Thinking about Uncertain Returns

- need to plan for different return scenarios
- think of returns being generated by a chance mechanism
- use historical series of returns to infer probabilities for future returns
- the *investment horizon* matters a lot for what these probabilities are
- potential problem: probabilities for tomorrow’s returns might depend on current return, or other variables (e.g. boom, recession)
- actually, returns not well predictable over short periods of time, but over long periods there is predictability
Risks to Bondholders (1)

One-Period Nominal Return for a bond with coupon payment $C$, price $P$:

- if maturity is one period, then price is known, nominal return can be foreseen (except perhaps if bankruptcy risk)

- if maturity is longer than one period, price fluctuations imply that bond is a risky asset!

- long bond prices fluctuate if one-period interest rate changes (higher short rate $\rightarrow$ bond prices go down)
Risks to Bondholders (2)

- Real Return on a bond, with expected rate of inflation:

- ‘ex post’ (with realized inflation), Fisher equation true by definition

- ‘ex ante’ (with expected inflation), it is hard to test, because expected inflation is not observed

- but: many countries have indexed bonds, so real returns become observable

- in countries with high inflation risk, firms often borrow in foreign currency
Probability Distributions

Suppose a random variable, such as a return $R$, can take $N$ values, $R_1, R_2, ..., R_N$, with probabilities $\pi_1, \pi_2, ..., \pi_N$, respectively.

We summarize its behavior by

- **Mean** $E[R] = \sum_{n=1}^{N} \pi_n R_n$

- **Measures of Risk**
  - Variance $\text{var}(R) = \sum_{n=1}^{N} \pi_n (R_n - E[R])^2$
  - Standard Deviation $\sigma(R) = \sqrt{\text{var}(R)}$
Risk Return Tradeoff

- Mean-standard-deviation diagram shows risk and expected return of possible portfolios.
- Every asset (or portfolio) can be represented by one point.
- Given a risky and a riskless asset, we can generate any portfolio on a straight line through the two assets.
- Points between the two assets correspond to positive portfolio weights.

Preferences determine the optimal mix of risky and riskless asset.
Several Risky Assets

Consider two stocks

<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>Probability</td>
<td>.5</td>
<td>.5</td>
</tr>
<tr>
<td>Pepsi return $R^P$</td>
<td>.15</td>
<td>-.15</td>
</tr>
<tr>
<td>Coke return $R^C$</td>
<td>0</td>
<td>.2</td>
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</tbody>
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Expected Returns: $E[R^P] = 0$ \quad $E[R^C] = .1$

Standard Deviations

$\text{StD}(R^C) = \sqrt{.5(.1)^2 + .5(-.1)^2} = .1$

$\text{StD}(R^P) = \sqrt{.5(.-.15)^2 + .5(.15)^2} = .15$

→ Coke looks more attractive than Pepsi, but: combining the two can give a riskless portfolio (pick $w^P = .4, w^C = .6$)!
Diversification

Bottom Line: by holding several risky assets, risk can be reduced (diversification)

- expect to see people spreading wealth across many assets

- how ‘good’ an asset looks depends not only on own risk and return; for example, emerging markets are attractive because of low correlation with US index,

- puzzle: not a lot of cross-border equity holdings