1. (Simplified version of Heckscher-Ohlin model). Consider a country which can produce two goods: cloth (C) and food (F) using two inputs: labor (L) and land (T). Production of each good requires inputs to be used in fixed proportions as follows (these are called Leontief technologies):

To produce cloth: 3 units of labor and 1 unit of land are required for each unit of cloth.
To produce food: 1 unit of labor and 3 units of land are required for each unit of food.

Let $\bar{L}, \bar{T}$ represent the total amount of labor and land available in the economy, let $P_C, P_F$ denote the prices of output and let $W, R$ denote the prices of labor and land respectively.

a) Find the production possibility frontier (ppf) for this economy and sketch it (Hint: it is defined by two linear inequalities that restrict total labor, and total land, usage).

(i) Show how an increase in the supply of land shifts the ppf and the production point where both factors are fully used (in this simple version, there is a unique production point that represents full employment of both inputs).

b) Find input prices $\{W, R\}$ in terms of output prices, assuming both goods are produced and both factors fully used. (Hint: setting price equal to marginal cost for each good gives two equations in two unknowns; solve these equations for factor prices. For example, if $W$ is the wage rate and $R$ the rental rate on land, the cost to produce each unit of food is: $3R + W$. Thus, if food is produced: $P_F = MC_F = (3R + W)$).

(i) Show how an increase in the price of cloth will affect factor prices.

c) Assume two countries (the US and Mexico) have identical tastes and technology, but the U.S. has more land and Mexico has more labor. Assuming the relative demand (ratio of demand for cloth to demand for food) is independent of income, discuss how autarky goods prices and factor prices differ between the two countries, then discuss how trade will affect factor prices in each country. Will factor prices be equalized between the two countries?

d) Modify the above model by assuming US productivity in both sectors double (implying it requires half as much of each input in each sector). Thus, in the US

To produce cloth: 1.5 units of labor and 0.5 units of land are required for each unit of cloth.
To produce food: 0.5 units of labor and 1.5 units of land are required for each unit of food.

Mexican technology remains unchanged. Viewed in a Ricardian context, the US has an absolute advantage in both goods but a comparative advantage in neither good (due to technology).

i) Show how this doubling of productivity in the US affects its autarky output prices and factor prices.

ii) Assuming free trade between the US and Mexico, find: (i) the pattern of trade and (ii) how trade affects factor prices in each country. Does free trade lead to factor price equalization, given different technology?
2. (The Standard Trade Model, Chapter 5) Chapter 5 shows that the potential gains from trade do not depend upon the reason for trade. For this problem, assume there are two goods (C, F), two countries of roughly equal size (US, Japan), and that each country has a strictly concave production possibility frontier (standard case). Assume tastes are identical but the production technologies differ so that in autarky the US has the lower relative price of good F.

a) Assuming free trade, state the pattern of trade and show both countries (overall) gain from trade. Also show that – starting from autarky – it is possible to increase world output of both goods.

b) Suppose that, in both the U.S. and Japan, the production of clothing causes pollution, which damages only local citizens (i.e., pollution from Japanese clothing production damages only Japanese citizens, and pollution from U.S. clothing production damages only U.S. citizens.) Also assume neither government does anything to correct this pollution. Under this scenario must the U.S. gain from trade? Must the Japanese gain from trade? Explain your answer.

c) Assume no pollution. Show how a U.S. export subsidy on food affects world prices, Japanese welfare and U.S. welfare, as well as production within the U.S.

d) Suppose the U.S. both subsidizes food exports and taxes clothing imports. If the export subsidy and import tariff are set at the same rate, what is the overall effect of this policy?

3. (Labor migration, Ch. 7) The simplest model to use to think about labor migration is one where there is only one good and labor is the only (variable) input. To illustrate, assume food (the only output) is produced using both labor and land according to the following production function:

\( Q_f^{us} = 20 \left( T_f^{us} \right)^{1/2} \left( L_f^{us} \right)^{1/2} \)

where \( T_f^{us} \) is US land input and \( L_f^{us} \) is labor input in the US.

The amount of land in the U.S. is fixed at \( T_f^{us} = 100 \). Labor demand is found by equating the marginal value product of labor to the wage rate \( (W) \). For simplicity, let \( P_f = 1 \) so labor demand is:

\( MPL = \frac{\partial Q_f^{us}}{\partial L_f^{us}} = 10 \left( \frac{T_f^{us}}{L_f^{us}} \right)^{1/2} \) \( \rightarrow P_f \left[ 10 \left( \frac{T_f^{us}}{L_f^{us}} \right)^{1/2} \right] = W \rightarrow \left( L_f^{us} \right)^d = \frac{100T_f^{us}}{W^2} \)

Finally, the rent (or profits) earned by U.S. landowners is output minus payments to labor:

\( R = Q_f^{us} - WL_f^{us} = 20 \left( T_f^{us} \right)^{1/2} \left( L_f^{us} \right)^{1/2} - \left[ 10 \left( \frac{T_f^{us}}{L_f^{us}} \right)^{1/2} \right] L_f^{us} = 10 \left( \frac{L_f^{us}}{T_f^{us}} \right)^{1/2} \)

In the above, I have substituted for \( Q_f^{us} \) from equation (1) and \( W \) from equation (2).

a) Show how increases in the labor supply \( (L_f^{us}) \), given \( (T_f^{us}) = 100 \), affect the wage rate and the return to land owners (the rent on land).
b) Let the total domestic U.S. labor force be $L^u^s = 25$. Find the equilibrium output, wage rent and rent on land in the US, assuming full employment (so $L^m_f = L^u^s = 25$).

c) Assume the Mexican wage rate is 5 and that Mexican workers will move to the U.S., if allowed, as long as US wages are at least as high as in Mexico. Let $I$ denote the number of immigrant workers from Mexico, so the total number of workers in the US is: $L^u^s_f = (L^u^s_f + I) = (25 + I)$.

i. How does immigration from Mexico affect U.S. wages and the return on land? In the US who will favor immigration and who will oppose it?

ii. US net national income $(Y^n)$ is US output less wages paid to Mexican immigrants $(W^m)$:

$$Y^n = Q^n_f - W^m I = 20(100)^{1/2} (25 + I)^{1/2} - W^m I$$

If Mexican immigrants are paid the same wage as US workers, how does immigration affect US net income? Explain.

iii. Would US net income be higher if Mexican workers were paid the US wage or the Mexican wage? If employers have a choice, which wage would they rather pay?

iv. If you allow free immigration between the US and Mexico, what ultimately will happen to the wage difference between the two countries?

d) Suppose all workers in the US – native US workers and immigrant workers, receive free health insurance from the US government that is financed by a tax on landowners. If there were no restrictions on immigration, must the US gain (i.e., will US net national income increase) by allowing immigrant workers? Explain.

e) Finally, suppose the production function for food in Mexico is:

$$Q^m_f = 10\left(T^m_f\right)^{1/2} \left(L^m_f\right)^{1/2}$$

where $T^m_f = 100$, is the amount of fixed land in Mexico, $L^m_f$ is the number of workers employed in Mexico and the Mexican population is $L^{mex} = 25$.

i. Let $I$ denote the number of workers who move from Mexico to the US, so that the work force in the respective countries becomes:

$$L^u^s_f = (L^u^s_f + I) = (25 + I); \quad L^m_f = (L^m_f - I) = (25 - I)$$

Starting from $I=0$, how does immigration from Mexico to the US affect total world output $(Q^u^s_f + Q^m_f)$?

ii. Calculate the level of immigration that maximizes total world output.