Problem Set #3 Solutions

(1) The intercept tells you the expected birthweight of a child whose mother never smoked while pregnant. Thus,

\[ E(BirthWeight|Cigs = 0) = 119.6. \]

Since there 16 ounces per pound, this suggests that the average birthweight for mothers who do not smoke is about 7.5 pounds. (This seems sensible). Also note that there are likely to be lots of women in the sample who never smoke, and thus we would expect to get a good estimate of the intercept in this application.

The slope on Cigs is negative, suggesting cigarette consumption while pregnant lowers birthweight. Specifically, for every one cigarette consumed per day, birth weight is expected to be lowered by .05 ounces.

(b) \[ E(Birthweight|Cigs = 20) = 119.6 - .0514(20) \approx 118.6. \]

Thus, we expect that birthweight will fall by about one ounce, on average, for women who smoke one pack of cigarettes per day.

(c) It is difficult to argue that this picks up a causal relationship. Women who smoke while pregnant probably possess lots of other characteristics that are different from women who do not smoke. For example, their access to prenatal care, eating habits, exercise habits, alcohol consumption patterns, etc. may be different from women who do not smoke. In addition, these omitted factors probably influence birthweight. So, before arguing a causal link here, one should probably at least attempt to control for these other factors.

(2) The intercept here is not reliable. It implies that individuals with no income have negative amounts of consumption. While we might interpret this as “borrowing,” this is a stretch, since no data points in the sample have negative values of consumption!
The problem here is that no individuals in the data have income values near zero, and so it is difficult to estimate the intercept of this relationship.

(b) Income can either be consumed, or used for other purposes, such as saving. If all income is consumed, we would expect the intercept to equal zero and the slope to equal one. When only a portion of income is consumed, we would expect the slope coefficient to be less than one.

(c) 

\[ \text{Consumption} = -124.84 + 0.853(30,000) = 25,465. \]

Thus, we would expect that an individual earning $30,000 per year will consume $25,465 of that income.

(d) The necessary income amount is obtained by solving the equation

\[ \text{Income} = (50,000 + 124.84)/0.853 = 58,763. \]

(4.1) Throughout this exercise, let \( TS \) denote the test score and \( CS \) denote class size.

(a) We seek \( E(TS|CS = 22) \):

\[
E(TS|CS = 22) = \hat{\beta}_1 + 22\hat{\beta}_2 = 520.4 - 22(5.82) = 392.36
\]

(b) Note that (in terms of the population regression function):

\[ E(TS|CS = 23) = \beta_1 + (23)\beta_2 \quad \text{and} \quad E(TS|CS = 19) = \beta_1 + (19)\beta_2. \]

Therefore, the difference, denoted \( \Delta \), from going from a class size of 19 to a class size of 23 is

\[
\Delta \equiv E(TS|CS = 19) - E(TS|CS = 23) = \beta_1 + (19)\beta_2 - [\beta_1 + (23)\beta_2] = -4\beta_2.
\]
Replacing population parameters with their estimated values, we find

\[ \hat{\Delta} = -4(-5.82) = 23.28. \]

So, test scores are predicted to be about 23.23 points larger when the class size is at its smaller value of 19. (Note: you could have defined \( \Delta \) as the difference between a class size of 23 - a class size of 19, in which case your point estimate would be -23.28).

(e) We are given \( CS = 21 \).

Our OLS formula for the intercept implies:

\[ \hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}, \]

or in this particular case:

\[ \hat{\beta}_1 = TS - \hat{\beta}_2 CS \Rightarrow TS = \hat{\beta}_1 + \hat{\beta}_2 CS. \]

Based on what is given in this problem, we can evaluate the right hand side of this final expression to obtain:

\[ TS = 520.4 - 5.82(21) = 395.85. \]

(4.4) We are asked to show that the intercept estimator is unbiased, given that the slope estimator is unbiased. Our intercept estimator is defined as

\[ \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}. \]

Since

\[ \bar{y} = \beta_0 + \beta_1 \bar{x} + \bar{u}, \]

it follows that

\[ \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \]

\[ = \beta_0 + \beta_1 \bar{x} + \bar{u} - \hat{\beta}_1 \bar{x} + \bar{u} \]

\[ = \beta_0 + (\beta_1 - \hat{\beta}_1) \bar{x} + \bar{u} \]
To establish unbiasedness, we must take the expectation of this expression:

\[
E(\hat{\beta}_0) = E\left[\beta_0 + (\beta_1 - \hat{\beta}_1)x + \bar{u}\right] \\
= E(\beta_0) + E[(\beta_1 - \hat{\beta}_1)x] + E(\bar{u}) \\
= \beta_0 + E[(\beta_1 - \hat{\beta}_1)x] + E(\bar{u}) \\
= \beta_0 + E[(1/n)\sum_i u_i] \\
= \beta_0 + (1/n)\sum_i E(u_i) \\
= \beta_0
\]

The third step follows since we regard \(x\) as a constant (a better way to do this is to apply the law of iterated expectations, but we do not take up that level of generality here). The fourth step applies the fact that \(\hat{\beta}_1\) is unbiased, while the fifth notes that \(E(u_i) = 0\) by the assumptions of our regression model.