

Homework Assignment 4 solution.

1. (3 points) Suppose that you have \$1000 to invest and there are two financial instruments that you can purchase. The returns on each of these instruments depend on oil prices in the following manner:

Instrument A:

Oil Prices	Probability	Return
High	0.2	30%
Average	0.6	10%
Low	0.2	-10%

Instrument B:

Oil Prices	Probability	Return
High	0.2	-5%
Average	0.6	10%
Low	0.2	25%

The price of one unit of each financial instrument is \$100.

- (a) You consider two investment strategies: either invest all your \$1000 in instrument A or invest all your \$1000 in instrument B. For each of these strategies compute the expected return and variance of the return. If you are risk averse, which one you would choose?

The expected returns are:

$$\text{A: } 0.2 \cdot 30 + 0.6 \cdot 10 + 0.2 \cdot (-10) = 10\%$$

$$\text{B: } 0.2 \cdot (-5) + 0.6 \cdot 10 + 0.2 \cdot (25) = 10\%$$

The variances are:

$$\text{A: } 0.2 \cdot (30-10)^2 + 0.6 \cdot (10-10)^2 + 0.2 \cdot (-10-10)^2 = 160$$

$$\text{A: } 0.2 \cdot (-5-10)^2 + 0.6 \cdot (10-10)^2 + 0.2 \cdot (25-10)^2 = 90.$$

Both investments have the same expected return, however, the investment B is less risky (as measured by the variance of returns). Any risk-averse investor would choose B over A.

- (b) Now suppose that you can buy any combination of the two instruments (10 total) that you like, e.g. you can buy 6 of A and 4 of B, or 1 of A and 9 of B. Propose an investment strategy (a combination of the two instruments) that any risk averse investor would choose over both strategies in part (a)? (You have to show why any risk averse investor would choose it.)

Consider the following investment strategy – put 50% of your money in A and 50% in B (buy 5 of each instrument). The returns on this investment can be determined by applying the appropriate weights (50%, 50%) to the respective returns on A and B for each level of oil prices:

50/50 Strategy:

Oil Prices	Probability	Return
High	0.2	$0.5*(30\%) + 0.5*(-5\%) = 12.5\%$
Average	0.6	$0.5*(10\%) + 0.5*(10\%) = 10\%$
Low	0.2	$0.5*(-10\%) + 0.5*(25\%) = 7.5\%$

The expected return and variance of this investment strategy:

Expected return: $0.2*12.5 + 0.6*10 + 0.2*7.5=10\%$

Variances: $0.2*(12.5-10)^2 + 0.6*(10-10)^2 + 0.2*(7.5-10)^2=2.5$

The 50/50 investment strategy has exactly the same expected return as A and B, but it is much less risky than either of them. You can check that any combination of A and B would beat either A or B in terms of risk. This strategy is called hedging – investing in instruments returns on which move in opposite directions – when one posts low returns, the other delivers high returns. It is not always possible to find such instruments to diversify away most of the risk.

- (c) Now suppose that Instrument B is not available. Instead, you can invest in an instrument C, returns on which do not depend on oil prices and depend on the weather in Iowa:

Instrument C:

Weather in Iowa	Probability	Return
Cold	0.2	35%
Average	0.6	10%
Hot	0.2	-15%

Without doing any calculations, explain how this instrument can be valuable to a risk-averse investor.

Assuming that weather in Iowa is not related to oil prices, we can use instrument C to diversify part of the risk. The strategy involving investing in instruments with unrelated returns – when the return on one instrument is low, the return on the other instrument may be high – is called risk spreading. For example, if we invest in B and C, the risk of this investment strategy will be lower than risks of both B and C individually. The reason this happens is that the probability of low returns (-5% for B and -15% for C) for *both* instruments simultaneously is much lower ($0.2*0.2=0.04$ or 4%) than the probabilities of low returns on each instrument individually (20%). Again, this is possible because the events that trigger low returns for different instruments are different and unrelated, i.e. you may have hot weather in Iowa (low return on C) and low oil prices (high return on B). In situations like this low returns on C will be compensated by high returns on B.

2. (3 points) Read the policy debate on inflation policy at http://www.swlearning.com/economics/policy_debates/inflation.html and briefly (one or two paragraphs) summarize what you've read.
3. (2 points) Explain how a car insurance company can eliminate most of the risk that it faces by selling a large number of policies. What is the difference between this type of risk and the risk associated with insuring homes on the coast of Gulf of Mexico?

Risk associated with each individual driver can be diversified away by selling a large number of policies. The fundamental reason this is possible is that the individual risks are not related to each other, i.e. the fact that Joe hit other car while parking doesn't mean that Jane will do the same. One of the interpretations of the probability (to get into an accident) is the proportion of similar people that actually get into accidents among millions that drive. The more people a company covers – the better it will be able to predict the proportion of them that will get into an accident. The risk comes from the fact that we never know whether each individual person will end up in an accident. By being able to accurately predict the proportion of people getting in accidents, the insurance company faces very little risk.

Now, if you consider insuring homes on the Gulf of Mexico, the individual risks are no longer unrelated. If one home gets destroyed, there is a high chance that it happened because of the hurricane, which in turn means that home on the same block will most likely be destroyed as well. In other words, if a hurricane hits (it happens with certain probability, but it is almost impossible to say with certainty weather it will happen in each individual year) most homes in the area are affected. If it doesn't hit, no homes will be destroyed. It is in a way similar to insuring just one driver, not many drivers. To diversify this risk, an insurance company might have to look for other areas, where the probability of something bad happening to homes is unrelated to hurricanes – it may insure homes in California (where the risk is not related to hurricanes but is related to forest fires). The chance that a major hurricane will hit and there will be major forest fires in California is lower than each will happen individually.

4. (2 points) Suppose that the interest rate on 3-month Treasury Bills is currently 5%. The interest rate on a 3-month corporate bond is 7%. Now assume that the Fed chairman made an announcement that Fed will not try to stabilize inflation for the next couple of years. What do you think will happen to the yields on the Treasury bills and this corporate bond?

Clearly, the inflation risk will be higher because it will be more difficult to predict inflation in the future. The point of this question is that inflation risk is not idiosyncratic (affecting one bond), it is systemic (affects the whole economy). The required risk premiums will rise for all bonds, not just private bonds. In other words, this risk is not specific to any single type of bonds. So we expect yields on both T-Bills and corporate bonds to rise by a similar amount.