1. Consider two savings accounts. Bank A pays 8% compounded annually, and Bank B pays 7.5% compounded daily. Based on effective annual rates, which bank would you prefer?

2. You are taking out a $100,000 loan to be repaid over 25 years in 300 monthly payments.
   1. If the interest rate is 16% per year, what is the amount of the monthly payment?
   2. If you can only afford to pay $1,000 per month, how large a loan could you take?
   3. If you can afford to pay $1,500 per month and need to borrow $100,000, how many months would it take to pay off the mortgage?

3. Assume that you are 40 years old and wish to retire at age 65. You expect to be able to average a 6% annual rate of interest on your savings over your lifetime (both prior to retirement and after retirement). You would like to save enough money to provide $8,000 per year beginning at age 66 in retirement income to supplement other sources (social security, pension plans, etc.). Suppose you decide that the extra income need be provided for only 15 years (up to age 80). Assume that your first contribution to the savings plan will take place one year from now. How much must you save each year between now and retirement to achieve your goal?

4. You are the boss of a fund management company. The firm currently offers two products, a ‘US MARKET’ fund, which invests exclusively in US stocks, and a FIFTY-FIFTY™ fund, which has half of its assets in riskless bonds, the other half in US stocks. The return on the US stock index has a mean $\mu_{US}$ of 15%, and a standard deviation of $\sigma_{US} = .1$. The riskless interest rate is $r = .05$.

You have become aware of interesting opportunities in Mexico. The research department of the firm has determined that the mean return on stocks in Mexico is $\mu_{MEX} = .2$, with a standard deviation of $\sigma_{MEX} = .2$. Also, the returns in Mexico and the US are uncorrelated ($\rho_{US,MEX} = 0$). You have called a staff meeting to discuss new products that the firm might want to offer.

(i) One employee claims that it is possible to have a fund that
   1. invests only in Mexican stocks and riskless bonds, and
   2. yields a higher mean return than the FIFTY-FIFTY™ fund, for the same level of risk (standard deviation)

   Show that he is wrong.

(ii) Another employee proposes a USMEX fund which invests 60% in the US index and 40% in Mexico. He claims that this fund has a higher return than the US MARKET fund, for the same risk.

   Show that he is right.

(iii) Show that it is possible to construct a ‘GLOBAL’ fund that
   1. invests in all three types of assets (US stocks, Mexican stocks, riskless asset), and
   2. yields a higher mean return than the FIFTY-FIFTY™ fund, for the same level of risk (standard deviation).
Give the explicit portfolio weights of the three assets in the GLOBAL fund.

Hint: make use of the work by the employee in part (ii)!

Answers

**Problem 1.** Since Bank A compounds annually, the rate $i_A = .08$ is already the effective annual rate. For Bank B, we have to convert. The microperiod here is a day, so we first need the daily rate

$$i_{D,daily} = \frac{.075}{365} = .000205$$

Now we compound daily to get the effective annual rate, call it $i_D$. This is given by the future value of one dollar at the end of the year, minus one.

$$i_D = (1 + i_{D,daily})^{365} - 1 = .0777$$

So Bank A is offering the better deal, even if we take compounding into account.

**Problem 2.** This is an annuity problem.

*Part a.* The number of periods is $n = 300$ and the face value is $PV = 100,000$. We have an annual interest rate of 16%, so the monthly rate is $i = \frac{.16}{12} = .0133$. Remark: different rounding at this stage might give slightly different answers for later parts. We are looking for the payment

$$pmt = \frac{PV \times i}{1 - (1 + i)^{-n}} = \frac{100,000 \times .0133}{1 - (1 + .0133)^{-300}} = 1356$$

*Part b.* Again $n = 300$ and $i = .0133$. But now the payment $pmt = 1000$ is given and we are looking for the present value

$$PV = \frac{pmt}{i} \left(1 - \frac{1}{(1 + i)^n}\right) = 73,590$$

*Part c.* Now $PV = 100,000$ and $pmt = 1500$ are fixed. We want the number of periods:

$$n = -\frac{\ln(1 - \frac{PV \times i}{pmt})}{\ln(1 + i)} = 164.8$$

This is not an integer; so the 165th payment is not completely needed.

**Problem 3.** *Part 1.* This problem is solved in two steps. First, we need to figure out how much money we need to have at the end of the working life in order to enjoy $8,000 per year throughout retirement. Then in the second step we can find out how much we need to save every year to accumulate the relevant amount during working life

**STEP 1.** This is an annuity problem. In retirement we need a payment of $pmt = 8000$ for $n = 15$ periods, with an interest rate $i = .06$. We are looking for the present value: this is the money we need to have in the bank at the end of working life. The formula gives
\[ PV = \frac{8000}{.06} \left(1 - \frac{1}{(1 + .06)^{15}}\right) = 77,697.99 \]

STEP 2. So we know that we need to have $77,697.99 at the end of working life. Since we want to save the same amount every year, we can think of this number as the future value \( FV \) of an annuity. The payment corresponds to the amount saved each year; this is what we’re looking for. We are given the number of periods \( n = 65 - 40 = 25 \) years of working life) and the interest rate \( i = .06 \). We use the formula for the payment and the relationship between future and present value \( PV = (1 + i)^{-n} FV \)

\[ pmt = \frac{PV \cdot i}{1 - (1 + i)^{-n}} = \frac{(1 + i)^{-n} \cdot FV \cdot i}{1 - (1 + i)^{-n}} = 1416 \]

Problem 4. Question (i). First, we need the mean and standard deviation of the FIFTY-FIFTY fund.

\[ \mu_{50} = .15 \cdot .5 + .05 \cdot .5 = .1 \]
\[ \sigma_{50} = .5 \cdot .1 = .05 \]

Next, we need to find the mean return of a portfolio that has the same standard deviation as the FIFTY-FIFTY fund. The weight on Mexican stocks in such a portfolio, say \( w_i \), satisfies

\[ .05 = .2w_i \]

or \( w_i = .25 \). The mean is then

\[ \mu_i = .25 \cdot .2 + .75 \cdot .05 = .0875 < .1 = \mu_{50} \]

This shows the employee is wrong.

Question (ii). We compute the variance of the USMEX fund. The variance of US stocks is \( \nu_{US} = .01 \) and the variance of Mexican stocks is \( \nu_{MEX} = .04 \). We then have

\[ \nu_{USMEX} = (.6)^2(.01) + (.4)^2(.04) = .0036 + .0064 = .01 \]

The USMEX fund has indeed the same variance (and so the same standard deviation) as the US MARKET fund, \( \sigma_{USMEX} = .1 \). Its mean return is

\[ \mu_{USMEX} = .6 \cdot .15 + .4 \cdot .2 = .17 > .15 = \mu_{US} \]

This shows the employee is right.

Question (iii). There are several ways to answer this. The simplest is perhaps to directly use what the employee in (ii) has already figured out: the USMEX fund has a better mean return for the same risk than the US MARKET fund. There should now be a combination of the USMEX fund and the riskless asset that has a higher mean return than the FIFTY-FIFTY fund.

Use \( w_{USMEX} \) to denote the weight of the USMEX fund in the such a combination portfolio. We
want to have

$$\sigma_{50} = \sigma_{GLOBAL} = w_{USMEX}\sigma_{USMEX}$$

$$\cdot 05 = .1w_{USMEX}$$

so if $w_{USMEX} = .5$, then we have the same risk as the FIFTY-FIFTY$^{TM}$ fund. The new portfolio has 50% bonds and 50% of the USMEX fund. But this is the same as 50% bonds and $(50\%)(60\%)=30\%$ US stocks and $(50\%)(40\%)=20\%$ Mexican stocks. These are the portfolio weights for the GLOBAL fund. The mean return of the new fund is

$$\mu_{GLOBAL} = w_{USMEX}\mu_{USMEX} + (1 - w_{USMEX})\mu$$

$$= .5 \cdot .17 + .5 \cdot .05$$

$$= .11$$

$$> .1 = \mu_{50}$$

This shows that the GLOBAL fund has a higher mean return.