1. Let $Y$ be a Bernoulli random variable with success probability $Pr(Y = 1) = p$, and let $Y_1, \ldots, Y_n$ be i.i.d. draws from this distribution. Let $\hat{p}$ be the fraction of successes (1s) in this sample. It is straightforward to show that

- $\hat{p} = \bar{Y}$.
- $var(\hat{p}) = \frac{p(1-p)}{n}$.

Suppose that you are given results from a survey of 900 households in which they were asked whether they favor the so-called “public option” as part of health care reform. In that survey, 500 households said yes (i.e., $Y_i = 1$), whereas the other half said no (i.e., $Y_i = 0$).

a. Use the survey results to estimate $p$.

b. Use the estimator $\hat{p}(1-\hat{p})/n$ to calculate the standard error of your estimator.

c. What is the p-value associated with the hypothesis test $H_0 : p = 0.5$ vs. $H_1 : p \neq 0.5$; i.e., that the population is evenly split over the issue.

d. What conclusion would you reach regarding the null hypothesis based on your results.

e. Suppose that instead of a sample of 900 households, the survey used only 9 households, 5 of which favored the public option. Does your conclusion regarding the null hypothesis change?

2. In the past, a chemical plant has a stated goal of producing an average of 1100 pounds of chemical per day. The records for the past year, based on 260 operating days, show the following:

- $\bar{Y} = 1060$ pounds per day.
- $s_Y = 340$ pounds per day. The managers of the firm wish to test whether or not the average daily production differs from its stated goal.

a. Give the appropriate null and alternative hypotheses.

b. Compute the necessary p-value to test the hypothesis using the available data.

c. Construct a 90% confidence interval for the sample mean.

3. The breaking ability of two types of automobiles was compared. Random samples of 64 automobiles were tested for each type. The recorded measurement was the distance required to stop when the brakes were applied at 40 miles per hour. The computed sample means and variances were:

- $\bar{Y}_A = 100$ feet, $s^2_A = 102$.
- $\bar{Y}_B = 109$ feet, $s^2_B = 87$.

The automobile manufacturers are interested in whether the two vehicle types have the same mean stopping distance.

a. Give the appropriate null and alternative hypotheses.

b. Compute the necessary p-value to test the hypothesis using the available data.

c. Construct a 95% confidence interval for the difference between the two sample means.