Distinction Between Interest Rates and Returns

**Rate of Return**

\[ RET = \frac{C + P_{t+1} - P_t}{P_t} = i_c + g \]

where:

\[ i_c = \frac{C}{P_t} = \text{current yield} \]

\[ g = \frac{P_{t+1} - P_t}{P_t} = \text{capital gain} \]
### Key Facts about Relationship Between Interest Rates and Returns

#### Table 2  One-Year Returns on Different-Maturity 10%‐Coupon‐Rate Bonds When Interest Rates Rise from 10% to 20%

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return (2 + 5) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>−49.7</td>
<td>−39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>−48.4</td>
<td>−38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>−40.3</td>
<td>−30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>−25.9</td>
<td>−15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>−8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated using Equation 3.
Maturity and the Volatility of Bond Returns

• Only bond whose return = yield is one with maturity = holding period

• For bonds with maturity > holding period, \( i \uparrow \)
P\( \downarrow \) implying capital loss

• Longer is maturity, greater is price change associated with interest rate change
Maturity and the Volatility of Bond Returns

- Longer is maturity, more return changes with change in interest rate
- Bond with high initial interest rate can still have negative return if $i \uparrow$
- Prices and returns more volatile for long-term bonds because have higher interest-rate risk
- No interest-rate risk for any bond whose maturity equals holding period
Distinction Between Real and Nominal Interest Rates

Real Interest Rate
Interest rate that is adjusted for expected changes in the price level

\[ i_r = i - \pi^e \]

1. Real interest rate more accurately reflects true cost of borrowing
2. When real rate is low, greater incentives to borrow and less to lend

if \( i = 5\% \) and \( \pi^e = 3\% \) then:

\[ i_r = 5\% - 3\% = 2\% \]

if \( i = 8\% \) and \( \pi^e = 10\% \) then

\[ i_r = 8\% - 10\% = -2\% \]
U.S. Real and Nominal Interest Rates

Interest Rate (%)

Nominal Rate

Estimated Real Rate

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Thinking about Uncertain Returns

- need to plan for different return scenarios
- think of returns being generated by a chance mechanism
- use historical series of returns to infer probabilities for future returns
- the investment horizon matters a lot for what these probabilities are
- potential problem: probabilities for tomorrow’s returns might depend on current return, or other variables (e.g. boom, recession)
- actually, returns not well predictable over short periods of time, but over long periods there is predictability
Risks to Bondholders (1)

One-Period Nominal Return for a bond with coupon payment $C$, price $P$:

- if maturity is one period, then price is known, nominal return can be foreseen (except perhaps if bankruptcy risk)

- if maturity is longer than one period, price fluctuations imply that bond is a risky asset!

- long bond prices fluctuate if one-period interest rate changes (higher short rate $\rightarrow$ bond prices go down)
Risks to Bondholders (2)

- Real Return on a bond, with *expected* rate of inflation:
  - ‘ex post’ (with realized inflation), Fisher equation true by definition
  - ‘ex ante’ (with expected inflation), it is hard to test, because expected inflation is not observed
- but: many countries have indexed bonds, so real returns become observable
- in countries with high inflation risk, firms often borrow in foreign currency
Probability Distributions

Suppose a random variable, such as a return $R$, can take $N$ values, $R_1, R_2, ..., R_N$, with probabilities $\pi_1, \pi_2, ..., \pi_N$, respectively. We summarize its behavior by

- Mean $E[R] = \sum_{n=1}^{N} \pi_n R_n$
- Measures of Risk
  - Variance $\text{var}(R) = \sum_{n=1}^{N} \pi_n (R_n - E[R])^2$
  - Standard Deviation $\sigma(R) = \sqrt{\text{var}(R)}$
Risk Return Tradeoff

- mean-standard-deviation diagram shows risk and expected return of possible portfolios
- every asset (or portfolio) can be represented by one point
- given a risky and a riskless asset, we can generate any portfolio on a straight line through the two assets
- points between the two assets correspond to positive portfolio weights.

Preferences determine the optimal mix of risky and riskless asset.
Several Risky Assets

Consider two stocks

<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.5</td>
<td>.5</td>
</tr>
<tr>
<td>Pepsi return $R^P$</td>
<td>.15</td>
<td>-.15</td>
</tr>
<tr>
<td>Coke return $R^C$</td>
<td>0</td>
<td>.2</td>
</tr>
</tbody>
</table>

Expected Returns:  
\[ E[R^P] = 0 \quad E[R^C] = .1 \]

Standard Deviations  
\[ \text{StD}(R^C) = \sqrt{.5(.1)^2 + .5(.1)^2} = .1, \]
\[ \text{StD}(R^P) = \sqrt{.5(-.15)^2 + .5(.15)^2} = .15 \]

→ Coke looks more attractive than Pepsi, but: combining the two can give a riskless portfolio (pick $w^P = .4, w^C = .6$)!
Diversification

Bottom Line: by holding several risky assets, risk can be reduced (diversification)

- expect to see people spreading wealth across many assets

- how ‘good’ an asset looks depends not only on own risk and return; for example, emerging markets are attractive because of low correlation with US index,

- puzzle: not a lot of cross-border equity holdings