Econ 455 Lapan
Fall 2011

Answers - Problem Set 1
(Producer and consumer surplus)

1. Consider a firm with total costs:

$$TC(y) = \left\{10y + y^2\right\}; \quad y \geq 0$$

a) The firm's profits, as a function of output and price $\pi(y) = py - TC(y) = \left\{py - 10y - y^2\right\}$.

i. **Find the output level, $y^*(p)$, that maximizes profits.**

To find the output level, maximize profits with respect to output (y):

$$\frac{d\pi(y)}{dy} = \left\{p - [10 + 2y]\right\} = 0 \to \quad y^* = 0, \quad p \leq 10; \quad y^* = \left(\frac{p-10}{2}\right), \quad p \geq 10.$$

ii. Show that this output level (supply curve) is the firm's marginal cost curve.

The firm's marginal cost is: $MC = \frac{dTC}{dy} = 10 + 2y$ so:

$p = MC = 10 + 2y \to y^* = \frac{p-10}{2}, \quad p \geq 10$; $y^* < 0$ does not make economic sense so for $p < 10$, the firm does not produce any output $(p < MC)$.

iii. Use the firm's supply curve to calculate cost increase when output increases from 20 to 30
See figure on next page. The firm's supply curve is shown as MC in figure on next page
So, from class, the change in cost is the area under the MC curve between the two output levels:

$$TC(30) - TC(20) = Area\left\{20, A, B, 30\right\} = \left\{\int_{y=20}^{30} (10 + 2y)\, dy\right\} = 600$$

{You can ignore the integral, or terms in bracket, if it is not familiar}. The area under the MC is a trapezoid, with area base times average height (or break area up into a rectangle and triangle if that is easier). Doing the calculation yields 600.

If you use the cost curve: $TC(30) = 10*30 + (30)^2 = 1200;$

$TC(20) = 10*20 + (20)^2 = 600$ so $TC(30) - TC(20) = 1200 - 600 = 600$

iv. Use your answer to part (i) to calculate maximized profits:

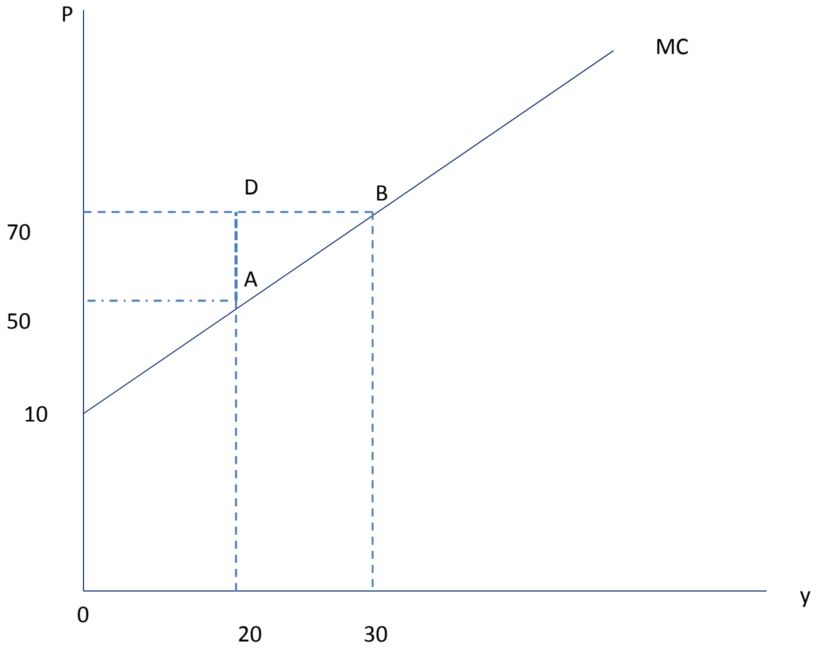
$$\pi^*(p) = py^*(p) - 10y^* - \left(y^*\right)^2 = (p-10)\left(\frac{p-10}{2}\right) - \left(\frac{p-10}{2}\right)^2 = \left(\frac{p-10}{2}\right)^2, \quad p \geq 10;$$

Profits are zero for $p \leq 10$ since *y* is zero.

b) Show how much the firm's profits increase when price increases from 50 to 70.
As discussed in class, this is the (change in) producer surplus, given by area {50,A,B,70}.
Hence, the change in profits is 500**.**

You can check the answer by using the maximized profit function. From earlier:

$$\pi^* = \left(\frac{p-10}{2}\right)^2; \ \pi^*(70) = \left(\frac{60}{2}\right)^2 = 900; \quad \pi^*(50) = 400; \ \text{Thus:} \quad \pi^*(70) - \pi^*(50) = 500$$



c) Modify part (b) by assuming the firm **must** continue to sell its original output of *20* units in the old market at a price of *50*, but is allowed to sell additional units in this new (e.g., a foreign) market at a price of *70*. If it sold 10 units there at a price of *70* (and continues to sell the 20 units in its home market at a price of 50), how much would its profits increase? **Give a numerical answer and show graphically how to calculate this answer.**

In this case the firm does NOT get the area {50,A,D,70} – which is what I called "windfall profits" in class. Hence the profit gain is only area ADB = 100.

Another way to understand the answer is that when it sells **an additional 10 units** at a price of 70, it gains revenue of 700 (area of rectangle {20,D,B,30}). But its costs increases by the area under the MC curve (which we previously calculated to be 600). **Thus, the additional output, at the higher price, increases profits by 100.**

2. A consumer has the (quasi-linear) utility function: $U(x, y) = x + 70y - (y^2/2)$. I is income, and (P_x, P_y) denote prices the individual pays for goods *x* and *y*, respectively.

a) Write the budget constraint, set up the utility maximization problem and derive the individual's demand functions (maximize utility subject to the budget constraint).

The budget constraint is: $I - P_x x - P_y y = 0 \rightarrow x = (I/P_x) - (P_y/P_x)y$

The simplest way to solve is by substituting the budget constraint into the utility function:

$U(x,y) = \dfrac{(I - P_y y)}{P_x} + 70y - (y^2/2)$ thus: $\dfrac{dU}{dy} = \dfrac{-P_y}{P_x} + (70 - y) = 0 \rightarrow y^d = 70 - \rho$ where $\rho \equiv \left(\dfrac{P_y}{P_x}\right)$, is the relative price of *y* (in terms of *x*). To guarantee both *x* and *y* are non-negative we requires $\rho \le 70,$ and $\dfrac{I}{P_x} \ge \rho y = \rho(70 - \rho)$. We assume both conditions hold.

An alternative way to get the same demand curves is to set the marginal rate of substitution equal to the price ratio:

$\left(\dfrac{MU_y}{MU_x}\right) = \left(\dfrac{P_y}{P_x}\right) \rightarrow \dfrac{(70 - y)}{1} = \left(\dfrac{P_y}{P_x}\right) = \rho$, and use the budget constraint to proceed as above.

i. **Find the individual's maximized utility by substituting the demand solutions back in to the utility function {this is called the person's *indirect utility function*}.**

Note that, with this solution, **maximized utility** is:

$$U^*(I, P_x, P_y) = \frac{I}{P_x} - \rho y^* + 70y^* - \left[(y^*)^2/2\right] = \frac{I}{P_x} + \frac{(70-\rho)^2}{2} \text{ since } y^* = (70-\rho).$$

The function above is called the "**indirect utility function**" and it shows the individual's maximized utility as a function of prices and income.

ii. **How does an increase in the price of good *y* affect maximized utility? Take the (partial) derivative of the indirect utility function with respect to p_y; what does this equal?**

An increase in P_y will make the person worse off, of course; for a finite change, we can measure this loss by the area next to the demand curve between the two prices. For a small price change, the loss is **approximated by** $y \cdot \Delta P_y$: that is, by the amount consumed times the (small) price change. Taking the (partial) derivative of the indirect utility function with respect to P_y:

$U^*(I, P_x, P_y) = \dfrac{I}{P_x} + \dfrac{(70-\rho)^2}{2}; \quad \dfrac{\partial U^*}{\partial P_y} = \dfrac{-(70-\rho)}{P_x}$. If we let $P_x = 1$ then this is the (negative) of the demand for good *y*. Formally: $y^* = \dfrac{-(\partial U^*/\partial P_y)}{(\partial U^*/\partial I)}$ - this is called Roy's identity (you do not need to know this).

b) **Currently a consumer with income $I = 5000$ can buy goods at prices: $P_x = 1, P_y = 50$. A new mall allows the person to buy good *y* at a price $P_y = 40$**

i. Find the consumer's purchases at this price. Will the consumer be better or worse off as a result of the opening of this store?

Using the demand curve $y^* = (70-\rho)$, at $P_y = 50,\ y^* = 20$; at $P_y = 40,\ y^* = 30$;
Clearly the consumer is better off to be able to buy at a lower price. If he (she) continued to purchase the same quantity (20) at a price of 40 instead of 50 he (she) would save **200.** Thus, the gain in consumer surplus must be at least 200 – i.e., the consumer would be willing to pay at least 200 to shop in the store. Of course, the actual amount would be higher.

ii. **If the store charges individuals *$F* to shop in the store, what is the maximum amount this individual would be willing to pay to shop in the store? Give a numerical answer**.

Using the indirect utility function, if the person pays a fee *F* to join the store, and buys good *Y* at the price $P_y = 40$ (his) her utility will be (since $P_x = 1$):

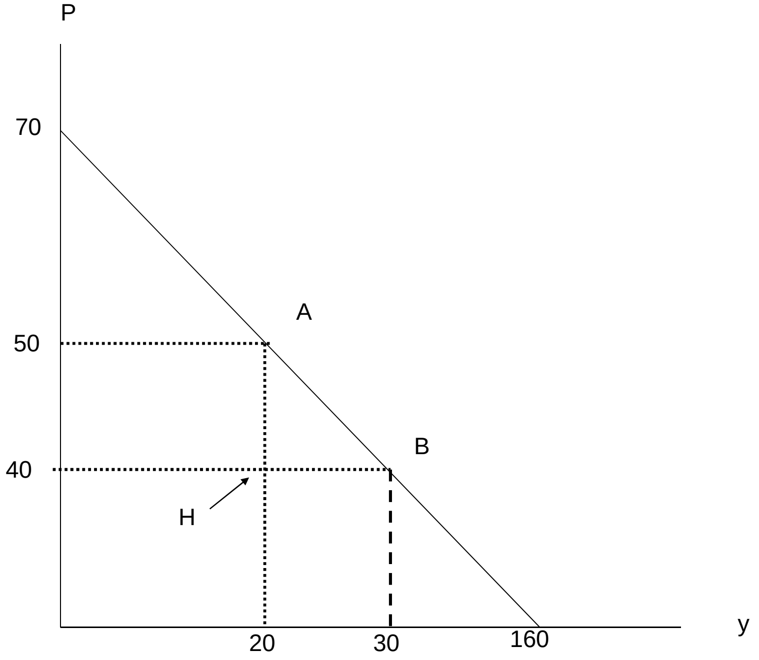
$$U^*\left(I-F, P_x, P_y\right) = \frac{I-F}{P_x} + \frac{(70-\rho)^2}{2} = 5000 - F + \frac{(70-40)^2}{2} = 5450 - F$$

Before the discount store, she paid a price of *50* and had utility of :

$$U^*\left(I, P_x, P_y\right) = 5000 + \frac{(70-50)^2}{2} = 5200$$

Thus, the maximum the person would pay is: $5450 - F = 5200 \rightarrow F^{\max} = 250$.

iii. Using the demand curve, show graphically how to calculate your answer for part ii. What is this area called?



There are two ways to calculate this gain. The simple way is to "remember" that the area next to the demand curve, between two prices, represents the change in "consumer surplus" which – assuming no income effect – represents the individual's maximum willingness to pay for a price decrease. ***Hence, for this example, the consumer surplus is the area of the trapezoid*** **{50,A,B,40} or**

Consumer surplus = $10\cdot\frac{(20+30)}{2}=250$, the same amount as calculated above.

The other way to calculate this gain (which is the way to prove that the area next to the demand curve actually measures consumer surplus) is to note the following:

The consumer gains from the price decrease in two ways:

1. The original quantity purchased (20) costs less, for a saving of $(50-40)\cdot 20=200$. *This is the windfall gain to the consumer, the amount she gains* ***if she does not change the amount purchased.***

2. The other gain is due to the increased purchases. The consumer increases purchases from 20 to 30 units; ***since the height of the demand curve (the marginal rate of substitution) gives the value to the consumer of each additional unit,*** the area **under** the demand curve, corresponding to the quantity change, represents the *maximum* amount the consumer would pay for that **additional quantity**. Hence:

Gross value additional consumption = Area of {20,A,B,30}= $10\cdot\left(\frac{40+50}{2}\right)=450$

Cost of additional consumption = $40\cdot 10=400$

Net gain from additional consumption = $450-400=50$

Thus, overall the consumer gains two ways: (1)by purchasing the original quantity at a lower price, and (2)by adjusting her purchases to reflect this new price. The total gain is: $200+50=250$ which is, of course, the same answer reached above.

3. {Efficiency of markets}. Use the supply curve from question 1 and the demand curve from question 2 and **find the equilibrium market price and output level for good *y*.** {In essence, you are assuming there are a lot of identical producers and consumers, but it is simpler to just work with one of each}.

The supply curve from question 1 is: $y^s=\frac{(p^s-10)}{2}$ where p^s is the price sellers receive

The demand curve from question 2 is: $y^d=70-p^d$; where p^d is the price consumers pay (and $P_x=1$)

If there are no taxes or subsidies then $p^s=p^d$ and equilibrium implies $y^s=y^d$. Thus, the equilibrium is: $y^s=y^d\rightarrow\frac{p-10}{2}=70-p\rightarrow p^e=50,\quad y^e=20$, where the superscript "e" stands for equilibrium.

a) **What output level and price would a dictator choose to maximize the sum of consumer and producer surplus.**

The height of the demand curve represents the value to the consumer of one more unit, whereas the supply curve represents the cost to the producer of one more unit. Thus:
$p^d=70-y$ is the marginal value of good *y* to the consumer and $p^s=((y-10)/2)$ is the marginal cost. Hence, the difference between the marginal value (MV) to the consumer minus the marginal cost (MC) to the producer is:

$MV - MC = \{70 - y\} - ((y-10)/2) = ((150 - 3y)/2)$.

Thus, for $y < 50$ increasing *y* increases the total surplus whereas for $y > 50$ decreasing *y* would increase the surplus. Thus, the optimal output level (in a closed economy) is **50 –** which is the competitive equilibrium.

b) Show how a government production subsidy of 30 per unit sold affects: (i)equilibrium consumer price (p) and producer price (p+30); (ii)equilibrium output; (iii)consumer surplus and (iv)producer surplus. Calculate the change in producer and consumer surplus and the cost to the government (taxpayers) of the subsidy. Discuss how the subsidy affects overall efficiency. Show graphically this change in efficiency.

The production subsidy effectively raises the price producers receive and hence shifts the supply curve down by the amount of the subsidy. Hence: $p^s = p^d + 30$. Dropping the superscript "d" (***p* stands for the consumer price**), the new supply curve is: $y^s = \frac{((p+30)-10)}{2} = \left(\frac{p}{2} + 10\right)$ since, when the consumer price is *p*, producers receive $(p + 30)$. In the graph below the new supply curve is the dotted positively sloped line. The new equilibrium is:

$$S = D \rightarrow \left(\frac{p}{2} + 10\right) = (70 - p) \rightarrow p' = 40; (p' + 30) = 70; \quad y' = 30$$

Thus, as a result of the subsidy:

(i)The consumer price falls from 50 to 40; BUT the price producers receive, including the subsidy, increases from 50 to 70.

(ii)Equilibrium output increases from 20 to 30

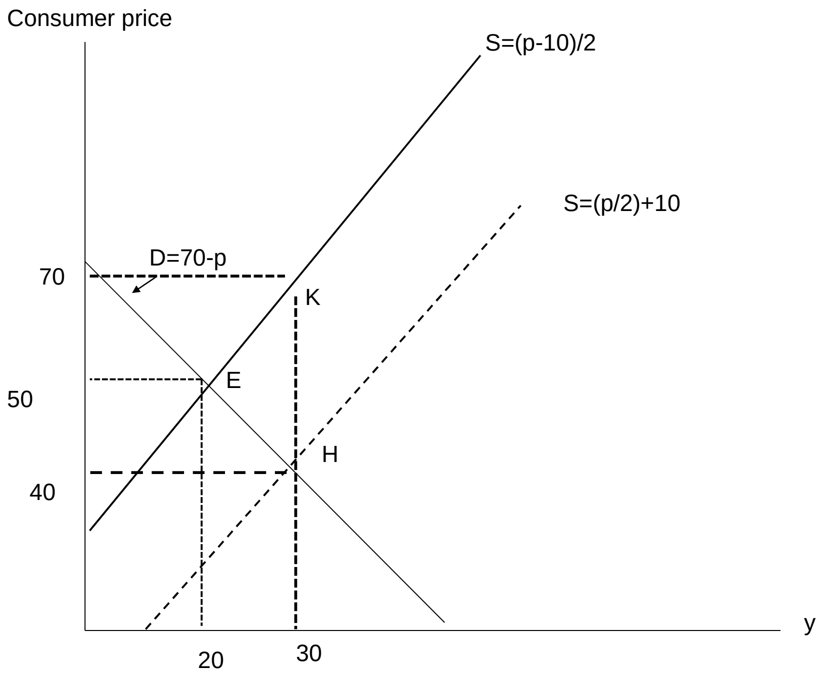
(iii)Consumer surplus increases, of course, because of the lower price. The increase in consumer surplus is area {50,E,H,40}= 250

(iv)Producer surplus increases because the price (including the subsidy) increases from 50 to 70; hence, the increase in producer surplus is area {50,E,K,70}= 500

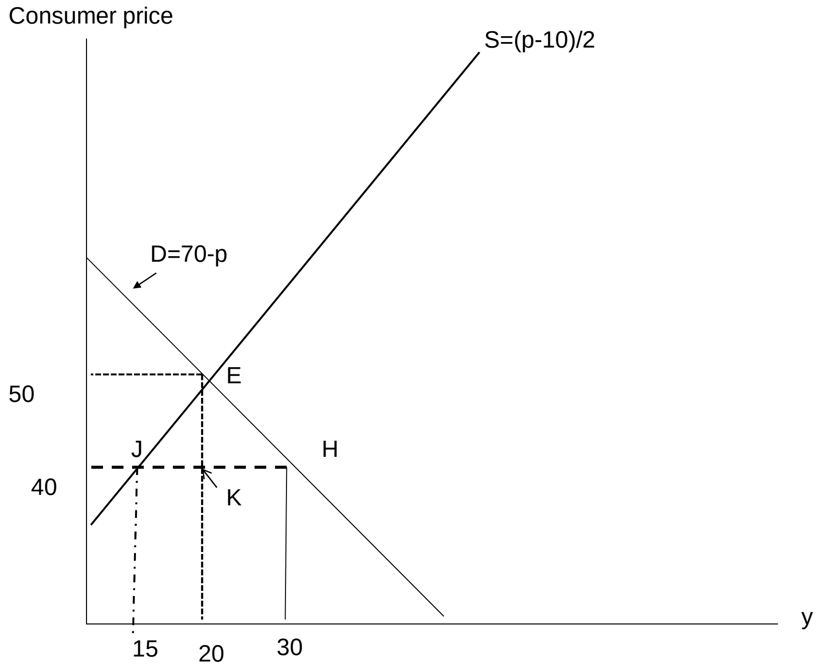
Does anybody lose? Sure, the government – or taxpayers – who have to pay for this subsidy. The cost of the subsidy is 30 times the output level or: **Subsidy Cost = 30*30=900**

Overall, then the net gain is negative = Increase Consumer Surplus + Increase Producer Surplus – Cost of Subsidy = **-150.**

This area is equal to the area of triangle {E,K,H}. Why? Because the area represents the amount by which the real costs associated with the extra 10 units of output exceed the value to consumers of that additional amount – i.e., the area between the marginal cost curve and the marginal value curve.



c) Suppose there is no subsidy but the government allows trade with the rest of the world. The world price is 40, and the country – since it is small – can import or export as much of the good as it wants at this world price. Show how international trade at $p^w = 40$ affects consumer surplus, producer surplus, and overall efficiency. Show the net gain – or loss – from this trade graphically and give an economic explanation of why this area represents the overall welfare change from trade.
(see next page)



If trade takes place at a world price of 40, and there are no import or export barriers the domestic price (for both producers and consumers) must fall to 40. Thus, consumer and producer price both decrease to 40 (from 50), consumption increases to 30, production decreases to 15, and imports are 15**.**
Producers lose area {50,E,J,40} = 175
Consumers gain area {50,E,H,40} =250
Increase in sum of producer and consumer surplus = area {J,H,E} = 75
This increase in welfare is associated with the **decrease in production and the increase in consumption due to trade.**

Output decreases from 20 to 15; the cost saving from reducing output is the area under the MC curve between y=15 and y=20, which is area {15,J,E,20}; while the cost of buying this output on the world market = area {15,J,K,20}. Hence, the **area of {J,E,K}** represents the gain (cost saving) from importing that output rather than producing it domestically.

Similarly, consumption increases from 20 to 30; the value to consumers of that consumption is given by the area under the demand curve, area of {20,E,H,30}. The cost to consumers of the additional output is world price (40) times the amount of increased consumption (10), or the area {20,K,H,30}. Overall, then, **given the lower price,** consumer**s gain area {K,E,H}** due to **increasing** consumption.

Hence, the overall gain from trade is area **{J,E,H}.** From an economic perspective, it is crucial to understand that the overall gain comes not because price changes (per se), but because quantity changes in response to this price change.