Understanding Interest Rates

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Notes on Mishkin Chapter 4: Part A
(pp. 68-80)

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Mishkin Chapter 4: Part A -- Selected Key In-Class Discussion Questions and Issues

🔹 Five basic types of **debt (or credit market) instruments**. Who pays what, to whom, and when?

🔹 Why is **present value (PV)** considered to be one of the most important concepts in finance?

🔹 Why is **yield to maturity (YTM)** considered to be the most important measure of an interest rate?

🔹 **PV** and **YTM** -- **what’s the connection**?

🔹 **Illustrations**
Five Basic Types of Debt Instruments

1. Simple Loan Contracts
2. Fixed-Payment Loan Contracts
3. Coupon Bond
4. (Zero-Coupon) Discount Bond
5. Consol (or Perpetuity)
Type 1: (One-Year) Simple Loan Contract

• Borrower issues to lender a contract stating a loan value (principal) $LV$ and interest payment $I$. 

• Today the borrower receives $LV$ from lender.

• One year from now the lender receives back from the borrower an amount $LV+I$.

• *Example: One-Year Deposit Account*
  Deposit $LV = $100;  Interest payment $I = $10
  Borrower’s end-of-year payment = $100 + $10.
Type 2: Fixed Payment Loan Contract

• Today a borrower issues to a lender a contract with a stated loan value LV ($), an annual fixed payment FP ($/Yr), and a maturity of N years.

• Today the borrower receives LV from the lender.

• For the next N successive years, the lender receives from borrower the fixed payment FP.

• FP includes principal and interest payments.

Example: 30-year fixed-rate home mortgage
Type 3: Coupon Bond

• Today a seller offers for sale in a bond market a bond with stated annual coupon payment $C/yr$, face (or par) value $F ($), and a remaining maturity of $N$ years.

• Today the bond seller receives from a buyer a price $P ($/bond) as determined in the bond market.

• For next $N$ successive years, the bond holder receives the fixed annual payment $C$ from original bond issuer.

• At maturity, the bond holder also receives the face value $F$ from the original bond issuer.

Examples: 30-year corporate bond, U.S. Treasury notes (1-10yrs) and bonds ($\geq 10$yrs)
Type 4: Discount Bond

• Today a seller offers for sale in a bond market a bond with a stated face value $F$ ($) and remaining maturity of $N$ years.

• Today the bond seller receives from a buyer a price $P$ ($/bond) as determined in the bond market.

• At the end of $N$ years the bond holder receives the face value $F$ from the original bond issuer.

• Example: Treasury Bills  Maturity < 1yr., typically offered in 1mo., 3mo., & 6 mo. maturities. The U.S. Treasury stopped offering 1yr (52-week) bills in 2001.
Type 5. Consol (or Perpetuity)

- Today a seller offers for sale in a bond market a bond with a stated annual coupon payment $C/\text{Yr}$ and no maturity date (i.e., bond exists “in perpetuity”).

- Today the bond seller receives from a buyer a price $P$ ($/\text{bond}$) \textit{as determined in the bond market}.

- In each future year the bond holder receives the coupon payment $C$ \textit{from the original bond issuer}.

- \textit{Example:} Consols were originally issued by UK in 1751, and remain a small part of UK’s debt portfolio.
Interest Rates and the Yield to Maturity

- **Interest Rate**: Measure of cost of borrowing money
- The most important interest rate that economists calculate is the “Yield to Maturity” (YTM):

  \[
  \text{YTM for an asset } A = \text{The interest rate } i \text{ that equates the “current value” of } A \text{ with the “present value” of all future payments received by the owner of } A
  \]

  - What does “Current Value (CV)” mean?
  - What does “Present Value (PV)” mean?
Calculating Present Value (PV)

- **PV** is the value today of future received money.
- Suppose the annual interest rate is $i$.
- The present value of $100 to be received $N$ years in the future is
  \[ PV = \frac{100}{(1+i)^N} \]
  Why?
- If $PV = \frac{100}{(1+i)^N}$ is deposited today, and left to accumulate interest for $N$ years, the amount at end of $N$ years is
  \[ (1+i)^N \cdot \left[ \frac{100}{(1+i)^N} \right] = 100 \]
Timing of Payments

Cannot directly compare payments received at different points in time:

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>100</td>
<td>$100/(1+i)$</td>
<td>$100/(1+i)^2$</td>
<td>$100/(1+i)^N$</td>
</tr>
</tbody>
</table>

$100$
Numerical Examples

• If $i = 10\%$, and $1$ is received one year from now,
  \[ PV = \frac{1}{(1+.10)^1} \]
  \[ PV \approx 0.91 \]

• If $i = 10\%$, and $1$ is received two years from now,
  \[ PV = \frac{1}{(1+.10)^2} \]
  \[ PV \approx 1/1.21 \]
  \[ PV \approx 0.83 \]

• If $i = 10\%$, $4$ is received at end of year 1, and $5$ is received at end of year 2, the PV of ($4, 5$) is
  \[ \frac{4}{(1+.10)} + \frac{5}{(1+.10)^2} \approx 3.64 + 4.15 \approx 7.79 \]
Numerical Examples…Continued

• Suppose the annual interest rate is $i = 10\%$.

• You will receive $3$ at the end of one year, $5$ at the end of 3 years, and $110$ at the end of eight years.

• Your payment stream is
  \[(3, 0, 5, 0, 0, 0, 0, 0, 110)\]

• The PV of your payment stream is
  \[$3/(1+.10) + 5/(1+.10)^3 + 110/(1+.10)^8$\]
Yield to Maturity Again

- The “Yield to Maturity (YTM)” on a debt instrument A is defined as follows:

  \[ \text{YTM on } A = \text{The interest rate } i \text{ that equates the “current value” of } A \] \text{ with the present value (PV) of all future payments received by the owner of } A \]

- \textbf{Current Value (CV) of } A = \text{Amount someone is actually willing to pay today to own } A.\]

- CV is determined either by loan contract terms or through a market process.
YTM for (One Year) Simple Loans: Example

- LV = Loan value (Principal) = $1000
- Maturity N = 1 Year
- Interest Payment I = $10
- Current Value (CV) for loan contract = LV
- Equate CV with PV of total payment stream:

  \[ CV = \$1000 = \left[ \frac{\$1000}{1+i} + \frac{\$10}{1+i} \right] = PV \]

- The value of i that solves this formula is the YTM for the simple loan:

  \[ i^* = \frac{\$10}{\$1000} = 0.01 \text{ (1 \%)} \]
YTM for 1-Year Simple Loans: General Formula

- Loan value = LV
- Maturity = 1 Year
- Interest Payment = I
- Current Value (CV) = LV

Equate CV with PV of total payment stream:
\[ LV = \frac{[LV + I]}{(1+i)} \]

The value of i that solves this formula is the YTM:
\[ i^* = \frac{I}{LV} \]
YTM for a Fixed Payment Loan: Example

- Loan value (LV) = $1000
- Annual fixed payment FP=$126 for 25 years
- Current Value (CV) = LV
- Equate CV with PV of total payment stream:
  \[ $1000 = \frac{126}{1+i} + \frac{126}{(1+i)^2} + \ldots + \frac{126}{(1+i)^{25}} \]
- The value of i that solves this formula is the **YTM** for the fixed payment loan:
  \[ i^* \approx 0.12 \ (12\%) \]
YTM for a Fixed Payment Loan: General Formula

\[ CV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \ldots + \frac{FP}{(1+i)^N} \]

- **CV** = Loan Value (LV)
- **FP** = Annual fixed payment
- **N** = Number of years to maturity
- The value \( i^* \) that satisfies this formula is the **YTM** for the fixed payment loan
YTM for a Coupon Bond: Example

- A coupon bond has an annual coupon payment \( C = $100 \), a face value \( F = $1000 \), and it matures in 10 years.
- The current price of the bond is \( P = $1200 \).
- Current Value (CV) = $1200.
- The YTM is the value of \( i \) that solves \( CV = PV \):
  \[
  $1200 = \frac{100}{1+i} + \frac{100}{(1+i)^2} + \ldots + \frac{100}{(1+i)^{10}} + \frac{1000}{(1+i)^{10}}
  \]
- The YTM is \( i^* = 0.07135 \) (7.135%).

YTM for a Coupon Bond: General Formula

\[ P = \frac{C}{1+i^*} + \frac{C}{(1+i^*)^2} + \ldots + \frac{C}{(1+i^*)^N} + \frac{F}{(1+i^*)^N} \]

- \( P \) = Bond market price = Current Value (CV)
- \( C \) = Annual coupon payment
- \( F \) = Face value
- \( N \) = Maturity
- Solve formula for \( i^* = YTM \)

\* Note there is an INVERSE relationship between the bond market price \( P \) and the YTM \( i^* \) “all else equal” (that is, for any given face value \( F \), coupon payment \( C \), and maturity \( N \))
Inverse Relationship Between Price $P$ and YTM for a Coupon Bond

Table 1  Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)  

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>

NOTE: Coupon Rate = C/F

Four Interesting Facts in Table 1:
1. The bond price $P$ and the YTM are negatively related.
2. When $P$ equals the face value $F=$1000, the C/F (10%) equals the YTM.
3. $P/F > 1$ implies C/F (10%) > YTM.
4. $P/F < 1$ implies C/F (10%) < YTM.
A simple way to remember relationship among P, F, YTM, and Coupon Rate C/F:

- Consider the coupon bond formula for YTM $i^*$ for N=1:
  $$P = \frac{C}{(1+i^*)} + \frac{F}{(1+i^*)} = \frac{F+C}{(1+i^*)}$$

- Divide each side by the face value F
  $$P/F = \frac{(1 + C/F)/(1+i^*)}$$

- It follows that
  $$P/F > 1 \text{ if and only if } C/F > i^*$$
  $$P/F = 1 \text{ if and only if } C/F = i^*$$
  $$P/F < 1 \text{ if and only if } C/F < i^*$$
YTM for a One-Year Discount Bond

• Face value \( F \)

• Maturity \( N=1 \)

• Current Value \( CV = P \) (bond market price)

• YTM is the value \( i^* \) that solves the formula
  \[ P = \frac{F}{1+i^*}, \quad \text{or equivalently,} \quad i^* = \frac{F - P}{P} \]

• **Example:** If \( P=900 \) and \( F=1000 \), then
  \[ i^* = \frac{(1000 - 900)}{900} \approx 0.11 \quad (11\%) \]
YTM for a Consol (or Perpetuity)

• Consol has fixed coupon payment C forever

• As explained in Mishkin (footnote 3, page 77, 2nd Bus School Edition), for any given i,
  \[ \text{PV of (C,C,C,\ldots) = } \frac{C}{i} \]

• Current Value (CV) = P (market price)

• The YTM is the value \( i^* \) that solves
  \[ P = \frac{C}{i^*} \]

• Therefore \( i^* = \frac{C}{P} \)
The Power of the YTM Concept

• Suppose you observe a person today buying a coupon bond (C=$100, F=$1000, N=10) at a current market price P=$1200.

• You then calculate that the YTM is $i^* = 0.07135$

• How might $i^*$ be used to estimate what CV the same person would be willing to pay today for a discount bond with face value F=$3000 and maturity N=2?

• Can estimate $CV = \frac{3000}{[1+i^*]^2} \approx 2,613.70$