Answers to Problem Set #1

1. Let $MNB_1^1$ and $MNB_2^1$ be the marginal net benefits of periods 1 and 2 in scenario 1, and $MNB_1^2$ and $MNB_2^2$ be those in scenario 2. Since $MC_2$ is lower in scenario 2, we know

$$MNB_1^1 = MNB_1^2$$

$$MNB_2^1 < MNB_2^2$$

We can then graph the resource allocation problems for both scenarios as follows:

\[
\begin{align*}
\text{PV}[MNB_1^1] &= \text{PV}[MNB_1^2] \\
\text{PV}[MNB_2^2] \\
MUC_1^2 & \quad \text{MUC}_1^1 \\
q_1^{2*} & \quad q_1^{1*}
\end{align*}
\]

$q_1^{1*}$ and $q_1^{2*}$ are the resource allocated to period 1 in scenarios 1 and 2 respectively. Since $q_1^{2*} < q_1^{1*}$, more resource is allocated to the second period in the second version than in the first version. The marginal user cost is higher in the second version.

Intuition: in the second scenario, you will extract more in period two because the marginal cost of extraction becomes lower. Similarly, the opportunity cost of extraction in period one is higher, since the forgone second period benefit is higher. This leads to a higher marginal user cost.

2. I will present the math approach. You can also use graphs

(a) According the static efficiency criterion, we set $MB_1=MC_1$ and $MB_2=MC_2$. That is,
\[
\begin{align*}
150 - 2q_1 &= q_1 \\
300 - q_2 &= q_2
\end{align*}
\]

Thus, the optimal solution is \( q_1^* = 50 \), and \( q_2^* = 150 \).

However, the total required resource to satisfy the static efficiency is 200 tons of coal, higher than the actual stock of 100. Static efficiency is inadequate.

(b) For dynamic efficiency, we know \( \text{MNB}_1 = \text{MB}_1 - \text{MC}_1 = 150 - 3q_1 \), and \( \text{MNB}_2 = 300 - 2q_2 \). The dynamic efficiency rule requires \( \text{MNB}_1 = \text{PV}[\text{MNB}_2] \). Thus we have

\[
\begin{align*}
150 - 3q_1 &= \frac{300 - 2q_2}{1.2} \\
q_2 &= 100 - q_1
\end{align*}
\]

Substituting \( q_2 = 100 - q_1 \) into the first equation, and multiplying both sides by 1.2, we get

\[1.2(150 - 3q_1) = 300 - 2(100 - q_1)\]

from which we get \( q_1^d = 14.29 \). We then know \( q_2^d = 85.71 \).

We see that the quantity extracted goes up over time. This is different from our example in class where the quantity extracted goes down over time. The reason is that in this question, the marginal benefit of using the resource goes up in the second period. (Remember that in our class example, the demand curve for the resource remains the same in both periods.) Since the resource is much more valuable in period two, you want to extract more in that period even with discounting.

(c) The efficient price in each period equals the marginal benefit of using the resource in that period. Thus,

\[
\begin{align*}
p_1 &= \text{MB}_1 = 150 - 2q_1 = 121.42 \\
p_2 &= \text{MB}_2 = 300 - 2q_2 = 214.29.
\end{align*}
\]

(d) The marginal user cost in each period equals the marginal net benefit in that period. Thus

\[
\begin{align*}
\text{MUC}_1 &= \text{MNB}_1 = 150 - 3q_1 = 107.13 \\
\text{MUC}_2 &= \text{MNB}_2 = 300 - 2q_2 = 128.58.
\end{align*}
\]

You can verify that the marginal user cost increases at the rate of interest, 20%.

(e) Now the new marginal cost includes both the marginal extraction cost and the marginal environmental cost. Based on the new marginal cost \( \text{MC}^* \), we know the new marginal net benefits as:

\[
\begin{align*}
\text{MNB}_1^* &= 150 - 2q_1 - (2q_1 + 90) = 60 - 4q_1 \\
\text{MNB}_2^* &= 300 - q_1 - (2q_2 + 90) = 210 - 3q_2.
\end{align*}
\]
Again, we first see whether there is resource scarcity, that is, whether static efficiency is sufficient. Setting \( MNB_1^* = 0 \) and \( MNB_2^* = 0 \), we get \( q_1^* = 15 \), and \( q_2^* = 70 \). The total resource extracted is 85, which is less than the total stock. Thus when environmental cost is appropriately accounted for, we do not have a resource scarcity, and static efficiency is equivalent to dynamic efficiency.

The efficient prices again equal the marginal benefits. That is:
\[
\begin{align*}
p_1^* &= MB_1 = 150 - 2 \times 15 = 120 \\
p_2^* &= MB_2 = 300 - 70 = 230.
\end{align*}
\]
Since less coal is extracted due to environmental costs, the price of coal is now higher.

The marginal user costs are zero, since the resource is not scarce anymore. Note that MUC measures opportunity cost of extraction in a certain period. When there is enough resource, the opportunity cost is zero.

3. The time of starting moon and asteroid mining depends on the following factors: the cost (including both marginal cost and possibly fixed cost) of such operation, the marginal cost of mining on earth, and the availability of other substitutes with lower extraction costs. If substitutes exist in either production or consumption, we may never start mining the moon given the high cost. But if there are no substitutes (that is, the mineral is irreplaceable), and if economically feasible technologies have been developed, we will start moon mining at the point where the total marginal cost of extraction on earth (marginal extraction cost plus the marginal user cost) equals that on the moon.

If currently we know that a new technology has been developed that reduces greatly the cost of moon mining, we should increase our current resource use. That is, the price of the resource should drop now. Of course, the marginal user cost still rises at the interest rate if the marginal extraction cost is independent of the resource stock.