Jim's optimal after-tax consumption.

The height of any point on the graph is equal to [Income-(consumption of CMs)*(price of CMs)]. The difference between points M and E2 is the price of CMs).

The height of M= $50-25*0.5=50-12.5=37.5$;

The height of E2= $50-25*1=25$;

The tax revenues= $37.5-25=12.5$.



d. Compute the Equivalent Variation of this tax?

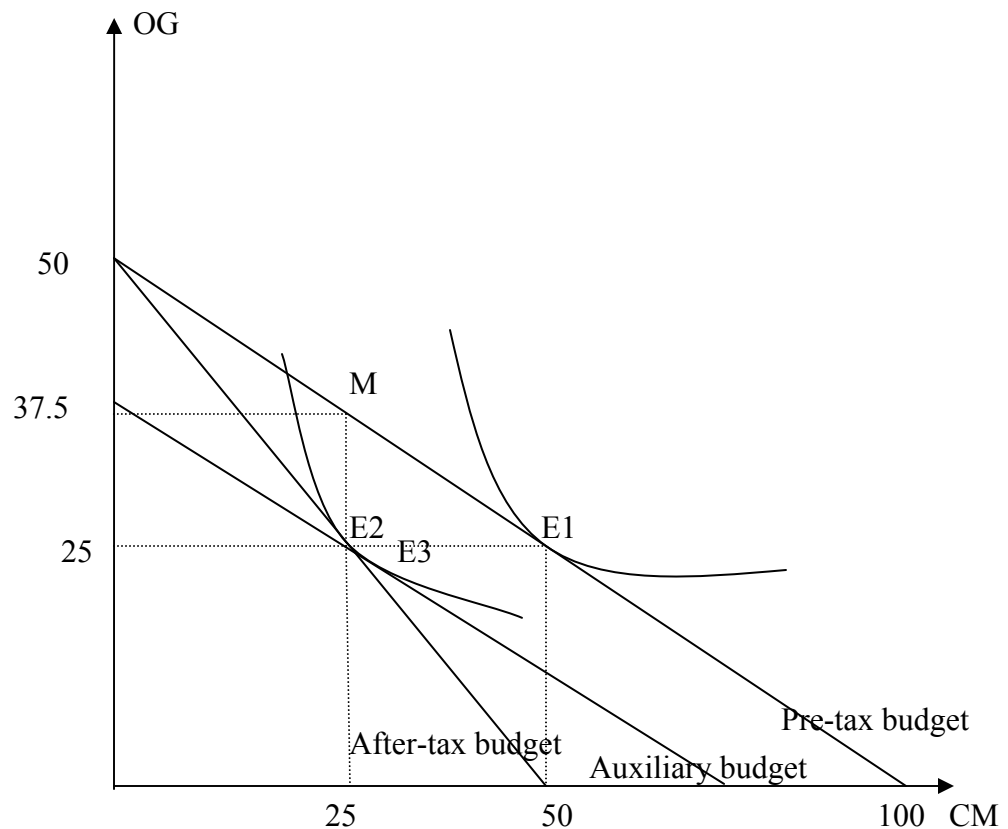
Refer to the graph below. To compute the Equivalent Variation (EV), we need to construct an auxiliary budget line. The two important properties of this budget line are that (1) it is parallel to the pre-tax budget (which means that we use pre-tax prices to construct it) and (2) it touches the new after-tax indifference curve (which means that Jim is indifferent between being on the after-tax budget and being on this auxiliary budget). We denote the optimal allocation on this auxiliary budget by E3.

Once again, we know the slope of this new budget line (determined by pre-tax prices), but we have to determine the level of income associated with it. We denote this level of income by I' . The optimal consumption levels are given by:

$(CM)^* = 0.5 * (I' / 0.5) = I'$, $(OG)^* = 0.5 * (I' / 1) = I' / 2$. The key point to note here is that we use pre-tax prices. As already mentioned the utility of allocation E3 should be the same as utility of allocation E2 (they lie on the same indifference curve). We therefore have:

$U(E2) = 25 = U(E3) = (I')^{0.5} (I' / 2)^{0.5} = I' (0.5)^{0.5}$. This leads to $I' = 25 / (0.5)^{0.5} = 25 / 0.707 = 35.36$.

The EV = Income associated with original budget - Income associated with auxiliary budget = $50 - 35.36 = 14.64 > 12.5$ (tax revenues). The excess burden is the difference between the two: $14.64 - 12.5 = 2.14$.



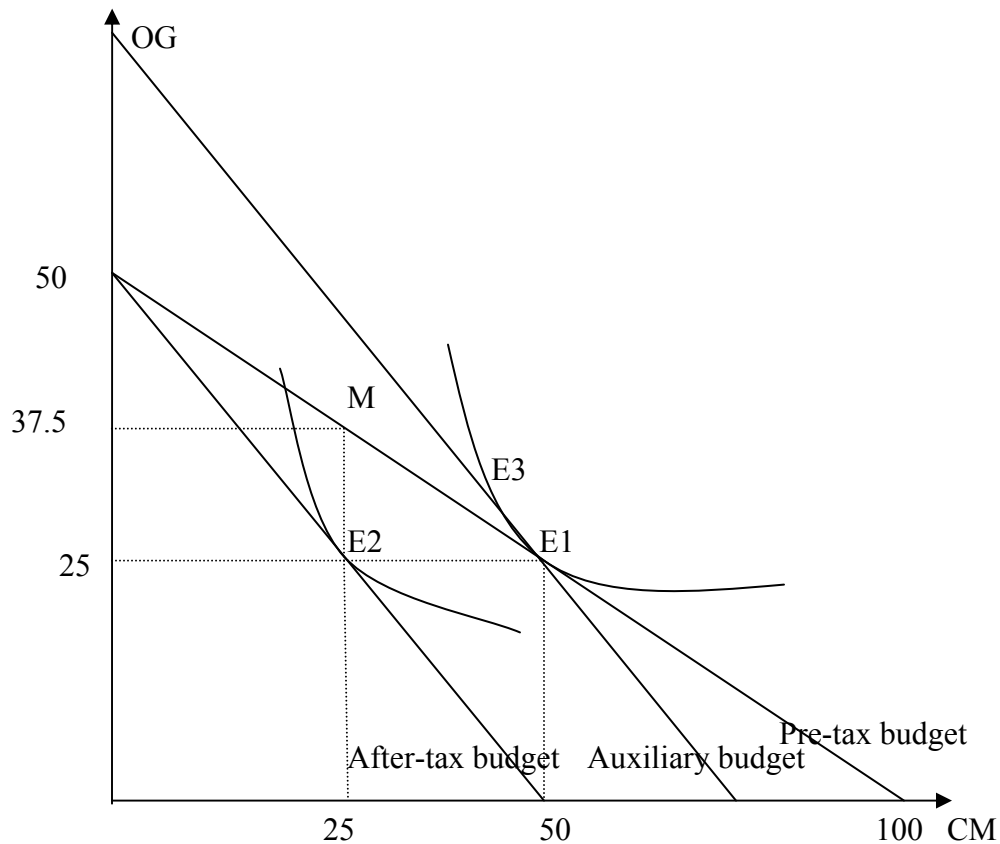
e. Compute the Compensating Variation of this tax?

Refer to the graph below. To compute the Compensating Variation (CV), we need to construct an auxiliary budget line in different way from part (d). The two important properties of this budget line are that (1) it is parallel to the after-tax budget (which means that we use after-tax prices to construct it) and (2) it touches the original pre-tax indifference curve

(which means that Jim is indifferent between being on the original pre-tax budget and being on this auxiliary budget). We denote the optimal allocation on this auxiliary budget by E3. Once again, we know the slope of this new budget line (determined by after-tax prices), but we have to determine the level of income associated with it. We denote this level of income by I'. The optimal consumption levels are given by: $(CM)^* = 0.5 * (I'/1) = I'/2$, $(OG)^* = 0.5 * (I'/1) = I'/2$. The key point to note here is that we use after-tax prices. As already mentioned the utility of allocation E3 should be the same as utility of allocation E1 (they lie on the same indifference curve). We therefore have:

$$U(E1) = (50)^{0.5} (25)^{0.5} = 25(2)^{0.5} = U(E3) = (I'/2)^{0.5} (I'/2)^{0.5} = I'/2. \text{ This leads to } I' = 2 * 25 * (2)^{0.5} = 50(2)^{0.5} = 70.71.$$

The CV = Income associated with auxiliary budget - Income associated with original budget = 70.71 - 50 = 20.21 > 12.5 (tax revenues). The excess burden is the difference between the two: 20.21 - 12.5 = 7.71.



f. Compare your answers to (c), (d), and (e).

We have the following relationship $CV > EV > \text{tax revenues}$. The excess burden is bigger if we use CV as a measure of damage that this tax introduces.