

**Homework Assignment 2 solution.**

1. (2 points) Consider an annuity that makes yearly payments  $C$  for as long as someone lives. What would happen to the purchase price of the annuity as
  - a. The age of the purchaser (at the time of purchase) goes up. **The annuity will become cheaper because there will be fewer payments that will have to be made.**
  - b. The size of the monthly payment rises. **The annuity will become more expensive.**
  - c. The health of the purchaser improves. **The annuity will become more expensive because there will likely be more payments.**
  - d. What is the value of this annuity today, if it pays \$100 a year, and the interest rate is 5%? **We have to calculate the present value of all payments. Since it is unknown when the holder of this annuity will die, we'll assume that the payments will be made forever (this assumption is fairly innocuous because the present value of payments made for a substantial number of years, like 40 or 50, is very close to the present value of payments made forever). The present value of all payments is equal to  $PV = \$100/0.05 = \$2000$ .**
  
2. (2 points) Most of college students face substantial uncertainty about their future incomes. How would you design a financial instrument that would allow you to insure against future income risk?

**It is possible to imagine the following financial instrument (this is by no means the only possible answer):**

**A college student pays an insurance company (or any other entity willing to insure students against the income risk) a fixed sum of money at the time of purchase of this instrument (let's say, \$1000). The insurance company promises to make a single payment to this student if his starting salary is below \$35,000/year. The payment is equal to the 50% of the difference between the actual salary (which could be zero) and \$35,000. Student gets nothing if his salary is above \$35,000.**

**The question didn't ask for it, but I wanted to point out the following: The problem with such an insurance contract is that it seriously damages incentives to work hard in college and to try to get a good job that pays well. In other words, why would anyone work hard, if he's going to get paid if he's unemployed? This problem is called Moral Hazard (we discussed it briefly in class).**

3. (2 point) Nicole found a new job after graduating from an MBA program. This job has two salary options: (1) either get a \$100,000 sign-up bonus and

\$90,000/year salary or (2) \$110,000/year salary. Market interest rate is 6%. For how many years will Nicole have to work to justify taking option (2)?

**A straightforward way to solve this problem would be to calculate present value of each pay option for all years starting with one and continuing until the present value is equal for a specific number of years. Easier way is to recognize that there are two differences between the two options. One difference is that the first option pays a \$100,000 sign-up bonus and the second option doesn't. The present value of this sign-up bonus is equal to \$100,000 because it is given immediately. The second difference is that the option (2) pays \$20,000/year more than option (1). The present value of getting \$20,000/year over the next n years should be equal to \$100,000. We should determine what n is. One way to solve for n is to use formula:**

**$PV = [C/i] * [1 - 1/(1+i)^n]$ , where  $PV = \$100,000$ ,  $C = \$20,000$ ,  $i = 0.06$ . Using this formula, we can determine that the number of years it takes,  $n = 6.12$  (or 7 full years).**

- (2 points) Simon thinks about starting up a software company. He'd have to buy one computer now and one computer in 5 years to replace the older one. The price of a computer is \$1000 now and \$1100 in 5 years. Simon expects to get \$300 a year in revenues for the next ten years. There is no inflation. The market interest rate is 5%. Should Simon undertake this project?

**We can use the same formula as in the previous problem:  $PV = [C/i] * [1 - 1/(1+i)^n]$ , where  $C = \$300$ ,  $i = 0.05$ ,  $n = 10$ , so the present value of all revenues is  $PV_r = \$2,316$ . The present value of costs is  $PV_c = 1000 + 1000/1.05^5 = \$1,783 < \$2,316$ . The discounted revenues are higher than the discounted costs, so the project should be undertaken.**

- (2 points) Jim wants to buy a house for \$100,000. Bank offers him a 30-year mortgage with fixed interest rate 7% (fixed in this case means that the interest rate won't change until mortgage loan is repaid in 30 years). Jim would have to make 360 monthly payments of about \$651 each. What would the monthly payment be if the interest rate were 8% instead of 7%?

**There are two methods to go about this problem.**

**First involves calculating a monthly interest rate first and then using formula  $PV = [C/i] * [1 - 1/(1+i^m)^n]$ , where  $PV = \$100,000$ ,  $i^m$  is the monthly interest rate,  $n = 360$ . We have to find  $C =$  monthly payment.**

**Monthly interest rate is found from:  $(1+i^m)^{12} = 1+i$ , where  $i^m$  is monthly interest rate and  $i$  is yearly interest rate. So,  $1.08 = (1+i^m)^{12}$ . Using this formula we get that  $i^m = 0.6434\%$ .**

**This method allows you to get the exact answer:  $C = \$714.39$ .**

The second method involves using spreadsheet software like MS Excel. Instead of calculating a monthly interest rate directly, you can use yearly interest rate, but you have to be careful how you discount monthly payments using yearly rate. For example, first month's payment will be discounted using factor  $1/(1+0.07)^{(1/12)}$ . The key here is to raise your yearly % rate to the power 1/12 and not one because you use years as your measure of time. Next payment will be discounted using factor  $1/(1+0.07)^{(2/12)}$ . This is equivalent to calculating monthly interest rate (call it  $i^m$ ) first and then discounting using factors  $1/(1+i^m)^1$ ,  $1/(1+i^m)^2$  and so on.

In Excel you create three columns: one with your monthly payment (it's what we have to figure out -- but put some number there for now, like 650). The other column consists of just numbers 1 through 360 (months). The third column will contain the present value of each monthly payment. You'd have to enter formula there:

= "cell containing monthly payment" / (1.08) ^ ("cell containing the month's number" / 12). You enter the formula for the first cell only, Excel will automatically enter the formula for other cells if you drag your mouse over the whole column.

Then you sum all present values and see if it's close to \$100,000. If it's bigger than \$100,000, you put some lower value for your monthly payment column (like 620), if it's bigger, you increase the value. Try a few times until you get reasonably close to \$100,000. You should get the same answer: \$714.

The point of this exercise is to show that even seemingly modest increase in the interest rate increases your monthly payments for quite a lot. Alternatively, with this higher interest rate, the present value of monthly payments of \$651 is going to be lower than \$100,000.