Multiple Choice (5 points each): For each of the following, select the single most appropriate option to complete the statement.

1) The probability of an outcome

   a) is the number of times that the outcome occurs in the long run.
   b) equals $M \times N$, where $M$ is the number of occurrences and $N$ is the population size.
   c) is the proportion of times that the outcome occurs in the long run.
   d) equals the sample mean divided by the sample standard deviation.

2) The cumulative probability distribution shows the probability

   a) that a random variable is less than or equal to a particular value.
   b) of two or more events occurring at once.
   c) of all possible events occurring.
   d) that a random variable takes on a particular value given that another event has happened.

3) Two random variables $X$ and $Y$ are independently distributed if all of the following conditions hold, with the exception of

   a) $\Pr(Y = y \mid X = x) = \Pr(Y = y)$.
   b) knowing the value of one of the variables provides no information about the other.
   c) if the conditional distribution of $Y$ given $X$ equals the marginal distribution of $Y$.
   d) $E(Y) = E[E(Y \mid X)]$.

4) Assume that $Y$ is normally distributed $N(\mu, \sigma^2)$. To find $\Pr(c_1 \leq Y \leq c_2)$, where $c_1 < c_2$ and $d_i = \frac{c_i - \mu}{\sigma}$, you need to calculate $\Pr(d_1 \leq Z \leq d_2) =$

   a) $\Phi(d_2) - \Phi(d_1)$
   b) $\Phi(1.96) - \Phi(-1.96)$
   c) $\Phi(d_2) - (1 - \Phi(d_1))$
   d) $1 - (\Phi(d_2) - \Phi(d_1))$

5) An estimator is

   a) an estimate.
   b) a formula that gives an efficient guess of the true population value.
   c) a random variable.
   d) a nonrandom number.
6) An estimate is
   a) efficient if it has the smallest variance possible.
   b) a nonrandom number.
   c) unbiased if its expected value equals the population value.
   d) another word for estimator.

7) With i.i.d. sampling each of the following is true except
   a) \( E(\bar{Y}) = \mu_Y \).
   b) \( \text{var}(\bar{Y}) = \sigma^2_Y / n \).
   c) \( E(\bar{Y}) < E(Y) \).
   d) \( \bar{Y} \) is a random variable.

8) Among all unbiased estimators that are weighted averages of \( Y_1, \ldots, Y_n \), \( \bar{Y} \) is
   a) the only consistent estimator of \( \mu_Y \).
   b) the most efficient estimator of \( \mu_Y \).
   c) a number which, by definition, cannot have a variance.
   d) the most unbiased estimator of \( \mu_Y \).

Problems: Provide the requested information for each of the following questions. Be sure to show your work

9) (20 points) Following Alfred Nobel’s will, there are five Nobel Prizes awarded each year. These are for outstanding achievements in Chemistry, Physics, Physiology or Medicine, Literature, and Peace. In 1968, the Bank of Sweden added a prize in Economic Sciences in memory of Alfred Nobel. You think of the data as describing a population, rather than a sample from which you want to infer behavior of a larger population. The accompanying table lists the joint probability distribution between recipients in economics and the other five prizes, and the citizenship of the recipients, based on the 1969-2001 period.

<table>
<thead>
<tr>
<th></th>
<th>U.S. Citizen ((Y = 0))</th>
<th>Non-U.S. Citizen ((Y = 1))</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economics Nobel Prize ((X = 0))</td>
<td>0.118</td>
<td>0.049</td>
<td>0.167</td>
</tr>
<tr>
<td>Physics, Chemistry, Medicine, Literature, and Peace Nobel Prize ((X = 1))</td>
<td>0.345</td>
<td>0.488</td>
<td>0.833</td>
</tr>
<tr>
<td>Total</td>
<td>0.463</td>
<td>0.537</td>
<td>1.00</td>
</tr>
</tbody>
</table>

a) Compute \(E(Y)\) and interpret the resulting number.

b) Calculate and interpret \(E(Y \mid X = 1)\) and \(E(Y \mid X = 0)\).

c) A randomly selected Nobel Prize winner reports that he is a non-U.S. citizen. What is the probability that this genius has won the Economics Nobel Prize? A Nobel Prize in the other five disciplines?

10) (15 points) Find the following probabilities:

a) \(Y\) is distributed \(\chi^2_4\). Find \(Pr(Y > 9.49)\).

b) \(Y\) is distributed \(t_\infty\). Find \(Pr(Y > -0.5)\).

c) \(Y\) is distributed \(F_{4,\infty}\). Find \(Pr(Y < 3.32)\).
11) (15 points) The accompanying table gives the outcomes and probability distribution of the number of times a student checks her e-mail daily:

<table>
<thead>
<tr>
<th>Probability of Checking E-Mail</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Outcome (number of e-mail checks)</strong></td>
</tr>
<tr>
<td><strong>Probability distribution</strong></td>
</tr>
</tbody>
</table>

a) Calculate the c.d.f. for the above table.

b) What is the probability of her checking her e-mail between 1 and 3 times a day?

c) Of checking it more than 3 times a day?

12) (15 points) Adult males are taller, on average, than adult females. Visiting two recent American Youth Soccer Organization (AYSO) under 12 year old (U12) soccer matches on a Saturday, you do not observe an obvious difference in the height of boys and girls of that age. You suggest to your little sister that she collect data on height and gender of children in 4th to 6th grade as part of her science project. The accompanying table shows her findings.

<table>
<thead>
<tr>
<th>Height of Young Boys and Girls, Grades 4-6, in inches</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Boys</strong></td>
</tr>
<tr>
<td>( \bar{Y}_{Boys} )</td>
</tr>
<tr>
<td>57.8</td>
</tr>
</tbody>
</table>

a) Let your null hypothesis be that there is no difference in the height of females and males at this age level. Specify the alternative hypothesis.
b) Find the difference in height and the standard error of the difference.

c) Calculate the t-statistic for comparing the two means. Is the difference statistically significant at the 1% level?

13) (15 points) IQs of individuals are normally distributed with a mean of 100 and a standard deviation of 16. If you sampled students at your college and assumed, as the null hypothesis, that they had the same IQ as the population, then in a random sample of size

a) $n = 25$, find $\Pr(\bar{Y} < 105)$.

b) $n = 100$, find $\Pr(\bar{Y} > 97)$.

c) $n = 144$, find $\Pr(101 < \bar{Y} < 103)$. 