

### Solutions: Problem Set #4

(1) Taking the derivative of the objective function with respect to  $\beta_1$  gives the first order condition:

$$-2 \sum_{i=1}^n (y_i - \hat{\beta}_1) = 0.$$

Cleaning this expression up a bit, we obtain

$$\sum_{i=1}^n y_i - n\hat{\beta}_1 = 0$$

which implies

$$\hat{\beta}_1 = \bar{y} \equiv \frac{1}{n} \sum_{i=1}^n y_i.$$

The  $R^2$  for this regression is zero. This can be seen from the formula:

$$R^2 = \frac{\sum_i (\hat{y}_i - \bar{y})^2}{\sum_i (y_i - \bar{y})^2}.$$

Since  $\hat{y}_i = \hat{\beta}_1 = \bar{y}$  in this model with no explanatory variable, it follows that the numerator of the above expression is zero, and thus  $R^2$  equals zero.

**(2a)** The regression of number of weeks worked on education produced the following results:

$$\widehat{Weeks} = 17.08 + 1.42Ed.$$

The coefficients are intuitively plausible - the coefficient on education is positive, and suggests that every additional year of schooling attained leads women to work about 1.4 extra weeks per year, on average.

(2b) For a woman with 12 years of education:

$$E(\widehat{Weeks}|Ed = 12) = 17.08 + 1.42(12) = 34.12.$$

For a woman with 16 years of education:

$$E(\widehat{Weeks}|Ed = 16) = 17.08 + 1.42(16) = 39.8.$$

(2c) The STATA code I used for this problem set, together with the raw regression output are included as a separate attachment to this set of solutions.

**(3a)** The regression of number of weeks worked on kids produced the following results:

$$\widehat{Weeks} = 45.60 - 12.63Kids.$$

For women without young children in the home, the expected number of weeks worked per year is about 45.6, which is reasonably close to full-time employment. This result seems sensible. Having young children in the home leads to a significant reduction in the expected number of weeks worked per year. Specifically, having small kids in the home reduces the expected number of weeks by about 12.6.