1. Consider a simplified version of the Heckscher-Ohlin model with the following technology:

**To produce cloth**: three units of labor and one unit of land are required per unit output of cloth.

**To produce food**: one unit of labor and three units of land are required for each unit of food.

a) Find the production possibility frontier (ppf).

Let $Q_c, Q_f$ denote the outputs of good $C$ and $F$, respectively. The resource constraints are:

**Labor**: \[ 3Q_c + Q_f \leq L \] since the technology implies: $L_C = 3Q_c$ and $L_F = Q_f$

**Land**: \[ Q_c + 3Q_f \leq T \] since the technology implies: $T_C = Q_c$ and $T_F = 3Q_f$

The following figure shows the production possibility frontier for this economy; the points on, or below, the line labeled “labor constraint” insure that labor employed is no larger than available labor (with full employment on that line), while the line labeled land constraint has the same interpretation. For this simplified economy, the only output level where both inputs are fully employed is where the two lines intersect, at point $V$, where output is:

$$Q_c = \frac{3L - T}{8}; \quad Q_f = \frac{3T - L}{8}$$

The feasible production set is the region bounded by: \{0, (T/3), V, (L/3)\}, and the production possibility frontier is the line segments described by: \{(T/3), V, (L/3)\}.
(i) Show how an increase in the supply of land shifts the PPF.

An increase in land shifts the land constraint outward, as shown by the dotted line in the figure. The point \( W \) represents the new output level where both factors are fully employed (in this simple version, there is a unique production point that represents full employment of both inputs).  

**Note that an increase in \( T \) leads to an increase in output of the land intensive good (F) and a decrease in output of the labor intensive good –as described in class and in the text.**

b) Find input prices \( \{W, R\} \) in terms of output prices, assuming both goods are produced and both factors fully used.

Using the technology, since each unit of \( C \) requires 3 units of labor and 1 unit of land, costs are:  
\[ MC_c = 3W + R \]. Similarly, for good \( F \), which uses 3 units of land and 1 unit of labor:  
\[ MC_f = 3R + W \].  

If goods are produced, price must equal marginal cost; thus:  
\[ P_c = 3W + R; \quad P_f = 3R + W \]. Solving for \( R, W \) in terms of output prices yields:  
\[ W = \frac{3P_c - P_f}{8}; \quad R = \frac{3P_f - P_c}{8} \]

(i) Note that an increase in the price of good \( C \) (the labor intensive good) leads to a decrease in the real return on land (i.e., \( \frac{R}{P_f} \)) decreases as \( P_c \) increases, and hence \( \frac{R}{P_c} \) must also decrease) and an increase in the real wage – i.e., both \( \frac{W}{P_c} \) and \( \frac{W}{P_f} \) increase; this is why trade, in this model, benefits the owners of one factor and hurts the owners of the other factor. Similarly, an increase in \( P_f \) would cause \( \frac{R}{P_f} \), \( i = c, f \) to increase and \( \frac{W}{P_f} \), \( i = c, f \) to decrease).

c) Assuming the US is land abundant and Mexico is labor abundant (but they have identical tastes and technology), find the pattern of trade and discuss its consequences.

As shown above, given prices, an increase in the supply of land increases output of the land intensive good (F) and decreases output of the labor intensive good (C). At given prices, this will create an excess supply of good F and an excess demand for good C. Hence, as the stock of land increases within an economy, the equilibrium price of the labor-intensive good increases.

Thus, the autarky relative price of good \( C \) will be higher in the US than in Mexico. This, from the previous part, implies that the wage rate will be higher in the US and the return on land will be higher in Mexico (i.e., in autarky \( \left( \frac{P_c}{P_f} \right)^{us} > \left( \frac{P_c}{P_f} \right)^{mex} \rightarrow W^{us} > W^{mex} \) and \( R^{us} < R^{mex} \)

Thus, with trade, the US will export F (the land-intensive good) and import C (the labor-intensive good).  

As a result of trade, \( \left( \frac{P_c}{P_f} \right) \) falls in the US and increases in Mexico.  

But, from (b), this implies that the wage rate falls in the US and rises in Mexico, while the return on land (R) rises in the US and falls in Mexico.
Finally, if free trade equalizes commodity prices and both goods are produced in both countries, it must equalize factor prices (see equations determining factor prices in (b)), provided technology is the same in the two countries. This is the “factor price equalization theorem”.

d) Modify the above model by assuming US productivity in both sectors double:

Cloth requires: 1.5 units of labor and 0.5 units of land are required for each unit of cloth.
Food requires: 0.5 units of labor and 1.5 units of land are required for each unit of food.

i) Show how this doubling of productivity in the US affects its autarky output prices and factor prices.

In Ricardian terms, while the US has an absolute advantage (technologically) in both goods, there is no comparative advantage due to technology. To see this specifically, we can re-derive the production possibility frontier for the US:

Labor: (1a) \(3Q_c/2 + Q_f/2 \leq L\) since the technology implies: \(L_c = (3Q_c/2)\) and \(L_f = (Q_f/2)\)

Land: (2a) \(Q_c/2 + 3Q_f/2 \leq T\) since the technology implies: \(T_c = (Q_c/2)\) and \(T_f = (3Q_f/2)\)

This yields the full employment point of: \(Q_c = \left(\frac{3L - T}{4}\right)\); \(Q_f = \left(\frac{3T - L}{4}\right)\)

Thus, at full employment, output of both goods double and thus the relative supply is unchanged. Hence, if demand for both goods also doubles (because income doubles) – so that relative demand is unchanged, the doubling of productivity in both sectors will not affect autarky relative goods prices and hence will not affect the pattern of trade between the US and Mexico. Turning to input prices, using the logic of part (b) of the answer, for the US we have:

\[ P_f = MC_f = (3W/2) + (R/2); \quad P_c = MC_c = (W/2) + (3R/2); \]

solving for input prices (in the US) in terms of output prices we have:

\[ W^{us} = \frac{3P_c - P_f}{4}; \quad R^{us} = \frac{3P_f - P_c}{4}, \]

whereas for Mexico (from part b):

\[ W^{mex} = \frac{3P_c - P_f}{8}; \quad R^{mex} = \frac{3P_f - P_c}{8}. \]

Thus, we see that – given output prices – the doubling of productivity in both sectors in the US leads to a doubling of the real return to both factors.

ii) Under free trade between the US and Mexico, find: (i) the pattern of trade and (ii) how trade affects factor prices in each country.

Since the doubling of productivity in both sectors in the US leaves relative autarky prices unchanged, it follows that the pattern of trade is still determined by factor endowments – so the US will export food and Mexico cloth. Trade still lowers the real return to labor (raises the real return to land) in the US since it lowers the price of the labor-intensive good, whereas the opposite happens in Mexico. However, trade will not lead to factor price equalization because
technologies are different. As the above example shows, the real return to both factors will be twice as high in the US as in Mexico.

2. Consider the standard trade model with two goods (C, F) and two countries (US, Japan). Assume in autarky the US has the lower relative price of good F.

a) State the pattern of trade and show that both countries gain.

Since the autarky relative price of good F is lower in the US, under free trade the US will export food and import clothing. The demonstration that both countries gain from trade is shown in the picture below (which shows the impact from the US perspective).

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The point M represents the autarky production and consumption point for the US. The dashed line, which represents the world price under trade, shows that the world relative price of F is higher than the autarky relative price of F. As a result, US production moves to Q (more F produced, less C) and consumption occurs somewhere along the dotted line (TQT) – which represents balance of trade equilibrium. Since it is possible to consume more of all goods than in autarky (point M), the country must be potentially better off (though not everybody in the country must gain). The main point is that by allowing the country to trade at a different price than the autarky price, the change in production makes consumption points available to it that dominate the autarky consumption point.

The demonstration that world output of both goods can increase is as follows. In autarky:

\[
\frac{dQ_c}{dQ_f}^{US} = \left( \frac{P_f}{P_c} \right)^{US} \left( \frac{P_f}{P_c} \right)^J = \left( \frac{dQ_c}{dQ_f} \right)^J \] which means that the opportunity cost of producing food is lower in the US in autarky. Thus, by having the US produce more food (less clothing) and Japan
more clothing (less food) it is possible to increase world output of both goods. This rationale is just the same as for the Ricardian model, except because production costs were constant in the Ricardian model, the logic implies that – in the Ricardian model – at least one country will specialize.

b) Suppose that, in each country, production of clothing causes pollution – which damages only local citizens. If there is no government policy, this means that in autarky good C is overproduced and its price is lower than it should be (if efficiency prevailed). This is because firms do not account for the pollution costs they impose on others, but they should.

Hence, the country that imports C (the US) gains from trade for two reasons – the usual reason for gains from trade PLUS the fact that trade lowers domestic pollution (as output of C falls). On the other hand, the country that exports C (Japan) could gain or lose from trade. The trade – given pollution – benefits Japan for the usual reason. However, because trade causes output of good C to increase, and because it was overproduced in autarky, Japan could lose (from the higher pollution levels). There, if there are market failures, you cannot be sure trade is beneficial.

c) The US export subsidy for food means that the domestic (relative) price of food in the US will exceed the world price: \( P^f_{us} = P^w_{world} (1 + s) \), where \( s \) is the subsidy rate (%). For Japan, the domestic price equals the world price. Given the world price, US food exports increase (because the US price rises); this decreases world food prices. Japan, an importer of food, gains from the US export subsidy. The US, on the other hand, loses for two reasons – given world prices, the subsidy reduces US welfare and the lower world price, due to the subsidy, also hurts the US.

d) A US export subsidy tends to increase food production and exports, and raises the domestic relative price of food. A US import tariff raises the domestic relative price of clothing, and thus offsets the export subsidy. The overall impact depends on which is larger. Let \( s \) be the export subsidy rate (a %) and \( t \) the import tariff rate (also a %); then:

\[
\frac{P^f_{us}}{P^c_{us}} = \frac{P^w \left(1 + s\right)}{P^w \left(1 + t\right)} \qquad \text{and} \qquad P^w_{us} = P^w_{world} (1 + t)
\]

where \( P^w \) are world prices. Thus:

\[
\frac{P^f_{us}}{P^c_{us}} = \left( \frac{P^w}{P^w \left(1 + t\right)} \right) \left( \frac{1 + s}{1 + t} \right).
\]

If \( s = t \), then the two policies combined have no real effect.

3. (Labor migration) There is a single good, produced using land and labor. The amount of land in a country is fixed; labor may be mobile across countries. US technology is:

US: \( Q^us = 20 \left(T^us\right)^{1/2} \left(L^us\right)^{1/2} \);

Resource endowments are: \( T^us = T^us = 100; L^us = 25 \). Without factor movements, labor employment within an economy equals labor endowment.
(a) Show how increases in the US labor supply affect the wage rate and the rent on land.

From profit maximization, labor demand is solution to: \( P \cdot MPL = P \left( \frac{\partial Q}{\partial L} \right) = w \). Let \( P = 1 \); This implies:

\[
w = \left( \frac{\partial Q^m}{\partial L} \right) = 10\left( T_{f}^{1/2} \right) \left( L_{f}^{1/2} \right) = 10\left( T_{f}^{1/2} \right) \left( L_{f}^{1/2} \right)
\]

The above figure shows the labor demand and given the fixed labor supply, the equilibrium wage. As the labor supply increases (shifts to the right) it is fairly obvious that the real wage falls.

There are two ways to calculate the return to land. Either land is paid its marginal value product (so \( R = P (\partial Q/\partial T) \)) or land receives total output less what is paid to labor.

\[
\bar{R}T = PQ - WL \rightarrow \bar{R} = P \left( \frac{Q - (w/P)L}{T} \right) \rightarrow \bar{R} = Q - \left( \frac{\partial Q}{\partial L} \right) L
\]

Under constant returns to scale these two concepts of rent are the same: i.e., \( T (\partial Q/\partial T) + L (\partial Q/\partial L) = Q \). Thus:

\[
\frac{R^m}{P} = \left( \frac{\partial Q^m}{\partial T} \right) = 10\left( T_{f}^{1/2} \right) \left( T_{f}^{1/2} \right)
\]

Thus, as the US labor force increases the return to land increases. In the figure, in the initial situation the return to land is the area under the labor demand curve above the equilibrium wage – i.e., the area \( \{A, (w/P)^e, E\} \). When the labor supply increases, this area increases to \( \{A, (w/P)'^e, G\} \); the land rents increase both because landowners pay lower wages and also because the additional workers employed produce more than they are paid.
(b) With $L^{us} = L^{fs} = 25$, then

\[
\frac{w}{P} = 10 \left( T^{us} \right)^{1/2} \left( L^{us} \right)^{-1/2} = 10(100)^{1/2} (25)^{-1/2} = 20; \quad \frac{R}{P} = 10 \left( T^{us} \right)^{-1/2} \left( L^{us} \right)^{1/2} = 5
\]

Finally, US income is: $Y = Q = 20 \left( T^{us} \right)^{1/2} \left( L^{us} \right)^{1/2} = 20(100)^{1/2} (25)^{1/2} = 1,000$. Note that this is the same as adding up factor payments: $Y = WL^{us} + RT^{us} = 20 \times 25 + 5 \times 100 = 1,000$

(c) Let Mexican workers move to the US. If $I$ represents the number of immigrant workers, then:

$L^{fs} = (L^{us} + I) = (25 + I)$.

i. Clearly, as $I$ increases, US wages fall but the return on land rises. Landowners will favor immigration (for economic reasons) whereas workers will oppose it.

ii. If Mexican workers are paid the US wage, how does immigration affect the US? US net national income is output less wages paid to immigrants, thus:

\[
Y^{net} = Q - W^m \cdot I = 10 \left( T^{us} \right)^{1/2} (25 + I)^{1/2} - W^m \cdot I
\]

If the immigrants are paid the US wage, then: $W^m = 10 \left( T^{us} \right)^{1/2} (25 + I)^{-1/2}$

(for simplicity, we use output as the numeraire, so $P \equiv 1$). Simplifying:

\[
Y^{net} = 20 \left( T^{us} \right)^{1/2} (25 + I)^{1/2} - \left[ 10 \left( T^{us} \right)^{1/2} (25 + I)^{-1/2} \right] \cdot I = \left[ 10 \left( T^{us} \right)^{1/2} (25 + I)^{-1/2} \right] [50 + I]
\]

It is fairly easy to see that immigration must raise US income. Formally, you would differentiate $Y^{net}$ with respect to $I$:

\[
\frac{dY^{net}}{dI} = \left[ -5 \left( T^{us} \right)^{1/2} (25 + I)^{-3/2} \right] [50 + I] + \left[ 10 \left( T^{us} \right)^{1/2} (25 + I)^{-1/2} \right] = \left[ 5 \left( T^{us} \right)^{1/2} (25 + I)^{-3/2} \right] [I] > 0
\]

iii. Of course, from the US perspective, employers will be better off if the immigrant workers are paid the (lower) Mexican wage. And, given the number of immigrants, paying the guest workers a lower wage means a higher net income for the US (and lower for Mexico).

iv. If you allow free immigration between the two countries, and only economic factors dictate migration, then free immigration will equalize wages between the two countries.

(d) If all workers are given (free) health insurance paid by the US government, and if it is not financed by a tax on workers, then what happens is the cost to the US of employing an additional immigrant worker is the wage the worker receives PLUS the cost of the health insurance. Hence, it is possible that allowing immigration could lower the standard of living for the US since immigrant workers are receiving more than their marginal value product (it is like a wage subsidy).

Of course, in reality (but not in my question), these workers also pay taxes which would – at least
partly – offset the impact of “free” health insurance.

(e) Finally, suppose the production function for food in Mexico is:

\[ Q_f^m = 10 \left( T_f^m \right)^{1/2} \left( L_f^m \right)^{1/2} \]

where \( T_f^m = 100 \), is the amount of fixed land in Mexico, \( L_f^m \) is the number of workers employed in Mexico and the Mexican population is \( L_{\text{mex}} = 25 \).

i. Show how immigration affects total world output:

\[ Q^w = Q^{us} + Q^{mex} = 20 \left( T^{us} \right)^{1/2} \left( L^{us} + I \right)^{1/2} + 10 \left( T^{mex} \right)^{1/2} \left( L^{mex} - I \right)^{1/2} \]

where \( T^{us} = T^{mex} = 100 \) and \( L^{us} = L^{mex} = 25 \). Thus:

\[ Q^w = 20(100)^{1/2} (25 + I)^{1/2} + 10(100)^{1/2} (25 - I)^{1/2} = 200(25 + I)^{1/2} + 100(25 - I)^{1/2} \]

Verbally, since initially the marginal product of labor will be higher in the US, as labor moves from Mexico to the US world output must increase; this continues to occur until the marginal products are the same in the two countries. Hence:

\[
\frac{dQ^w}{dI} = 100(25 + I)^{-1/2} - 50(25 - I)^{-1/2} = 50(25 - I)^{-1/2} \left\{ 2 \left( \frac{25 - I}{25 + I} \right)^{1/2} - 1 \right\}
\]

Clearly, at \( I=0 \), this derivative is positive, indicating migration from Mexico to US increases world output.

ii. Maximizing world output entails setting \( \frac{dQ^w}{dI} = 0 \) (and checking that the second order condition holds, which it does). From above:

\[
\frac{dQ^w}{dI} = 0 \rightarrow 2 \left( \frac{25 - I}{25 + I} \right)^{1/2} = 1 \rightarrow 4(25 - I) = (25 + I) \rightarrow I = 15
\]

Thus, with \( I=15 \), \( L^{us} = 25 + 15 = 40 \), \( L^{mex} = 25 - 15 = 10 \) and the wage will be the same in both countries: \( W^{us} = 10(100)^{1/2} (40)^{-1/2} = 5(10)^{1/2} \); \( W^{mex} = 5(100)^{1/2} (10)^{-1/2} = 5(10)^{1/2} \)

Assuming no market failures, free immigration between the two countries will maximize world output.