

Problem Set 1 - Due Sept. 17, 2009  
(Producer and consumer surplus; review)

1. Consider a competitive firm with cost curve:  $TC(y) = 8 + 5y + (y^2/2)$  where 8 is fixed cost and  $y$  is the firm's output.
  - a) The firm's profits, as a function of output and price, are:  $\pi(y) = py - TC(y) = \left\{ py - 8 - 5y - (y^2/2) \right\}$ .
    - i. Find the output level,  $y^*(p)$ , that maximizes profits in the short run (fixed costs must be paid).
    - ii. Show that this output level (supply curve) is the firm's marginal cost curve.
    - iii. Calculate the firm's *maximized profits* as a function of price (i.e., evaluate profits at the output level that maximizes profits):  $\pi^*(p) = py^*(p) - 8 - 5y^*(p) - \left( \frac{[y^*(p)]^2}{2} \right)$
  - b) Find the firm's *long run* supply curve. (NOTE: In the short run, the firm must pay the fixed cost even if it produces no output. In the long run, the firm can shut down ( $y = 0$ ) and avoid the fixed cost so that the firm will not produce if  $\pi^*(p) < 0$  ).
  - c) Using the firm's **supply** curve, show (graphically and numerically) how much production costs increase when the firm increases output from  $y = 35$  to  $y = 45$ .
    - i. Use the cost curve to verify your answer.
  - d) The firm currently sells its output at a price of 40 but it has found new markets where it can sell all of its output at a price of 50. Assuming (competitive) profit maximization, use the supply curve to show how much profits increase due to this price change. (You can verify your result by using the original total cost curve and the definition of profits).
    - i. Modify part (d) slightly by assuming the firm **must** continue to sell its original output of 35 units in the old market at a price of 40, but is allowed to sell additional units in this new (e.g., foreign) market at a price of 50. If it sold 10 units there at a price of 50 (and continued to sell the 35 units in its home market at a price of 40), how much would its profits increase?

2. Consider a consumer who has the following (quasi-linear) utility function:

$$U(x, y) = x + (110y - y^2)$$

where  $(x, y)$  denotes the goods consumed by the individual. Let  $I$  denote the individual's income, and  $(P_x, P_y)$  denote the prices the individual pays for goods  $x$  and  $y$ , respectively.

- a) Write the individual's budget constraint, set up the utility maximization problem and **derive the individual's demand functions** for both goods (assume a solution where both goods are consumed).
  - i. Find the individual's **maximized utility** by substituting the demand solutions back in to the utility function {this is called the person's *indirect utility function*}.

- ii. Discuss how an increase in the price of good  $y$  affects maximized utility and interpret. Take the (partial) derivative of the indirect utility function with respect to  $p_y$ ; what does this equal?
- b) Currently a consumer, with income  $I=3000$ , can buy goods at the prices  $P_x = 1, P_y = 60$ . A new mall brings to town a discount store (Costco) which sells good  $y$  at the price  $P_y = 40$ .
- i. Find the consumer's purchases at this price. Will the consumer be better or worse off as a result of the opening of this store?
- ii. Suppose the store charges individuals a fee of  $\$F$  to shop in the store. What is the **maximum** amount this individual would be willing to pay to shop in the store? Give a numerical answer.
- iii. Using the demand curve, show graphically how to calculate your answer for part ii. What is this area called?
3. {Efficiency of markets}. Use the supply curve from question 1 and the demand curve from question 2 and **find the equilibrium market price and output level**. {In essence, you are assuming there are a lot of identical producers and consumers, but it is simpler to just work with one of each}.
- a) Suppose you were a dictator and you could choose the output level and price that would maximize the sum of producer and consumer surplus. What output level would you choose? How does it compare to the equilibrium output level you calculated above?
- b) Next, assume the government gives producers a production **subsidy** of **30 per unit sold** {thus, if consumers pay a price  $p$  for the good, **producers receive  $(p+30)$**  for each unit produced and sold). Show how this subsidy affects: (i)equilibrium consumer price ( $p$ ) and producer price ( $p+30$ ); (ii)equilibrium output; (iii)consumer surplus and (iv)producer surplus. Compare the total change in producer and consumer surplus to the cost to the government (and hence taxpayers) of the subsidy, and discuss how the subsidy affects overall efficiency. **Show graphically this change in efficiency.**
- c) Suppose there is no subsidy but the government now permits trade with the rest of the world. The world price is **50**, and the country – since it is small – can import or export as much of the good as it wants at this world price. Starting from the original equilibrium (with no subsidy), show how international trade, at the world price of 50, affects consumer surplus, producer surplus, and overall efficiency. Show the net gain – or loss – from this trade graphically and give an economic explanation of why this area represents the overall welfare change from trade.