1. (Factor movements) There are two countries, each producing one good (the same good) using land and labor with the following technology:

US: \( Q^{us} = 80(T^{us})^{1/2}(L^{us})^{1/2} \); Mexico: \( Q^{mex} = 40(T^{mex})^{1/2}(L^{mex})^{1/2} \)

Resource endowments are: \( T^{us} = 100; L^{us} = 64; T^{mex} = 100; L^{mex} = 64 \). Without factor movements, labor employment within an economy equals labor endowment.

(a) **Find and sketch labor demand, and calculate autarky factor prices and per capita income:**

From profit maximization, labor demand is solution to: \( P \cdot MPL \equiv P(\frac{\partial Q}{\partial L}) = w \). This implies:

For US: \( \frac{w}{P} = \frac{\partial Q^{us}}{\partial L} = 40(T^{us})^{1/2}(L^{us})^{-1/2} \rightarrow L^{us,d} = \frac{1600 \cdot T^{us}}{(w/P)^2} \)

For Mexico: \( \frac{w}{P} = \frac{\partial Q^{mex}}{\partial L} = 20(T^{mex})^{1/2}(L^{mex})^{-1/2} \rightarrow L^{mex,d} = \frac{400 \cdot T^{mex}}{(w/P)^2} \)

The above figure shows the labor demand and given the fixed labor supply, the equilibrium wage. Thus
US: \( L^d = L^* \rightarrow 64 = \frac{1600 \cdot 100}{(w/P)^2} \rightarrow \left( \frac{w^*}{P} \right) = \left( \frac{1600 \cdot 100}{64} \right)^{1/2} = 50 \)

Mexico: \( L^d = L^* \rightarrow 64 = \frac{400 \cdot 100}{(w/P)^2} \rightarrow \left( \frac{w^*}{P} \right) = \left( \frac{400 \cdot 100}{64} \right)^{1/2} = 25 \)

There are two ways to calculate the return to land. Either land is paid its marginal value product (so \( R = P(\partial Q/\partial T) \)) or land receives total output less what is paid to labor:

\[
\tilde{R}T = PQ - WL = P \left( \frac{Q - (w/P) L}{T} \right) \Rightarrow \tilde{R} = \frac{Q - (\partial Q/\partial L) L}{P} = P - \frac{\partial Q}{\partial L}
\]

Under constant returns to scale these two concepts of rent are the same: ie., \( T(\partial Q/\partial L) + L(\partial Q/\partial L) = Q \)

Thus:

\[
\frac{R^{us}}{P} = \left( \frac{\partial Q^{us}}{\partial T} \right) = 40 \left( L^{us} \right)^{1/2} \left( T^{us} \right)^{-1/2} = \frac{20 \cdot 64}{10} = 32; \quad \frac{R^{mex}}{P} = \left( \frac{\partial Q^{mex}}{\partial T} \right) = 20 \left( L^{mex} \right)^{1/2} \left( T^{mex} \right)^{-1/2} = \frac{160 \cdot 64}{10} = 16
\]

Finally, per capita income in each country is:

\[
US: \quad \frac{Q^{us}}{L^{us}} = 80 \left( \frac{L^{us}}{T^{us}} \right)^{1/2} = 800; \quad Mex: \quad \frac{Q^{mex}}{L^{mex}} = 40 \left( \frac{L^{mex}}{T^{mex}} \right)^{1/2} = 400
\]

Since resource endowments are the same in both countries, and the basic technology is the same except the US is twice as productive, all factor prices and income in the US are double their corresponding values in Mexico.

(b) Labor is free to move between countries, but takes the form of guest workers so the national population figures are unchanged:

(i) The number of workers in US is \( L^{us} + I \), in Mexico \( L^{mex} - I \). If wages are equalized:

\[
\frac{w^{us}}{P} = \left( \frac{\partial Q^{us}}{\partial L} \right) = 40 \left( T^{us} \right)^{1/2} \left( L^{us} + I \right)^{-1/2} = \frac{w^{mex}}{P} = \left( \frac{\partial Q^{mex}}{\partial L} \right) = 20 \left( T^{mex} \right)^{1/2} \left( L^{mex} - I \right)^{-1/2} \rightarrow
\]

\[
4 \left( L^{mex} - I \right) = \left( L^{us} + I \right) \text{ since: } T^{us} = T^{mex}. \text{ Hence:} \]

\[
l^* = \frac{4L^{mex} - L^{us}}{5} = \frac{192}{5} = 38.4 \rightarrow \left( L^{us} + I \right) = 102.4; \quad \left( L^{mex} - I \right) = 25.6
\]

(ii) As a result of the guest workers program, US wages fall and Mexican wages rise (and equalize) while the return on land in the US rises and the return on land in Mexico falls (so land returns move in the opposite direction in the two countries). The quantitative returns can be figured out by calculating the marginal product of each input in each country.

(iii) Before labor migration, US output and income were the same and: \( Q^{us} = Y^{us} = 800 \)

After labor migration, because there are more workers in the US, US output increases but some of that is paid to workers who return to Mexico so US output exceeds the income of US residents. Nevertheless, US income does increase since the increase in output exceeds the total wages paid to immigrants.
To calculate this, note that:

\[ W^{us} = 40 \left( \frac{T^{us}}{L^{us} + I} \right)^{1/2} \]

and thus:

\[ Y^{us} = Q^{us} - W^{us} = 80 \left( \frac{T^{us}}{L^{us} + I} \right)^{1/2} \left( L^{us} + I^* \right)^{1/2} - 40 \left( \frac{L^{us}}{L^{us} + I} \right)^{1/2} \cdot I^* = 40 \left( \frac{T^{us}}{L^{us} + I} \right)^{1/2} \left( 2L^{us} + I \right) \]

which increases as \( I \) increases. For the values \( T^{us} = 100, \ L^{us} = 64, \ I^* = 38.4 \) this yields:

\[ Y^{us} \approx 6577.5 \]

which is greater than output without immigration \( (= 6400) \). So, the immigration causes net US income to increase by 177.5, or per capita income by \( (177.5/64) \approx 2.77 \), a 2.77% increase.

(iv) If the guest workers become US citizens, then it is still possible to make everybody better off than they were before there was any immigration but if per capita income has to be the same for new citizens as for those living in the country before, then it is not possible to help everybody. Or, more succinctly, counting the new immigrants as part of the US population, US per capita income declines due to the immigration as:

\[ \left( \frac{Q^{us}}{L^{us} + I} \right) = 80 \left( \frac{T^{us}}{L^{us} + I} \right)^{1/2} \]

which is a decreasing function of \( I \).

(v) In discussing whether immigration benefits the US, one problem is that the definition of who the US is changes due to the immigration (I am not referring to cultural or language issues). Thus, as above, the previous US citizens and landowners can be made better off than they were before immigration and the immigrants can be made better off than they were in Mexico, but the average per capita income in the US – counting the new immigrants – does decline.

(c) If guest workers are paid more than their marginal product, then immigration could hurt the US. Specifically, if each guest worker receives a fixed payment of \( F \), then, using previous results:

\[ Y^{us} = Q^{us} - W^{us} = 40 \left( \frac{T^{us}}{L^{us} + I} \right)^{1/2} \left( 2L^{us} + I \right) - I \cdot F; \quad \left( \frac{\partial Y^{us}}{\partial I} \right) = 20 \left( \frac{T^{us}}{L^{us} + I} \right)^{1/2} \frac{I}{\left( L^{us} + I \right)^{3/2}} - F \]

A little immigration is sure to hurt the US and it is certain that, from the world’s perspective there will be too much immigration (if the goal is to maximize world output). If \( F > 9.63 \) any level of immigration hurts the US (you do not need to figure this last part out!)
2. Consider a small country (Thailand) with the following demand and supply curves for steel:

\[
\text{Supply} = 5(P_s - 300); \quad \text{Demand} = 4500 - 3P_s
\]

Assume Thailand can import steel at a given world price of: \( P_s = 400 \). Further, assume that Thailand imposes a tariff of \( t \) per unit of import.

a) **Show how: domestic price, consumption and production change as \( t \) increases. Also, calculate how consumer surplus, producer surplus, and government tariff revenue change as \( t \) increases.**

Given the world price, the tariff inclusive price of imports in Thailand is given by: \( P^d = 400 + t \); this will be the domestic price if it is less than the autarky price (i.e., if \( t \leq 350 \)). Note that for \( t > 350 \) the tariff is prohibitive, there are no imports, and the domestic price equals the autarky price of 750. Thus, assuming \( t < 350 \), we have:

\[
P^d = (400 + t); \quad D = 4500 - 3P^d = 3300 - 3t; \quad S = 5(P^d - 300) = 500 + 5t; \quad M = D - S = 2800 - 8t
\]

Consumption and imports fall, and production rises, as \( t \) increases. The changes in consumer and producer surplus are seen on the diagram on the next page. The increase in producer surplus is given by area \( \{400,A^*,A,(400+t)\} \), whereas the decrease in consumer surplus is given by: \( \{400,B^*,B,(400+t)\} \). Thus:

\[
\Delta PS = \frac{1}{2} t \cdot (500 + 500 + 5t) = 500t + \left(\frac{5t^2}{2}\right);
\]

\[
\Delta CS = -\frac{1}{2} t \cdot (3300 + 3300 - 3t) = -3300t + \left(\frac{3t^2}{2}\right)
\]

\[
TR = tM = t \cdot (2800 - 8t) = 2800t - 8t^2 \quad \text{is tariff revenue.}
\]

Thus, it is easily seen that producer surplus increases with the tariff, and consumer surplus decreases (for \( t < 350 \)), whereas tariff revenue increases with the tariff for \( t < 175 \), and then decreases thereafter.

Overall: \( \Delta Welfare = TR + \Delta PS + \Delta CS = -4t^2 \) so that the tariff lowers overall welfare.

(i) If \( t > 350 \), the tariff is prohibitive, no trade occurs and domestic price is 750.

b) **Compare the domestic equilibrium when \( t=200 \) to the case where there is no tariff, but there is an import quota of 1200 units.**

From part (a), with \( t=200 \), imports \( M = 2800 - 8t = 1200 \). Thus, a quota of 1200 and a tariff of 200 have identical effects on domestic price, consumption, production and imports. The only possible difference is the tariff revenue (which is 240,000 under the tariff). Under the quota, importers make 200 on each unit imported (the world price is 400 and the domestic price solves \( D - S = 6000 - 8P^d = 1200 \) which implies \( P^d = 600 \)). Hence, importers will earn excess profits of 1200*200=240,000 unless the quota licenses are auctioned off, in which case the two policies are identical.
c) Suppose Thailand subsidizes steel imports.

A subsidy is just a negative tariff, so for part (b) just let $t = -s$ and you get:

$$P^d = (400 - s); D = 4500 - 3P^d = 3300 + 3s; S = 5 \left(P^d - 300\right) = 500 - 5s; M = D - S = 2800 + 8s$$

(Note if $s > 100$ output is zero, not negative). So, as is apparent, the subsidy: (i) reduces domestic price; (ii) raises consumer surplus; (iii) lowers producer surplus; (iv) causes negative tax revenue – i.e., the government must spend money. And it reduces overall welfare. Going through the same exercises as for the tariff:

$$\Delta PS = \frac{-1}{2}s \cdot (500 + 500 - 5s) = -500s + \left(5s^2/2\right)$$

$$\Delta CS = \frac{1}{2}s \cdot (3300 + 3300 + 3s) = 3300s + \left(3s^2/2\right)$$

$$Govt\ Spending \left(= GS\right) = sM = s \cdot (2800 + 8s) = 2800s + 8s^2$$

$$\Delta Welfare = \Delta PS + \Delta CS - GS = -4s^2 < 0 \quad s \neq 0$$

i. Since the subsidy increases imports, there is no quota – that is, no restriction on imports – that will have the same effect. Of course, if the government decides how much to import (through a state trading agency, for example), then it can replicate the effects of an import subsidy.

ii. No, as seen above both lower welfare. The point is that for a small country with no market distortions (no inefficiencies) free trade is optimal and any divergence from free trade must lower welfare.
(d) Output depends on the price producers receive; for an importing country the price producers receive can be increased either by an import tariff, as above, or by direct production subsidies. Let \( P^f \) stand for the price the producer receives; then output is \( Q = 5(P^f - 300) \). To get an output of 1000 requires a producer price of 500: \( Q = 5(P^f - 300) = 1000 \rightarrow P^f = 500 \).

(i) So, a production subsidy of 100 is required to yield the required output since, with free trade, the firm sells its output at the (world) price of 400, and also receives the subsidy from the government:
\[
P^f = 400 + s = 500 \rightarrow s = 100.
\]
The change in producer surplus is the same as with a tariff of 100, and is the area \( \{400,A^*,A,400+t\} \), with \( t=100 \), in figure 1. The cost to the government (taxpayer) of the subsidy is \( s \cdot Q = 100,000 \).

\[
\Delta PS = 500t + \left(5t^2/2\right) = 50000 + 25000 = 75,000; \quad \text{Cost Subsidy} = 100,000; \quad \Delta W = -25,000
\]
The deadweight loss due to the subsidy is due to overproduction, which is area \( A^*EA \) in figure 1. But there is no deadweight loss due to changes in consumption because the production subsidy does not alter consumption.

(ii) From earlier, the deadweight loss with a tariff of 100 is: \( \Delta Welfare = -4t^2 = -40,000 \)
Thus, the tariff leads to a larger welfare loss – in order to meet the production target – than does the production subsidy. In addition to the deadweight loss due to overproduction, with a tariff there is the additional deadweight loss (due to reduced consumption) of area \( B^*FB \) in Figure 1.

(iii) The point is the policy that directly addresses the noneconomic objective is the best policy; hence, a production subsidy dominates a tariff to increase domestic output because the tariff also affects consumption. If the goal is to reduce consumption, this can be achieved by an import tariff (that raises domestic price for both producers and consumers) or by a consumption tax. Both reduce consumption, but the tariff has the (unwanted) effect of increasing output, which creates additional inefficiencies. Hence, the consumption tax is the best policy to reduce consumption of steel.

3. Next, consider the case of two large countries:

**US:**
\[
\text{Demand } = 180 - 10P^\text{us} \quad \text{Supply} = 20P^\text{us} \quad \text{where } P^\text{us} \text{ is the price of soybeans in the US;}
\]

**China:**
\[
\text{Demand } = 180 - 4P^\text{cs} \quad \text{Supply} = 6P^\text{cs} \quad \text{where } P^\text{cs} \text{ is the price of soybeans in China;}
\]

(a) **Find autarky prices:**

**US:**
\[
S^a = D^a = 30P^\text{us} - 180 = 0 \rightarrow P^a = 6
\]

**China:**
\[
S^c = D^c = 10P^\text{cs} - 180 = 0 \rightarrow P^c = 18
\]

(b) **Assuming free trade (no tariffs), find the equilibrium price and quantities traded.**

World equilibrium requires:
\[
(S^\text{us} - D^\text{us}) + (S^\text{cs} - D^\text{cs}) = 0 \rightarrow 30P^\text{us} + 10P^\text{cs} - 360 = 0
\]

Free trade implies: \( P^\text{us} = P^\text{cs} = P^w \) (world price).

Combining the above two equations implies: \( 40P^w - 360 = 0 \rightarrow P^w = 9 \)
US exports = Chinese imports = \( (S_{\text{us}}^{ax} - D_{\text{us}}^{ax}) = 30P_{s}^{ax} - 180 = 90 \)

(c) **Show how a US import tariff of \( t \) affects the volume of trade, prices in China and the US, and US welfare.** Who pays for the US tax? Explain.

Drop the subscript \( (s) \) for soybeans, for simplicity; the US export tariff implies: \( P^c = P^{ax} + t \) if the product is to be sold in both the US and China. This equation, together with the world supply = world demand equation implies:

\[
30P^{ax} + 10P^c - 360 = 0 \rightarrow 30P^{ax} + 10(P^{ax} + t) - 360 = 40P^{ax} + 10t - 360 = 0 \rightarrow P^{ax} = 9 - (t/4) \\
P^c = 9 + (3t/4)
\]

Thus, even though the US imposes the tax, only \( \frac{1}{4} \) is paid by US citizens while \( \frac{3}{4} \) is paid by Chinese citizens. There is “partial incidence” of the tax. To calculate the welfare consequences for the US consider the figure below:

(i) Because the tariff decreases US price, consumers gain area \( (9,A,A^*,(9-t/4) \) and producers lose area \( (9,B,B^*,9-t/4) \). Calculating these areas gives:

\[
\Delta CS = (t/4)(90 + 1.25t) = 22.5t + (5/16)t^2; \\
\Delta PS = -(t/4)(180 - 2.5t) = -45t + (5/8)t^2
\]

On the other hand government tariff revenue is \( tX = t(90 - 7.5t) \), which is area \( \{J,K,B^*A^*\} \). Hence, the overall welfare impact is:

\[
\Delta W = \text{Area}\{J,K,M,L\} - \text{Area}\{A,L,A^*\} - \text{Area}\{B,M,B^*\} = \text{Rectangle 3} - \text{Triangle 1} - \text{Triangle 2}
\]

The last two areas are the familiar losses due to overconsumption and underproduction in the US due to the export tariff. **What is new is the first area** – which represents the gains to the US because China is paying the US more for soybeans – i.e., the increased price times the amount exported at the price. This
gain to the US is a loss to China – a transfer from China to the US because China pays part of the US tax. In terms of the numbers given her:

\[ \Delta W^{US} = \text{Tariff Revenue} + \Delta CS + \Delta PS = 90t - 7.5t^2 + 22.5t + (5/16)t^2 - 45t + (5/8)t^2 = 67.5t - (105/16)t^2 \]

The US gains from any tariff such that \( \Delta W > 0 \rightarrow 0 < t < (72/7) \).

(ii) For \( t=4 \), US price falls by 1, Chinese price rises by 3, US consumer surplus increases by 95, US producer surplus decreases by 170, US exports are 60, US tariff revenue is 240 and the change in US welfare is: \( \Delta W = 95 - 170 + 240 = 165 \). Clearly the US gains from the export tariff.

The answer is different from that found in question 2 because in 2 world price was not affected by the tariff. In this problem, the US increases the world price of its exports through the tariff and this is beneficial to the US, though obviously detrimental to China. In essence, the US is exploiting its world monopoly power by using this export tariff.

(iii) What would happen if the import tariff of 4 were replaced by a quota of 60 units?

With an export quota of 60, if it binds, we have:

US: \( S^{US} - D^{US} = 30P_s^{US} - 180 = 60 \rightarrow P_s^{US} = 8; \quad \text{China: } D^C - S^C = 180 - 10P^C_s = 60 \rightarrow P^C_s = 12 \)

Thus, with US exports limited, there would be a price gap of 4 between US and Chinese prices. Whoever had the right to export from the US (the holders of the quota licenses) would make excess profits of 4. Thus, the only difference between the tariff of 4 and the export quota of 6 is that the government revenue under the tariff becomes excess profits for the exporters under the quota. If the quota is auctioned off, then the two policies are equivalent.

(iv) Find the US export tariff that maximizes US welfare.

From part (i) above we have: \( \Delta W = 67.5t - (105/16)t^2 \)

Maximizing with respected to \( t \):

\[ \frac{d(\Delta W)}{dt} = 135 \frac{2}{2} - \frac{105}{8}t = 0 \rightarrow t^* = \frac{36}{7} \approx 5.14 \]

As stated above, free trade is not optimal for the US because its policies affect world price. Thus, it has the ability to act like a monopolist on world markets. But, as with monopoly, even though the monopolist can increase profits by restricting output, the loss to consumers exceeds the gains to the firm – there is a deadweight loss. The same is true of the US export tariff – the loss to the Chinese exceeds the gains to the US, creating a deadweight loss from the tariff.

(d) Show how the US tariff affects Chinese welfare.

For the Chinese there is no tariff revenue, so the higher world prices they face results in lower welfare. Breaking it down by producers and consumers:
Impact on China of US export tariff

The US export tariff causes world price (and hence Chinese price) to increase from 9 to \( 9 + (3/4)t \). This causes Chinese consumption to fall, production to increase, and imports to decrease. The welfare impact on each group is:

\[
\begin{align*}
\Delta PS &= \text{Area}\{9, A^*, A, (9 + (3/4)t)\} = \frac{3t}{4} \left( 54 + \frac{9t}{4} \right) \\
\Delta CS &= -\text{Area}\{9, B^*, B, (9 + (3/4)t)\} = -\frac{3t}{4} \left( 144 - \frac{3t}{2} \right) \\
\Delta W^c &= \Delta PS + \Delta CS = -\frac{135t}{2} + \frac{45t^2}{16}
\end{align*}
\]

In terms of the figure, the deadweight loss to China is \( \text{Area}\{A^*, A, B, B^*\} = \text{Rectangle 3} + \text{triangle 4} + \text{triangle 5} \).

If you compare the figures for the US and China, you see that area “3” is a transfer from China to the US because of higher export prices, while triangles 1 & 2 for the US, and triangles 4 & 5 for China measure the overall inefficiency (or deadweight loss) due to the US policy. In terms of equations:

\[
\Delta W^{us} + \Delta W^c = -\frac{15t^2}{4} < 0
\]

i. A Chinese import tariff, by raising domestic prices in China, would reduce the demand for imports and thus lower the world price of soybeans. Thus, just as the US gains from an export tariff, China can gain from an import tariff. But note that, in each case, the benefits to the winner are smaller than the losses to the loser. Hence, it is possible that each country tries to act as a
monopolist (a monopsonist in the case of China, since it is a buyer), but by doing so both countries are worse off. However, neither country has the incentive to \textit{unilaterally} ends its tariff.

(c) \textbf{How does the tariff affect world welfare?}

As shown in part (d), it leads to a decline in world welfare because the volume of trade falls and a wedge is driven between production costs in the US and China, and the value to consumers in the two countries of soybeans.

i. \textbf{Why doesn’t the US unilaterally eliminate its tariff?}

The simple point is that what is good for the world as a whole need not be good for the US \textit{without} compensation. Thus, if the US unilaterally eliminates its tariff, the US loses even though China gains even more. Without compensation of some sort, the US will be unwilling to lower tariffs. This is one reason why tariffs are often reduced as a result of international agreements rather than lowered unilaterally by countries (especially for larger countries. Smaller countries, with no ability to affect world prices, do not have the same incentive to maintain trade barriers).

\[
D^c = 180 - 2P^c = 90 + (t + \theta); \quad S^c = 4P^c = 180 - 2(t + \theta); \quad X^c = 90 - 3(t + \theta)
\]