

Problem Set 3 - Due November 4, 2011

1. (Simplified version of Heckscher-Ohlin model). Consider a country which can produce two goods: cloth (C) and manufactures (M) using two inputs: labor (L) and capital (K). Production of each good requires inputs to be used in **fixed proportions** as follows (these are called Leontief technologies):

**To produce cloth (C):** 2 units of labor and 2 units of capital are required for each unit of C.

**To produce manufactures (M):** 2 units of labor and 4 units of capital are required for each unit of M.

Let  $\bar{L}, \bar{K}$  represent the total amount of labor and capital available in the economy, let  $P_c, P_m$  denote the prices of output and let  $W, R$  denote the prices of labor and capital respectively.

- a) Find production costs and hence output price (price=marginal cost) for each good in terms of factor prices  $(W, R)$ . {Hint: To produce  $Q_m$  units of  $M$  requires  $2Q_m$  workers and  $4Q_m$  units of capital and total costs are  $\{WL_m + RK_m\}$ .
- i. Use your answers to solve for inputs prices in terms of output prices. How will an increase in the price of cloth affect factor prices?
- b) Find the production possibility frontier (*ppf*) for this economy and sketch it. (**Hint:** it is defined by two linear inequalities that state labor (capital) used cannot exceed supply. Use the hint from part (a) and apply the same reasoning for factors used in the cloth sector).
- i. Show how an increase in the supply of capital shifts the *ppf* and the production point where inputs are fully used (in this simple version, there is a unique production point that represents full employment of both inputs).
- c) Assume two countries (the US and China) have identical tastes and technology, but the U.S. has more capital and China has more labor. Assuming the relative demand (ratio of demand for cloth to demand for manufactures) is independent of income, **discuss how autarky goods prices and factor prices differ between the two countries, then discuss how trade affect factor prices in each country. Will factor prices be equalized between the two countries?**
- d) Modify the above model by assuming US productivity in both sectors doubles. Thus, in the US:

**To produce cloth:** 1 unit of labor and 1 unit of capital are required for each unit of C.

**To produce manufactures:** 1 unit of labor and 2 units of capital are required for each unit of M.

Chinese technology remains unchanged. Viewed in a Ricardian context, the US has an absolute advantage in both goods but a comparative advantage in neither good (due to technology).

- i. Find how this US productivity change affects its autarky output prices and factor prices.
- ii. Assuming free trade between the US and China, what will the pattern of trade be? Will free trade equalize factor prices? Will trade eliminate the pressure for factor migration?

2. (Factor movement problem). Consider a very simple model with two countries (US, Mexico) and only one good. Output of this good, in each country, depends upon technology, the amount of land, and the amount of labor in each country. Assume the following production technology:

$$\text{US: } Q^{us} = 40(T^{us})^{1/2}(L^{us})^{1/2}; \quad T^{us} = 225; \quad L^{us} = 100$$

$$\text{Mexico: } Q^{mex} = 30(T^{mex})^{1/2}(L^{mex})^{1/2}; \quad T^{mex} = 100; \quad L^{mex} = 100$$

In the above equations,  $T^{us}$  and  $T^{mex}$  represent the (usable) land area in each country, and  $L^{us}$  and  $L^{mex}$  represent the population (= number of workers initially) in each country.

- (a) For each country, find and sketch the labor demand curve. Also, calculate the equilibrium wage, return on land and *per capita* income in each country (all measured in terms of output), assuming no labor movements between the two countries is allowed.
- (b) Suppose the U.S. government decides to allow *some* Mexicans to work in the U.S., but it controls (or limits) this number by requiring that each Mexican worker obtain a work visa. Let  $V$  denote the number of work visas issued by the U.S. government, and let  $I$  denote the number of Mexicans who work in the U.S. Since a visa is required for Mexicans to work in the U.S.,  $I \leq V$ .

Suppose Mexicans decide where to work solely based upon where their income is higher, so that if visas are given away for free those who get the visa will choose to work in the U.S. if the U.S. wage is higher than the Mexican wage. Also, assume that, by law, any Mexican working in the U.S. must receive the same wage as a U.S. worker.

- i. Show how (i)U.S. wages; (ii)the return on land in the U.S.; (iii)Mexican wages; and (iv)the return on land in Mexico are determined as a function of the number of work visas ( $V$ ) issued by the U.S. How does total world output change as  $V$  increases. **Quantitative** answers are required.
- ii. If the number of visas is set very high, is it possible that not all are used? Explain.
- iii. **Calculate** how U.S. income, Mexican income, and world output change as  $V$  increases. Note that: U.S. income is U.S. output less wage income paid to Mexicans; Mexican income is Mexican output plus wages received by Mexican workers; and world output is the sum of output in each country. Thus:

$$Y^{us} = Q^{us} - W^{us} I = 40(T^{us})^{1/2}(L^{us} + I)^{1/2} - W^{us} I; \quad T^{us} = 225; \quad L^{us} = 100; \quad I \leq V$$

$$Y^{mex} = Q^{mex} + W^{us} I = 30(T^{mex})^{1/2}(L^{mex} - I)^{1/2} + W^{us} I; \quad T^{mex} = 100; \quad L^{mex} = 100$$

- iv. Suppose that the U.S. government auctions off the work visas, instead of giving them away for free. Let  $P$  be the price paid at auction for each visa, so that Mexicans who purchase a visa and work in the U.S. receive net income of  $(W^{us} - P)$ , while Mexicans who stay in Mexico receive net income of  $W^{mex}$ . Assuming Mexicans will work where net income is higher, show how: (1) $P$ ; (2)U.S. net income and (3)Mexican net income change as  $V$  increases. Since Mexican workers pay the U.S. government for the visa, U.S. and Mexican total income are now:

$$Y^{us} = Q^{us} - W^{us}I + PI = 40(T^{us})^{1/2}(L^{us} + I)^{1/2} - (W^{us} - P)I; \quad T^{us} = 225; \quad L^{us} = 100; \quad I \leq V$$

$$Y^{mex} = Q^{mex} + W^{us}I - PI = 30(T^{mex})^{1/2}(L^{mex} - I)^{1/2} + (W^{us} - P)I; \quad T^{mex} = 100; \quad L^{mex} = 100$$

- v. Does allowing unrestricted guest workers maximize U.S. income? Explain.
- (c) Suppose that each worker in the U.S. must pay a fixed fraction of his/her income in taxes, which are used to support national defense. Assume, for simplicity, that the guest workers receive no benefits from these taxes (no medical care, no schooling, etc.). If a guest worker program allows workers to freely choose where to work (and if Mexico has no such tax on workers), will free worker movement between the two countries maximize total output of the two countries? Explain.
3. **(Chapter 8, Trade Policy)** Consider a small country (e.g., Nicaragua) with the following demand and supply curves for sugar:
- $$\text{Supply} = 10P_s; \quad \text{Demand} = 180 - 2P_s$$
- Assume Nicaragua can export sugar at a **given world** price of:  $P_s = 40$ . Further, assume that Nicaragua imposes an **export tariff** of  $t$  per unit of export.
- (a) Show how: domestic price, consumption and production change as  $t$  increases. Also, calculate how consumer surplus, producer surplus, and government tariff revenue change as  $t$  increases. What happens to overall welfare in Nicaragua as  $t$  increases?
- i. If  $t > 25$ , what happens to Nicaragua's sugar exports?
- (b) Compare the domestic equilibrium when  $t = 15$  to the case in which there is an export quota of **120** units (but no tariff). How do exports, domestic price, production and consumption compare under the two plans? What happens to the tariff revenue? Which policy is better for the country?
- (c) Suppose, instead of an export tariff, the Nicaraguan government *subsidizes* exports at a rate of  $s$  per unit of export. Thus, for each unit of sugar exported, the exporter receives **40** from the foreign buyer **and** also receives  $s$  from the government, so the total revenue, per unit exported, to the exporter is **(40+s)**. Show how this export subsidy affects: (i) domestic price; (ii) consumer surplus; (iii) producer surplus; (iv) government expenditures on the subsidy; and (v) overall welfare.
- i. Is there any export quota that would have the same effect as the export subsidy?
- (d) Suppose the government's goal is to reduce domestic consumption of sugar, in order to cut back on obesity and diabetes. It can accomplish this goal with either an export subsidy or a consumption tax. Which is the better tool? Demonstrate your answer by comparing the welfare implications of an export subsidy of 15 and a consumption tax of 15.

4. (**No points** but a practice problem for people who want to see a more sophisticated version of the H-O model). Similar to question #1, suppose there are two goods (C and M) and two inputs, K and L. However, the technology for producing each good now allows smooth substitutability between the inputs. The production functions are:

$$Q_c = K_c^{1/3} L_c^{2/3}; \quad Q_m = K_m^{2/3} L_m^{1/3}$$

where  $\{K_c, L_c\}$  are the inputs (capital, labor) used in sector C and  $\{K_m, L_m\}$  are the inputs used in sector M. Let  $W$  denote the wage rate (price of L) and  $R$  the rental rate (cost of using K, capital). Finally, let  $P_c, P_m$  denote the output prices of goods C and M, respectively.

- (a) Derive the cost function for each good by minimizing cost for a given output level. *Which good is capital intensive? Why?* {Possible procedure for good C: Minimize  $(WL_c + RK_c)$  subject to the constraint  $Q_c = K_c^{1/3} L_c^{2/3}$ , which implies  $K_c = (Q_c)^3 (L_c)^{-2}$ ; thus, minimize:  $(WL_c + R(Q_c)^3 (L_c)^{-2})$  with respect to  $L_c$ . Use a similar procedure for good M).
- (b) **Given output prices**, show how an increase in the supply of labor changes output of each good. *Use your result to predict which good the labor-abundant country will export.*
- i. **Given output prices**, does the increase in input (labor) supply change factor prices? Explain.
- (c) Assuming both goods are produced (so that Price=Marginal Cost for each good), show how an increase in  $P_m$  will affect factor prices  $(W, R)$ . Does either factor price rise proportionally more than  $P_m$ ? If so, explain which one and why.
- (d) Use your answer to parts (b) and (c) to explain how trade will affect the distribution of income in a labor-abundant country (like China) and how it will affect the distribution of income in a capital-abundant country (like the U.S.). Will everybody gain from trade? Explain.
- (e) Assume the U.S. is capital-abundant. Which group in the U.S. is likely to favor import tariffs and which group is likely to oppose trade restrictions? Explain.