The Rule of Seventy
(http://www.econ.iastate.edu/econ355/choi/rule70.pdf)

This rule is often used to approximate the time required for a growing series to double. Let X be the initial value of a growing variable, and Y denote the terminal value at time t + n. The relationship between the two is given by

$$Y = X(1+g)^n, \tag{1}$$

where g is the annual growth rate. If one is interested in the time span required for X to double, $Y = 2X$, and

$$2 = (1+g)^n. \tag{2}$$

Taking natural logs, we get $\ln 2 = n \ln(1+g)$. Hence,

$$n = \frac{\ln 2}{\ln(1+g)}. \tag{3}$$

Equation (3) gives the exact time periods required for a growing variable to double its size. Let n* be the (accurate) solution to (3). Although accurate, it is not very convenient for everyday use when a calculator is not at hand.

One can approximate n using the definition of e^x.

From the Maclaurin series (expansion around x = 0) of e^x, we get

$$e^x = f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + ...$$

Since the derivative of $f(x) = e^x$ is itself,

$$f'(0) = f''(0) = f'''(0) = ... = e^0 = 1,$$

and

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3"} + ... + \frac{x^n}{n!} + R_n, \quad (4)$$

where the remainder term $R_n \to 0$ as $n \to \infty$.
Ignoring high order terms, for small x, it may be approximated by

$$e^x \cong 1 + x. \quad (5)$$

Taking logarithms of both sides, we get

$$x \cong \ln(1+x). \text{ [but } x > \ln(1+x) \text{]} \quad (6)$$

Using (6), equation (3) may be approximated as

$$n \cong \frac{\ln 2}{g} = \frac{0.693147}{g} \cong \frac{70}{g\%}. \quad (7)$$

This is the origin of the Seventy Rule. However, it should be remembered that second and higher order terms in (4) were truncated. The RHS of (6) can be written as

$$g^* = \ln(1+g) < g \quad (8)$$

Then

$$n = \frac{\ln 2}{\ln(1+g)} = \frac{\ln 2}{g^*} > \frac{\ln 2}{g}. \quad (9)$$

That is, the Rule of Seventy slightly underestimates the amount of time required for the growing variable to double its size. The following table shows that the Rule of Seventy is a better approximation of n* for $g \le 5\%$, but the Rule of Seventy Two is more accurate for $g > 5\%$. It is important to note that even the latter underestimates the actual value of n when the grow rate exceeds 10%. Thus, these rules are useful for $g \le 10\%$.

Growth rate g%	n (Rule of 70)	N (Rule of 72)	n* (accurate value)
1	70	72	69.66072
2	35	36	35.00279
3	23.33333333	24	23.44977
4	17.5	18	17.67299
5	14	14.4	14.2067
6	11.66666667	12	11.89566
7	10	10.28571	10.24477
8	8.75	9	9.006468
9	7.777777778	8	8.043232
10	7	7.2	7.272541

For other approximation of n, visit http://education.qld.gov.au/tal/kla/finance/files/doubles.doc.