1. Consider a consumer who has the following (quasi-linear) utility function:

\[ U(x, y) = x + (100y - y^2) \]

where \((x, y)\) denotes the goods consumed by the individual. Let \(I\) denote the individual’s income, and \((P_x, P_y)\) denote the prices the individual pays for goods \(x\) and \(y\), respectively.

a) Write the individual’s budget constraint, set up the utility maximization problem and derive the individual’s demand functions for both goods (assume a solution where both goods are consumed).

i. Find the individual’s maximized utility by substituting the demand solutions back in to the utility function {this is called the person’s indirect utility function}.

ii. Discuss how an increase in the price of good \(y\) affects maximized utility and interpret. Take the (partial) derivative of the indirect utility function with respect to \(p_y\); what does this equal?

b) Currently a consumer, with income \(I=4000\), can buy goods at the prices \(P_x = 1, P_y = 40\). A new mall brings to town a discount store (Costco) which sells good \(y\) at the price \(P_y = 30\).

i. Find the consumer’s purchases at this price. Will the consumer be better or worse off as a result of the opening of this store?

ii. Suppose the store charges individuals a fee of \(SF\) to shop in the store. What is the maximum amount this individual would be willing to pay to shop in the store? Give a numerical answer.

iii. Using the demand curve, show graphically how to calculate your answer for part ii. What is this area called?

c) Return to the original situation with \(I=4000, P_x = 1, P_y = 40\). Assume there is no Costco, but the individual may join an online club that sells good \(y\) (CDs, for example) at the price of \(P_y = 30\). However, this club requires the individual to buy at least 45 units of good \(y\).

i. If there were no minimum purchase requirement, how many units of good \(y\) would the person want to purchase?

ii. Will the person be better off joining the club (and purchasing 45 units) or not joining? If she chooses to join, what is the maximum fee she would be willing to pay to join this club?

2. Consider a consumer with the utility function: \(U(x, y) = x \cdot y\). Given income, \(I\), and prices \((P_x, P_y)\):

a) Write the budget constraint for the individual, set up the utility maximization problem and derive the individual’s demand functions for the two goods.
i. For these preferences, how does an increase in income affect the demand for good \( y \)? How does this result differ from that for Problem 1?

b) As in problem 1, suppose initially \( I = 4000, P_x = 1, P_y = 40 \). Suppose Costco moves to town and the person can buy good \( y \) at \( P_y = 30 \).

i. How many units of the good will she purchase and will she be better or worse off as a result of the price decrease?

ii. What is the maximum amount the person would pay for the right to shop at Costco?

iii. Can you calculate your answer to (ii) as the area next to the (Marshallian) demand curve? Why or why not?

3. Consider a competitive firm with cost curve: \( TC(q) = 8 + 10y + \left( y^2 / 2 \right) \) where 8 is fixed cost.

a) Find the firm’s marginal cost curve and, given output price, \( p \), the firm’s short run supply curve.

b) Find the firm’s long run supply curve. (NOTE: In the short run, the firm must pay the fixed cost even if it produces no output. In the long run, the firm can shut down (\( y = 0 \)) and avoid the fixed cost).

c) Using the firm’s supply curve, show (graphically and numerically) how much production costs increase when the firm increases output from \( y = 10 \) to \( y = 20 \).

i. Use the cost curve to verify your answer.

d) The firm currently sells its output at a price of 30 but it has found new markets where it can sell at a price of 40. Assuming (competitive) profit maximization, use the supply curve to show how much profits increase due to this price change. (You can verify your result by using the original total cost curve and the definition of profits).

4. {Efficiency of markets}. Use the demand curve from question 1 and the supply curve from question 3 and find the equilibrium market price and output level. {In essence, you are assuming there are a lot of identical producers and consumers, but it is simpler to just work with one of each}.

a) Show how a tax of 15 per unit sold affects: (i)equilibrium consumer and producer price; (ii)equilibrium output; (iii)consumer surplus and (iv)producer surplus. Compare the total change in producer and consumer surplus to the tax revenue raised and discuss how the tax affects overall efficiency.

b) Redo part (i) above for a production subsidy of 15 per unit produced and sold. Show how the subsidy affects (i)consumer and producer price; (ii)output; (iii)consumer surplus and (iv)producer surplus. Compare the total change in consumer and producer surplus to the cost to the government (taxpayers) of the subsidy. If a tax lowers efficiency, can we conclude a subsidy raises efficiency? Explain.