Homework Assignment 3 solution.

1. (2 points) Compute the yield to maturity of the following bonds:
   a. $100 face-value, 5-year, pure discount bond selling for $78. **The yield to maturity is the interest rate that makes the present discounted value of all future payments equal to the current price (internal rate of return on this bond).** In this case, the present value is equal to $100/(1+i)\^5$, where $i$ is the yield to maturity. This present value has to be equal to $78 – current price of this bond. So, $PV=\frac{100}{(1+i)^5}=78$. Solving for $i$ we get $i=5.09\%$.
   b. $100 face-value, 5-year, pure discount bond selling for $61. Here we have $PV=\frac{100}{(1+i)^5}=61$. Solving for $i$ we get $i=10.4\%$.
   c. $100 face-value, 1-year, coupon bond with coupon rate of 10\% selling for $102. **The only complication with coupon bonds is that we have to take into account coupon payments in addition to the face value repayment at maturity.** The total payment in one year is equal to the face value ($100) plus coupon payment (0.1*$100=$10). So the present value is $PV=\frac{110}{1+i}=102$. The corresponding yield to maturity is $i=7.8\%$.
   d. $100 face-value, 1-year, coupon bond with coupon rate of 8\% selling for $95. The total payment is equal to the face value ($100) plus coupon payment (0.08*$100=$8). The present value is $PV=\frac{108}{1+i}=95$. The corresponding yield to maturity is $i=13.7\%$.
   e. $100 face-value, 1-year, coupon bond with coupon rate of 6\% selling for $90. The total payment is equal to the face value ($100) plus the coupon payment (0.06*$100=$6). The present value is $PV=\frac{106}{1+i}=90$. The corresponding yield is $i=17.8\%$.

2. (2 points) You purchase a consol with annual coupon payments of $100, the interest rate is 8\%. One year later the interest rate has changed to 8.5\% and you decide to sell the consol. What is your one-year holding period return?

   **The holding period return is equal to**
   \[
   \text{Return} = \frac{\text{Coupon payment}}{\text{Price Paid}} + \frac{\text{Change in Price}}{\text{Price Paid}}.
   \]
   We have to determine the price of the bond at the time of purchase. The price has to be equal to the present value of all future payments. Since the coupon payments are going to be paid infinitely, the present value is equal to $PV=\frac{\text{Coupon}}{i}=\frac{100}{0.08}=$1250. The price in one year is going to be $PV=\frac{\text{Coupon}}{i}=\frac{100}{0.085}=$1176.47 (The payments are still going to continue infinitely in the future)
   So the change in price is equal to $1176.47-1250=-$73.53 (capital loss). There will be one coupon payment of $100 in one year.
   The holding period return is therefore:
Return = \$100/\$1250 – \$73.53/\$1250 = 0.021 (2.1\%). The rate of return is significantly less than the coupon rate (8\%) because the change in interest rate led to large capital loss.

3. (2 points) Suppose that US government decides to decrease its budget deficit. Draw a graph and explain what will happen to bond prices and interest rates. Government finances budget deficit by borrowing (issuing Treasury Bills and Treasury Bonds). If government decides to reduces its budget deficit (there are two ways – either tax more or spend less), it must mean that the government borrowing will be reduced. On the bond market graph the supply curve will shift to the left (from S to S’) leading to higher bond prices and lower interest rates.

4. (2 points) Imagine that the recent economic forecasts for the US economy for the next couple of years are really gloomy – analysts expect a major recession with reduction in output. Use graphs to explain what you think will happen to prices and interest rates of government and private bonds?

Recession will lead to the reduction in wealth, and consequently, in savings. It means that the demand for bonds (both government, and private) will decrease (the demand curve shifts to the left to D’). Government will probably start borrowing more because it cannot generate enough income with taxes (and assuming that spending remains at roughly the same level – it is politically very difficult to cut spending). The government borrowing was much higher during 2000/2001 recession than during economic upturn of the late 90-ies, when the borrowing was negative – government was running budget surpluses and
was considering buying out its long-term debt (30-year bonds). This means that the supply of government bonds will increase (the supply curve shifts to the right).

Government Bonds

As a result, the price of government bonds decreases and the interest rate rises.

Private companies will probably borrow less because there are fewer projects that need financing. The supply curve will shift to the left.

Private Bonds
As a result, the price of bonds can either increase or decrease (interest rate either fall or rise) depending on how far demand and supply shift relative to each other.

5. (2 points) Consider the following investment projects: (1) Invest $100 today and get $107 with probability=1. (2) Invest $100 today and get $102 with probability=0.5, or get $112 with probability=0.5. Compute expected returns and variances (risks) of each investment. Assuming that all investors are risk-averse, would anybody invest in project (2)? Now consider that another investment opportunity is available (3): Invest $100 today and get $102 with probability=0.5 or get $115 with probability=0.5. What is the expected return and variance of this project. Compare the risk and return of this project to the risk and return of project (1).

The expected returns on the investment projects are calculated as a sum of each possible return times the corresponding probability (it’s not crucial for this problem but you have to deduct the initial investment of $100 from each return—since the investment is the same for all projects, you can report gross numbers like $107 instead of $7):

(1) Return=$7*(probability of $7)=7*1=7.
(2) Return=$2*(probability of $2) + $12*(probability of $12)=$2*0.5 + $12*0.5=7.
(3) Return=$2*(probability of $2) + $15*(probability of $15)=$2*0.5 + $15*0.5=8.5.

The variances of returns on each investment project are calculated as a sum of (each return – the expected return) squared times the corresponding probability:

(1) Var=$(7 – expected return)^2*(probability of $7)= (7 – 7)^2*1=0
(2) Var=$(2 – 7)^2*0.5 + (12 – 7)^2*0.5= 25
(3) Var=$(2 – 8.5)^2*0.5 + (15 – 8.5)^2*0.5= 42.25.

No risk-averse investor will choose project (2) over project (1) because they offer exactly the same payoff on average, but project (2) is riskier (more uncertain as measured by the variance). When we compare project (3) to the other two, it is important to keep in mind that risk requires compensation, i.e. higher risk (variance) would require higher expected returns for any risk-averse investor to invest in it. Clearly, project (3) offers higher expected return. Whether this increase in the return is enough to compensate for the increased risk (variance) is largely an opinion question. In this particular case, it probably is because the lowest possible return ($2) is the same for projects (2) and (3), and it happens with the same probability. But the largest return is higher for the project (3) ($15 instead of $12), so probably everyone would invest in (3).