Econ 380
Practice For Midterm Exam 1

1. Consider the two period model discussed in class. Instead of oil, suppose we are concerned with allocating a limited supply of water over two time periods. Assume that the inverse demand equations for water in the two periods are given by:

\[ MB_1 = 150 - 2q_1 \]
\[ MB_2 = 450 - q_2 \]

where \( q_1 \) denotes the amount of water consumed in period 1 and \( q_2 \) denotes the amount consumed in period 2. The marginal cost of extracting and providing the water is given by:

\[ MC_1 = 50 \]
\[ MC_2 = 50 \]

If there is a total of 300 units of water available for the two periods and the discount rate is 33\% (i.e., \( r=.33 \)):

a. How much water would each time period wish to have, ignoring the other time period? That is, what is the statically efficient amount of water to consume in each period?

Static efficiency occurs in each period when we maximize the total net benefits. As we saw in class, this occurs when marginal benefits (MB) equals marginal cost (MC) (or equivalently, when MNB = 0). Mathematically, using the above equation, we need:

\[ MB_1 = MC_1 \]

or

\[ 150 - 2q_1 = 50 \]

or

\[ 2q_1 = 100 \]

or

\[ q_1 = 50 \]

Using the equations for period 2 will yield \( q_2 = 400 \). Since the two periods combined want 450 units of water, and there are only 300 units available, we have a scarcity problem.

This same solution can be found graphically, drawing the marginal cost and marginal benefit curves for the two periods.
We must now determine a way to allocate the water between the two periods.

b. How much would be allocated to the first period and how much would be allocated to the second period in a dynamically efficient allocation? Does the quantity used go up or down over time? Why?

This problem can be solved in a number of ways. I will show you the mathematical solution first. We have two pieces of information available to us in solving this problem. First, we know we will want to use up all of the water, since a shortage of water exists for both periods. Mathematically, this can be written:

\[ q_1 + q_2 = 300 \]

or

\[ q_2 = 300 - q_1. \]

The second piece of information we have is that we want the allocation to be dynamically efficient. As we saw in class, this occurs when the present value of the marginal net benefits in the two periods are equal. From the above equations, we have:

\[ \text{MNB}_1 = \text{MB}_1 - \text{MC}_1 \]

\[ = 150 - 2q_1 - 50 \]

\[ = 100 - 2q_1. \]

Similarly,
\[ MNB_2 = 450 - q_2 - 50. \]
\[ = 400 - q_2 \]

The present value of the marginal benefits in period 1 is just the marginal net benefits in period 1 (i.e., we do not discount the present):
\[ PV[MNB_1] = 100 - 2q_1. \]

However, we have to discount the marginal net benefits in period 2, because we do not receive those benefits until one period from the present. Thus,
\[ PV[MNB_2] = \frac{400 - q_2}{1 + .33} = 300 - .75q_2 \]

Since dynamic efficiency requires that the present value of the marginal net benefits be equal in the two periods, we have:
\[ 100 - 2q_1 = 300 - .75q_2 \]

Substituting in our resource constraint (i.e., \( q_2 = 300 - q_1 \)) gives us
\[ 100 - 2q_1 = 300 - .75(300 - q_1) \]

or
\[ 100 - 2q_1 = 300 - 225 + .75q_1 \]

or
\[ 25 = 2.75q_1 \]

or
\[ q_1 = 9.09 \]

and
\[ q_2 = 300 - q_1 = 290.91. \]

Notice that, unlike the problem we solved in class, the quantity used goes up over time, despite a very large discount rate. This happens because the demand for water (and the marginal benefits received from it) goes up substantially between time periods 1 and 2. One way to think about this case is that the higher marginal benefits in time period 2 may be due to the introduction of irrigation technology to the region. Suppose in period 1, no irrigation possibilities exist and water is used only for drinking and washing. If irrigation technology is available in period 2, the water also yields additional marginal benefits in the form of greater food production. More water is allocated to period 2 than period 1 because, while one has to wait for the benefits from period 2, the benefits are so large that the wait is worth while.

The same problem can be solved graphically using our two winged graphs.
c. What would be the efficient prices in the two periods?

The efficient price will be the value that buyers place in the last unit of water provided in each period (i.e., the marginal benefit). For period 1, \( q_1 = 9.09 \), so that the efficient price is \( MB_1 = 150 - 2(9.09) = 131.82 \). Similarly, \( q_2 = 290.91 \), so the efficient price for period 2 is \( MB_2 = 450 - 290.91 = 159.09 \).

d. What would be the marginal user's cost in each period?

The marginal user's cost is the marginal net benefit forgone in a period due to scarcity. For period 1, we have
\[
MUC_1 = MNB_1 \\
= 100 - 2q_1 \\
= 100 - 2(9.09) \\
= 81.82
\]

Similarly, for period 2 we have:
\[
MUC_2 = MNB_2 \\
= 400 - q_2 \\
= 400 - 290.91 \\
= 109.09
\]

Notice that the marginal user's cost does rise at the rate of interest (i.e., \( MUC_2 = (1 + .33)*MUC_1 \)).
2. A small town in Iowa has decided that they need to develop a street lighting system for their community. There are only two people in the town. Individual A's marginal benefits from the street lighting system are summarized by the inverse demand (or marginal benefits) equation

\[
MB_A = \begin{cases} 
45 - 3q & 0 \leq q \leq 15 \\
0 & q > 15 
\end{cases}
\]

while individual B's marginal benefits are given by

\[
MB_B = \begin{cases} 
30 - 2q & 0 \leq q \leq 15 \\
0 & q > 15 
\end{cases}
\]

where q measures the number of street lights installed in the community. The marginal cost of street lighting is \(MC = 10\).

1. How many street lights would be funded if the local authorities asked individuals A and B to pay for it? (Hint: allow for free-riding.) What is the total net benefit to society of this amount of street lighting? (You do not have to compute an actual number in this case, but can illustrate the area using a graph.)

Figure 1 below illustrates the problem. Both marginal benefit curves (MBA and MBB) and the marginal cost curve (MC) are graphed. If individual A is asked to purchase street lights, he or she should increase the number of street lights until the marginal benefit received just equals the marginal cost of the lights. In Figure 1, this corresponds to the point where MBA intersects MC, at \(q = 11.7\). Beyond that point (i.e., for \(q > 11.7\)), the marginal benefits to individual A from street lights are less than the marginal costs. Before that point (i.e., \(q < 11.7\)), the marginal benefits are greater than the marginal cost and \(q\).
should be increased.

In a similar manner, if we consider individual B, he or she will want to purchase street lights up to the point where \( MBB = MC \), which in Figure 1 corresponds to \( q = 10 \). However, if individual B realizes that A has already purchased 11.7 street lights, he or she will free-ride on individual A and purchase no additional street lights. With free-riding, \( q \) will end up at 11.7.

Now we have to compute the total net benefits from this number of street lights. The total net benefit to society is given by \( TNB = TB - TC \). However, it is important to keep in mind that there are two sources of benefits, those from individual A and those from individual B. Thus, \( TB = TBA + TBB \), so that \( TNB = (TBA + TBB) - TC \). Figure 2 illustrates each of these areas. Notice that the areas depicting the total benefits to A and B overlap. This is because we are dealing with a public good that both individuals can enjoy simultaneously. The costs, however, are incurred only once.
2. What is the socially optimal level of street lighting for this community? What is the total net benefit to society? Explain your findings.

In order to compute the socially optimal level of street lighting, we need to first compute the marginal benefits to society from street lighting. These marginal benefits are the sum of the marginal benefits to individual A and the marginal benefits to individual B. That is: \( \text{MBS} = \text{MBA} + \text{MBB} \). The addition of the two marginal benefit curves is illustrated graphically in Figure 3.
Mathematically, we have

\[ MB_s = \begin{cases} 
75 - 5q & 0 \leq q \leq 15 \\
0 & q > 15 
\end{cases} \]

The efficient level of police protection from society's perspective is then found by setting \( MBS = MC \). Doing this for the first part of the MBS equation (i.e., \( 0 \leq q \leq 15 \)) yields:

\[ 75 - 5q = 10 \]

or

\[ 65 = 5q \]

or

\[ q = 13. \]

Notice that this is larger than the free-riding solution of \( q = 11.7 \). We can also use the MBS to compute the total benefit to society. The TB to society of \( q=13 \) is the area under the MBS curve up to \( q = 13 \). The TC to society is the area under the marginal cost curve up to \( q = 13 \) and the TNB is the difference between TB and TC. The TNB to society is illustrated in Figure 4.]