Problem Set #1

(1) The following table gives the joint probability distribution $p(X, Y)$ of random variables X and Y.

	X		
Y	1	2	3
1	.02	.04	.08
2	.03	.18	.04
3	.04	.04	.08
4	.09	.18	.18

Determine the following:

(a) Do the entries of the table satisfy the conditions for a bivariate density function?

(b) The marginal (or unconditional) probability distributions of X and Y. [Note: These will be a collection of probabilities: the probabilities associated with the 3 values of X and the probabilities associated with the 4 values of Y].

(c) The conditional probability distributions $p(X|Y = 3)$ and $p(Y|X = 1)$. (Note: The first conditional probability distribution is the collection of three numbers, $\Pr(X = 1|Y = 3), \Pr(X = 2|Y = 3), \Pr(X = 3|Y = 3)$.)

(2) Let $D = 1$ denote the event that an adult male has a particular disease. In the population, it is known that the probability of having this disease is 20 percent, i.e., $\Pr(D = 1) = .2$

Now, suppose that an adult male has a son. Unlike the father's birth, new health policy now requires that all newborn males are tested for the disease. Suppose that a particular adult male's son is tested, and is confirmed not to carry this particular disease. Let $S = 0$ denote the event that son does not carry the disease.

Assume the following:

(a) If the father does, in fact, have the disease, the probability that his son will have the disease is 50 percent.

(b) If the father does not, in fact, have the disease, then the probability that his son will not have the disease is 100 percent.

What is the probability that the father has the disease, given that his son does not have the disease, i.e., $\Pr(D = 1|S = 0)$?

Extra Credit (Worth two percentage points on the first exam).

Now, suppose that this father has another son, and that, at birth, the second son is tested and is also known to be free of the disease. Let $S_1 = 0$ denote the event that the first son does not have the disease and $S_2 = 0$ denote the event that the second son does not have the disease.

Assuming, in addition to (a) and (b) above, that the test results are independent across children, given the disease status of the father, what is the probability that the father has the disease, given that *both* sons do not have the disease, i.e., $\Pr(D = 1|S_1 = 0, S_2 = 0)$?