Fall 2011 Harvey Lapan
Econ 455

Answers - Problem Set 3 – Due November 4, 2011

1. Consider a simplified version of the Heckscher-Ohlin model with the following technology:

**To produce cloth (C): 2** units of labor and **2** units of capital are required for each unit of C.
**To produce manufactures (M): 2** units of labor and **4** units of capital are required for each unit of M.

a) Find production costs and output price in terms of factor prices.
Let *W* be the wage rate and *R* the rental rate on capital. The hint gives the answer for manufactures; just apply the same reasoning to cloth.

$$TC(M) = Q_m(2W+4R) \to MC(M) = (2W+4R); \quad \text{Price} = MC \to P_m = (2W+4R)$$

$$TC(cloth) = Q_c(2W+2R) \to MC(cloth) = (2W+2R); \quad \text{Price} = MC \to P_c = (2W+2R)$$

i. Find factor prices in terms of output price. Show how an increase in $P_c$ affects *W,R*.

From above: $P_c = (2W+2R); \quad P_m = (2W+4R)$.
This is like two linear equations in two unknowns; one can invert this relationship and solve for *R:*
$P_c = 2W + 2R \to R = \frac{P_c - 2W}{2}$; substitute this in the relationship for manufactures:
$P_m = (2W+4R) = \left(2W + 4\left(\frac{P_c - 2W}{2}\right)\right) = (2P_c - 2W)$ or $W = \frac{2P_c - P_m}{2}$
Use the solution for *W* to solve for *R*: $R = \left(\frac{P_c - 2W}{2}\right) = \frac{P_c}{2} - \left(\frac{2P_c - P_m}{2}\right) = \frac{P_m - P_c}{2}$. Summarizing:
$W = \frac{2P_c - P_m}{2}$; $R = \frac{P_m - P_c}{2}$ and for both goods to be produced $2P_c > P_m > P_c$
How does an increase in $P_c$ affect factor prices?

$$\frac{\partial W}{\partial P_c} = 1 > 0; \quad \left(\frac{\partial W}{\partial P_c}\right)\left(\frac{P_c}{W}\right) = \frac{2P_c}{2P_c - P_m} > 1; \quad \frac{\partial R}{\partial P_c} = \frac{-1}{2} < 0$$

**Thus¸**an increase in the price of cloth lowers the return to *capital* and raises the wage rate (since cloth is labor-intensive and manufactures are capital-intensive). Moreover, the wage rate rises by more (in % terms) than does the price of cloth so the real wage increases in terms of either M or cloth.

b) Find the production possibility frontier (*ppf*).

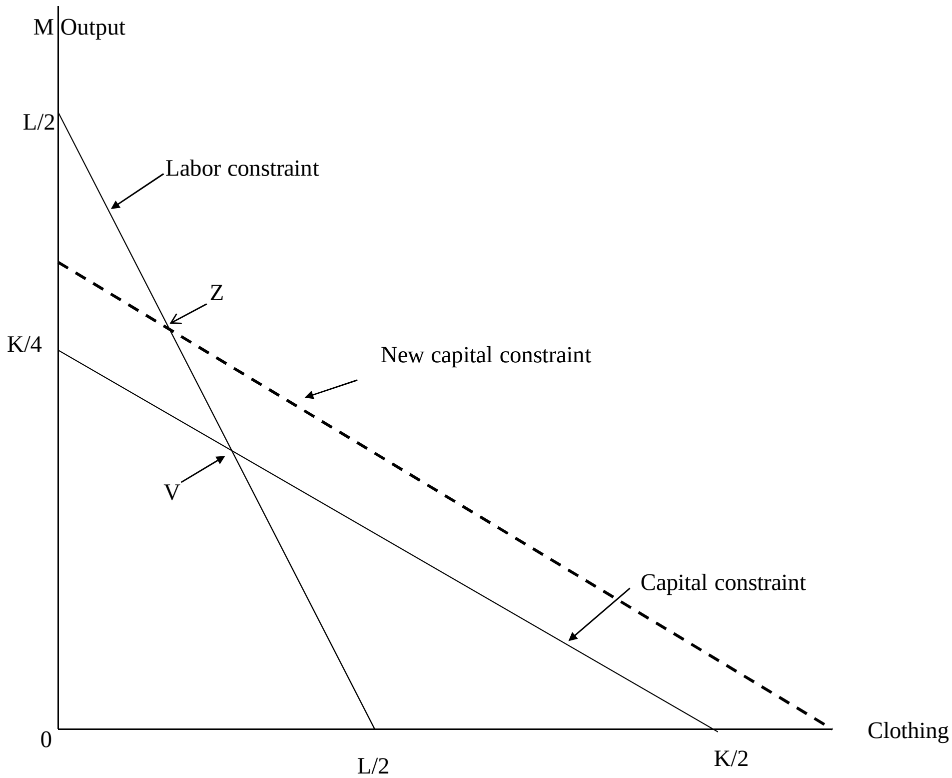
Let $Q_c$, $Q_m$ denote the outputs of good *C* and *M*, respectively. The resource constraints are:

**Labor: (1)** $2Q_c + 2Q_f \le \bar{L}$ since the technology implies: $L_c = 2Q_c$ and $L_m = 2Q_m$

**Capital: (2)** $2Q_c + 4Q_m \le \bar{K}$ since the technology implies: $K_c = 2Q_c$ and $K_m = 4Q_m$

The following figure shows the production possibility frontier for this economy; the points on, or below, the line labeled "labor constraint" insure that labor employed is no larger than available labor (with full employment on that line), while the line labeled capital constraint has the same interpretation. For this simplified economy, the only output level where both inputs are fully employed is where the two lines intersect, at point V, where output is:

$$Q_c = \frac{2\bar{L} - \bar{K}}{2}; \quad Q_m = \frac{\bar{K} - \bar{L}}{2}$$




The feasible production set is the region bounded by: {0,(K/4),V,(L/2)}, and the production possibility frontier is the line segments described by: {(K/4),V,(L/2)}.

(i)**Show how an increase in the supply of capital shifts the *ppf*.**

An increase in capital shifts the capital constraint outward, as shown by the dotted line in the figure. The point *Z* represents the new output level where both factors are fully employed (in this simple version, there is a unique production point that represents full employment of both inputs).
**Note that an increase in K leads to an increase in output of the capital intensive good (M) and a decrease in output of the labor intensive good –as described in class and in the text.**

**c) Assuming the US is capital abundant and China is labor abundant (but they have identical tastes and technology), compare autarky prices, then find the pattern of trade and discuss its consequences.**

As shown above, given prices, an increase in the supply of labor (in China) increases output of the labor intensive good (C) and decreases output of the capital intensive good (M). At given prices, this will create an excess supply of good C and an excess demand for good M. Hence, as the supply of labor increases within an economy, the equilibrium price of the labor-intensive good (cloth) decreases.

Similarly, an increase in the supply of capital (US) causes the equilibrium autarky relative price of the labor intensive good (cloth) to rise.

Hence, the autarky relative price of good C will be higher in the US than in China. This, from part (a) implies that the wage rate will be higher in the US and the return on capital will be higher in China (i.e., in autarky $\left(\frac{P_c}{P_m}\right)^{us} > \left(\frac{P_c}{P_m}\right)^{china} \rightarrow W^{us} > W^{china}$ and $R^{us} < R^{china}$

Thus, with trade, the US will export M (the capital-intensive good) and import C (the labor-intensive good). **As a result of trade**, $\left(P_c/P_m\right)$ falls in the US and increases in China.

But, from (a), this implies that the **wage rate falls in the US and rises in China, while the return on capital (R) rises in the US and falls in China.**

Finally, if free trade equalizes commodity prices **and both goods are produced in both countries**, it must equalize factor prices (see equations determining factor prices in (a)), *provided technology is the same in the two countries*. This is the "factor price equalization theorem".

d) Modify the above model by assuming **US productivity** in both sectors double, while Chinese technology remains unchanged. In the US:

C**loth requires: 1** units of labor and 1 unit of capital are required for each unit of cloth.
**Manufactures requires: 1** unit of labor and 2 units of capital are required for each unit of M.

**i)** Show how this doubling of productivity in the US affects its autarky output prices and factor prices.

In Ricardian terms, while the US has an absolute advantage (technologically) in both goods, there is no comparative advantage due to technology. To see this specifically, we can re-derive the production possibility frontier for the US:

**Labor: (1a)** $Q_c + Q_m \leq \bar{L}$ since the technology implies: $L_c = Q_c$ and $L_m = Q_m$

**Capital: (2a)** $Q_c + 2Q_m \leq \bar{K}$ since the technology implies: $K_c = Q_c$ and $K_m = 2Q_m$

This yields the full employment point of: $Q_c = \left(2\bar{L} - \bar{K}\right);\quad Q_f = \left(\bar{K} - \bar{L}\right)$

Thus, at full employment, output of both goods double – and thus the ***relative*** supply is unchanged. Hence, if demand for both goods also doubles (because income doubles) – so that relative demand is unchanged, the doubling of productivity in both sectors will not affect autarky *relative* goods prices and hence will not affect the pattern of trade between the US and China.

Turning to input prices, using the logic of part (a) of the answer, for the US we have:

$P_c = MC_c = W + R;\quad P_m = MC_m = W + 2R$; solving for input prices (in the US) in terms of output prices we have:

$W^{us} = \left(2P_c - P_m\right);\quad R^{us} = \left(P_m - P_c\right)$, whereas for China (from part a):

$$W^{china} = \frac{2P_c - P_m}{2}\ ;\quad R^{china} = \frac{P_m - P_c}{2}$$

Thus, we see that – given **output prices** – the doubling of productivity in both sectors in the US leads to a doubling of the real return to **both factors**.

ii) Will free trade equalize factors prices and remove the pressure for factor migration?

Since the doubling of productivity in both sectors in the US leaves relative autarky prices unchanged, it follows that the pattern of trade is still determined by factor endowments – so the US will export *M* and China cloth. Trade still lowers the real return to labor (raises the real return to capital) in the US since it lowers the price of the labor-intensive good, whereas the opposite happens in China. *However,* trade will not lead to factor price equalization because technologies are different. As the above example shows, *if trade equalizes goods prices* and both goods are produced in both countries, then the real return to both factors will be twice as high in the US as in China. Thus, there will still be pressure for factor movements.

2. (**Factor movements**) There is a single good (e.g., food), produced using land and labor. The amount of land in a country is fixed; labor may be mobile across countries. US and Mexican technology and resources are:

US: $Q^{us} = 40\left(T^{us}\right)^{1/2}\left(L^{us}\right)^{1/2};\quad T^{us} = 225;\quad L^{us} = 100$

Mexico: $Q^{mex} = 30\left(T^{mex}\right)^{1/2}\left(L^{mex}\right)^{1/2};\quad T^{mex} = 100;\quad L^{mex} = 100$

a) For each country, find and sketch the labor demand curve. Also, calculate the equilibrium wage, return on land and *per capita* income in each country (all measured in terms of output).

$$US: MPLabor = \left(\frac{\partial Q^{us}}{\partial L^{us}}\right) = \left[\frac{40}{2}\left(T^{us}\right)^{1/2}\left(L^{us}\right)^{-1/2}\right] \rightarrow P\left[20\left(\frac{T^{us}}{L^{us}}\right)^{1/2}\right] = W \rightarrow \left(L^{us}\right)^{demand} = \left(\frac{20P}{W}\right)^2 T^{us}$$

US land rent: $R = \dfrac{PQ^{us} - WL^{us}}{T^{us}} = 20\left(L^{us}\right)^{1/2}\left(T^{us}\right)^{-1/2} = P\dfrac{\partial Q^{us}}{\partial T^{us}}$

**Since there is only one good, you can set $P = 1$,** since GNP, wages and rental rates are all measured in terms of this good. Evaluating at $T^{us} = 225,\quad L^{us} = 100$:

$$W^{us} = \left[20\left(\frac{225}{100}\right)^{1/2}\right] = \left[20\left(\frac{15}{10}\right)\right] = 30\ ;\quad R^{us} = 20\left(\frac{100}{225}\right)^{1/2} = \frac{200}{15} = \frac{40}{3}$$

Similarly for Mexico:

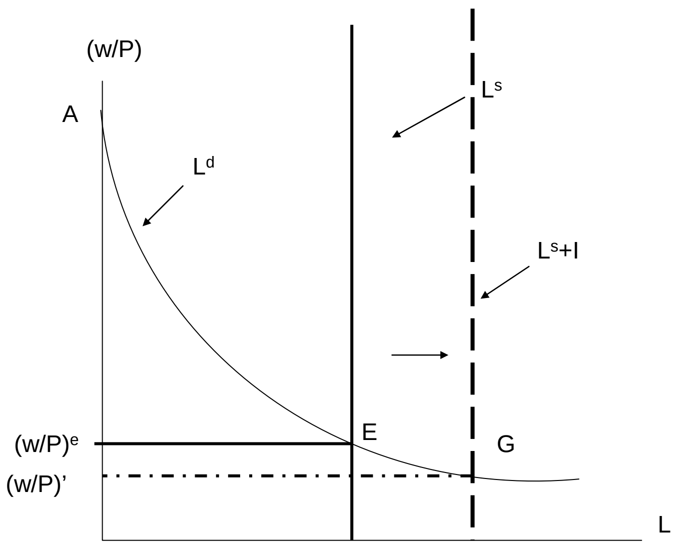
$$Mexico: \left(\frac{\partial Q^{mex}}{\partial L^{mex}}\right) = \left[15\left(\frac{T^{mex}}{L^{mex}}\right)^{1/2}\right] \rightarrow \left[15\left(\frac{T^{mex}}{L^{mex}}\right)^{1/2}\right] = W \rightarrow \left(L^{mex}\right)^{demand} = \left(\frac{15}{W}\right)^2 T^{mex}$$

Mexican land rent: $R = \dfrac{\partial Q^{mex}}{\partial T^{mex}} = 15\left(\dfrac{L^{mex}}{T^{mex}}\right)^{1/2}$

Evaluating at $T^{mex} = 100, \quad L^{mex} = 100$:

$$W^{mex} = \left[15\left(\frac{100}{100}\right)^{1/2}\right] = 15\,; \quad R^{mex} = 15\left(\frac{100}{100}\right)^{1/2} = 15$$

Below you see the labor demand curve for the US, and how immigration shifts the domestic labor supply. The labor demand curve for Mexico looks similarly; though emigration from Mexico (to the US) would reduce the labor supply in Mexico.




Wage Determination

The returns to land can be measured, in this diagram, as the "consumer surplus" since we can think of landowners as "renting" labor, and hence a lower labor price benefits landowners. In the diagram land rents, at the initial wage, would be given by {A,E,$(w/P)^e$}; if wages fall, as shown, land rents increase by the area {$(w/P)^e$,E,G,(w/P)'}

b) Assume the U.S. allows some Mexican workers to enter the U.S., but the number is limited by the number of visas issued (*V*). The actual number of guest workers has to be no larger than this $(I \leq V)$. Since only wages determine where people work, the actual number of guest workers (or immigrants) will equal the number of visas, as long as U.S. wages are at least as high as Mexican wages.

**For future purposes, let $L_{us} = L^{us} + I = 100 + I$ where $L_{us}$ is the number of people working in the U.S. and *I* are guest workers. Similarly, for Mexico $L_{mex} = L^{mex} - I = 100 - I$**

i. **First consider what would happen if there were no restrictions on labor movements; wages would be equalized so:** $W^{us} = \left[20\left(\frac{T^{us}}{L^{us}+I}\right)^{1/2}\right]; \quad W^{mex} = \left[15\left(\frac{T^{mex}}{L^{mex}-I}\right)^{1/2}\right]$

Where $L^{us}, L^{mex}$ is the population, and $\left(L^{us}+I\right), \left(L^{mex}-I\right)$ are the number of workers. Setting wages in the two countries equal to each other, and using the values for capital, we have

$$W^{us} = W^{mex} \rightarrow \left[20\left(\frac{T^{us}}{L^{us}+I}\right)^{1/2}\right] = \left[15\left(\frac{T^{mex}}{L^{mex}-I}\right)^{1/2}\right] \rightarrow \left(\frac{20}{15}\right)^2 \left(\frac{T^{us}}{L^{us}+I}\right) = \left(\frac{T^{mex}}{L^{mex}-I}\right) \rightarrow$$

$$\left(\frac{16}{9}\right)\left(\frac{225}{L^{us}+I}\right) = \left(\frac{100}{L^{mex}-I}\right) \rightarrow 4\left(L^{mex}-I\right) = \left(L^{us}+I\right) \rightarrow 5I = 4L^{mex} - L^{us} = 300; \quad I^* = 60$$

So, if $V < 60$ wages will not be equalized and the visa limit will be binding.

Thus, for $V < 60$, $I = V$ and we have

$$W^{us} = \left[20\left(\frac{T^{us}}{L^{us}+V}\right)^{1/2}\right]; \quad R^{us} = \left[20\left(\frac{L^{us}+V}{T^{us}}\right)^{1/2}\right] \rightarrow \frac{\partial R^{us}}{\partial V} > 0 > \frac{\partial W^{us}}{\partial V}$$

Similarly for Mexico,

$$W^{mex} = \left[15\left(\frac{T^{mex}}{L^{mex}-V}\right)^{1/2}\right]; \quad R^{mex} = \left[15\left(\frac{L^{mex}-V}{T^{mex}}\right)^{1/2}\right] \rightarrow \frac{\partial R^{mex}}{\partial V} < 0 < \frac{\partial W^{mex}}{\partial V}$$

In the US, US workers lose, landownders gain; the opposite happens in Mexico.

As for world output:

$$Q^{world} = Q^{us} + Q^{mex} = 40\left(T^{us}\right)^{1/2}\left(L^{us}+V\right)^{1/2} + 30\left(T^{mex}\right)^{1/2}\left(L^{mex}-V\right)^{1/2}$$

$$\frac{\partial Q^{world}}{\partial V} = \frac{\partial Q^{us}}{\partial L_{us}} - \frac{\partial Q^{mex}}{\partial L_{mex}} = 20\left(\frac{T^{us}}{L^{us}+V}\right)^{1/2} - 15\left(\frac{T^{mex}}{L^{mex}-V}\right)^{1/2}$$

$$= 20\left(\frac{225}{100+V}\right)^{1/2} - 15\left(\frac{100}{100-V}\right)^{1/2} > 0 \quad for \quad V < 60$$

As long as the marginal product of labor is higher in the US than in Mexico, the inflow of workers to the US will raise world output.

**ii.** As explained above, if $V > 60$, since it only takes 60 workers to equalize wages between the two countries, then not all the visas will be used. So, a sufficiently high visa quota is similar to having no restrictions at all.

**iii. Calculate how US and Mexican income change as V increases (the output change was done above).** From the definition of income in the problem set:

$$Y^{us} = Q^{us} - W^{us}V = 40\left(T^{us}\right)^{1/2}\left(L^{us}+V\right)^{1/2} - W^{us}V = 40\left(T^{us}\right)^{1/2}\left(L^{us}+V\right)^{1/2} - \left\{20\left(\frac{T^{us}}{\left(L^{us}+V\right)}\right)^{1/2}\right\}V$$

$$= 20\left(\frac{T^{us}}{\left(L^{us}+V\right)}\right)^{1/2}\left\{2\left(L^{us}+V\right)-V\right\} = 20\left(T^{us}\right)^{1/2}\left(L^{us}+V\right)^{-1/2}\left(2L^{us}+V\right)$$

**Hence:**

$$\frac{dY^{us}}{dV} = 20\left(T^{us}\right)^{1/2}\left\{\frac{-\left(L^{us}+V\right)^{-3/2}\left(2L^{us}+V\right)}{2}+\left(L^{us}+V\right)^{-1/2}\right\} = 10\left(T^{us}\right)^{1/2}\left(L^{us}+V\right)^{-3/2}V > 0$$

Another *mathematical* way to see the same thing is:

$$Y^{us} = Q^{us} - W^{us}V \rightarrow \frac{\partial Y^{us}}{\partial V} = \frac{\partial Q^{us}}{\partial L_{us}} - W^{us} - V\frac{\partial W^{us}}{\partial L_{us}} = -V\frac{\partial W^{us}}{\partial L_{us}} > 0; \quad \frac{\partial L_{us}}{\partial V} = 1 \text{ since}$$

$\frac{\partial Q^{us}}{\partial L_{us}} = W^{us}$ and $\frac{\partial W^{us}}{\partial L_{us}} < 0$. Thus, as more workers enter, they force down the U.S. wage which means what employers pay immigrant workers goes down. In this case the U.S. must gain from unrestricted immigration.

For Mexico, the situation is more complicated. The Mexican worker earns more in the U.S., but as more Mexicans work in the U.S. it drives down the wages those workers earn. Hence, Mexico will – in this situation – gain by limiting outmigration to the U.S. Formally:

$$Y^{mex} = Q^{mex} + W^{us}V \rightarrow \frac{\partial Y^{mex}}{\partial V} = \frac{-\partial Q^{mex}}{\partial L_{mex}} + W^{us} + V\frac{\partial W^{us}}{\partial L_{us}} = \left(W^{us} - W^{mex}\right) + V\frac{\partial W^{us}}{\partial L_{us}} < \left(W^{us} - W^{mex}\right)$$

since, for Mexico, $L_{mex} = 100 - V, \left(\partial L_{mex}/\partial V\right) = -1$. Since $\left(\partial W^{us}/\partial L^{us}\right) < 0$, Mexico's largest gain occurs when there is still a wage gap between the countries. If you substitute in the actual functional forms:

$$\frac{\partial Y^{mex}}{\partial V} == \left(W^{us} - W^{mex}\right) + V\frac{\partial W^{us}}{\partial L_{us}} = \left[20\left(\frac{T^{us}}{L^{us}+V}\right)^{1/2}\right] - \left[15\left(\frac{T^{mex}}{L^{mex}-V}\right)^{1/2}\right] + V\left[-10\frac{\left(T^{us}\right)^{1/2}}{\left(L^{us}+V\right)^{3/2}}\right]$$

$$= \left[20\left(\frac{225}{100+V}\right)^{1/2}\right] - \left[15\left(\frac{100}{100-V}\right)^{1/2}\right] + V\left[-10\frac{\left(225\right)^{1/2}}{\left(100+V\right)^{3/2}}\right] < 0 \quad at \quad V = 60$$

So Mexico would benefit by limiting migration – **given that the migrants receive the U.S.**

**wage.** If the migrant workers have to buy the visas, or are paid the Mexican wage, then the situation is reversed and the U.S. would have the incentive to limit (but not eliminate) inflows. As shown already world output would increase until marginal products are equalized.

**iv.** Now, if the visas are auctioned, the equilibrium price (value of the visa) is the wage gap:

$$P = \left(W^{us} - W^{mex}\right); \quad \frac{\partial P}{\partial V} = \left(\frac{\partial W^{us}}{\partial L_{us}}\right) + \left(\frac{\partial W^{mex}}{\partial L_{mex}}\right) < 0 \,.$$

Not surprisingly, the more visas sold, the lower their value. Furthermore,

$$Y^{us} = Q^{us} - W^{us}V + PV = Q^{us} - W^{mex}V; \rightarrow \frac{\partial Y^{us}}{\partial V} = \left(\frac{\partial Q^{us}}{\partial L_{us}}\right) - W^{mex} - V\left(\frac{\partial W^{mex}}{\partial L_{mex}}\right)\left(\frac{\partial L_{mex}}{\partial V}\right)$$

$$= W^{us} - W^{mex} + V\left(\frac{\partial W^{mex}}{\partial L_{mex}}\right) < \left(W^{us} - W^{mex}\right)$$

$$Y^{mex} = Q^{mex} + W^{us}I - PI = Q^{mex} + W^{mex}V; \quad \frac{\partial Y^{mex}}{\partial V} = \left(\frac{-\partial Q^{mex}}{\partial L_{mex}}\right) + W^{mex} + V\left(\frac{\partial W^{mex}}{\partial L_{mex}}\right)\left(\frac{\partial L_{mex}}{\partial V}\right)$$

$$= \left(-W^{mex}\right) + W^{mex} - V\left(\frac{\partial W^{mex}}{\partial L_{mex}}\right) > 0$$

The Mexican wage rises as V increases since there are fewer workers left in Mexico. Hence, in this case, Mexico gains as V increases while the U.S. gains from some migration but unrestricted migration will not maximize the U.S. gain.
World output always increases until wages are equalized, but whether both countries prefer limited or unrestricted migration depends on how this increased output is split between the countries.

**v.** As discussed above, whether unrestricted immigration is optimal for the U.S. depends on the net wages received by the guest workers. If they receive the U.S. wage and do not have to pay for visas, then unrestricted migration is best. If they receive the Mexican wage or have to buy the visa, then it pays to limit immigration. This latter argument is much like a monopoly argument for limiting output.

**c)** If guest workers pay taxes in the U.S. but not in Mexico, and if there are no limits on migration from Mexico to the U.S., then the equilibrium condition for Mexican migration looks like: $\left(W^{mex}\right) = \left(W^{us} - Tax\right)$. In this expression, think of "W" as representing the annual income of workers in each country (for the same number of hours of work) and *Tax* as the annual tax paid by guest workers. Now, in equilibrium, the U.S. wage will be higher than the Mexican wage, which means U.S. labor productivity will exceed Mexican labor productivity, which means that world output is not maximized. World output could be increased by larger migration from Mexico to the U.S.

3. **(Chapter 8, Trade Policy)** Consider a small country (Nicaragua) with the following demand and supply curves for sugar:

Supply = $10P_s$ ; Demand = $180-2P_s$

Nicaragua can export (or import) sugar at a **given world** price of: $P_s = 40$. Nicaragua has an export tariff of *t* per unit exports.

b) The autarky price in Nicarague is found from: $X = S - D = 12P_s - 180 = 0 \rightarrow P_s = 15$. So:

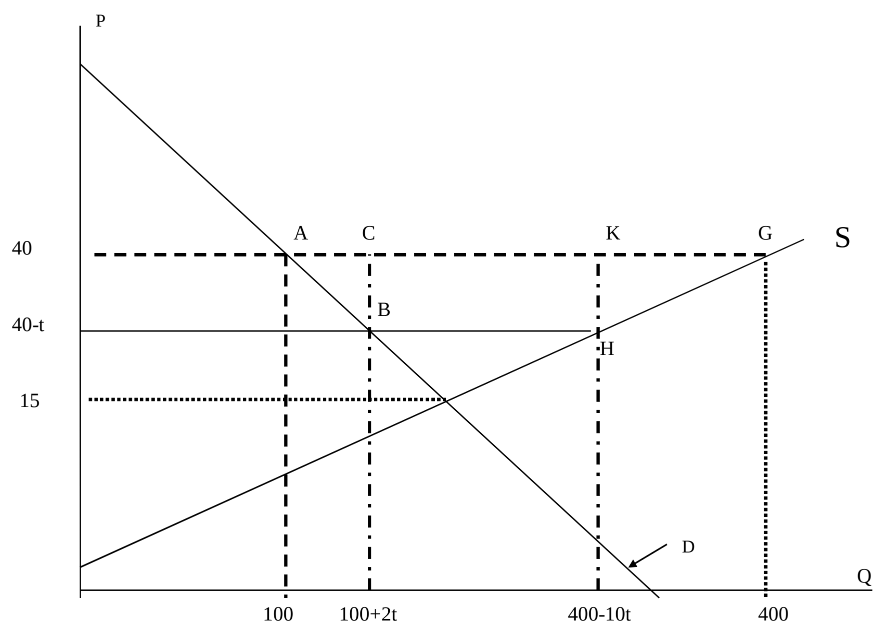
If $t \geq P^w - P^{aut} = 40 - 15 = 25 \rightarrow Exports = 0; \quad Q_s = D_s = 150, \quad P_s = 15$

If the tariff is less than 25, then trade will occur:

$t < 25; P_s = \left(P^w - t\right) = \left(40 - t\right) \rightarrow Q = 400 - 10t; D = 180 - 2\left(40 - t\right) = 100 + 2t; \quad X = Q - D = 300 - 12t$

So, as *t* increases, domestic price falls, production falls, consumption increases and exports fall.

To calculate the change in surpluses, it helps to use a graph:




The change in PS is area {40,G,H,40-t};
The change in CS is area {40,B,A,40}
Tax revenue is areas {C,K,H,B}. Hence

$\Delta PS = -\left(1/2\right)t \cdot \left(400 + 400 - 10t\right) = -400t + 5t^2; \qquad \Delta CS = \left(1/2\right)t \cdot \left(100 + 100 + 2t\right) = 100t + t^2$

$TR = tX = t \cdot \left(300 - 12t\right) = 300t - 12t^2$ is tariff revenue.

Producer surplus decreases with the tariff (only t<25 is relevant), consumer surplus increases, whereas tariff revenue increases with the tariff for $t < 12.5$, and then decreases thereafter. Overall:

$\Delta Welfare = TR + \Delta PS + \Delta CS = -6t^2$ so that the tariff lowers overall welfare.

(i)If *t>25*, the tariff is prohibitive, no trade occurs and domestic price is 15.

**b) Compare the domestic equilibrium when *t*=15 to the case where there is no tariff, but there is an export quota of 120 units.**

From part (a), with *t*=15, exports $M = 300 - 12t = 120$. Thus, an export quota of 120 and a tariff of 15 have identical effects on domestic price, consumption, production and exports; and hence they have identical effects on consumer and producer surplus. The only possible difference is the tariff revenue (which is 1800 under the tariff); under the quota, exporters make 15 on each unit exported and hence will earn excess profits of 1800, unless the quota licenses are auctioned off, in which case the two policies are identical. If the quotas are given to ***foreign*** importers, then the "revenue" from the tariff is lost to the country, and so the quota, in that case, would be inferior to the tariff for the exporting country.

c) Suppose the government *subsidizes* exports at a rate of ***s*** per unit of export. Show how this export subsidy affects Nicaragua (see Figure below):

(i)domestic price – domestic price for both consumers and producers increases to {40+s}

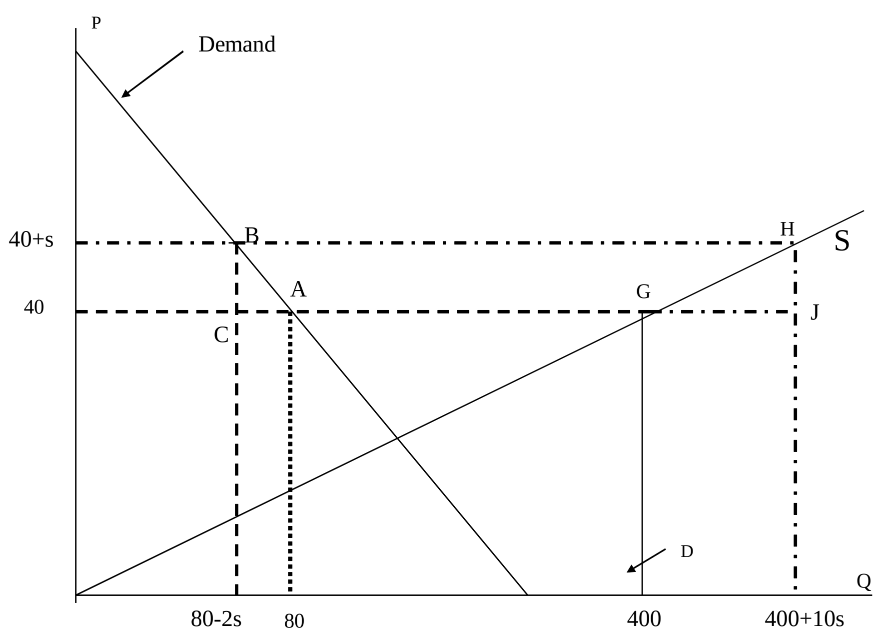
(ii)**Consumer surplus decreases -** due to the higher price - by area next to demand curve between two prices – area {40,A,B,(40+s)} in figure; **hence**: $\Delta CS = \frac{-s}{2}\{80 + 80 - 2s\} = \{-80s + s^2\}$

(iii)**Producer surplus** increases due to the higher price, by the area next to supply between two prices – area {40,G,H,(40+s)}. **Hence:** $\Delta PS = \frac{s}{2}\{400 + 400 + 10s\} = \{400s + 5s^2\}$

(iv)**Government expenditures**: Exports are: $S - D = 320 + 12s$

Hence: Cost to government = $s(320 + 12s) = (320s + 12s^2)$

(v)Impact overall welfare: $\Delta CS + \Delta PS$ – Government Expenditures = $-6s^2 < 0, \quad s \neq 0$




The deadweight loss is area ABC and area JGH.

i. No export quota can force exports to increase over the free trade level so there is no quantitative policy that has the same impact as an export subsidy.

d) **If the goal is to decrease sugar consumption,** an export subsidy accomplishes this by raising domestic price; but the export subsidy also causes domestic production to increase. With a consumption tax of 15, there is no change in producer surplus compared to free trade, whereas the change in consumer surplus is the same as with an export subsidy of 15. Thus, the welfare consequences of a consumption tax of 15 is:

$\Delta PS = 0$ **;**

$\Delta CS = -80s + s^2 = -1200 + 225 = -975$

**Tax Revenue =** $15 \cdot (50) = 750$

Welfare Loss = -225; this is area of triangle ABC.
(use part a, when s=15, to get the change in consumer surplus and consumption level). For the export subsidy, from part a, with s=15, we have

Welfare Loss with export subsidy = $-6s^2 = -1350$

Thus, the consumption tax which results in the same level of domestic consumption as the export subsidy has a lower welfare cost because we do not have the loss associated with producing more expensive sugar and selling it abroad at a price that is less than actual production costs. AND, of course, if the decreased sugar consumption lowers diabetes, and reduces health care costs, there is a positive benefit to offset the welfare loss (but this benefit is the same with the export subsidy and the consumption tax if it is due to reduced consumption).

4. **(No points –practice for those interested)** *(More sophisticated version of H-O model).* There are two goods (C and M) and two inputs (K and L). The production functions are:

$$Q_c = K_c^{1/3} L_c^{2/3}; \quad Q_m = K_m^{2/3} L_m^{1/3}$$

where $\{K_c, L_c\}$ are the inputs (capital, labor) used in sector *C* and $\{K_m, L_m\}$ are the inputs used in sector *M*. Let *W* denote the wage rate (price of L) and *R* the rental rate (cost of using K, capital). Finally, let $P_c$, $P_m$ denote the output prices of goods C and M, respectively.

(a) There are two ways to derive the cost function – one by substitution, and one by using non-linear programming (which involves using the Lagrangian function).

By substitution:

Let $Q_i = K_i^{\beta} L_i^{\alpha}$. Solving for labor yields: $L_i = \left(Q_i K_i^{-\beta}\right)^{1/\alpha} = \left(Q_i\right)^{1/\alpha} \left(K_i\right)^{-\beta/\alpha}$ .

Total costs are: $TC = WL_i + RK_i = RK_i + W\left(Q_i\right)^{1/\alpha}\left(K_i\right)^{-\beta/\alpha}$ (1)

Equation (1) expresses total costs as a function of factor prices, output and capital inputs. Cost minimization means choose the capital input that minimizes this expression. Thus:

$$\frac{d(TC)}{dK_i} = R - \left(\frac{\beta}{\alpha}\right) W \left(Q_i\right)^{1/\alpha} \left(K_i\right)^{-(\beta/\alpha)-1} = 0 \quad (2) \qquad \text{since: } \frac{d\left(K_i\right)^{\theta}}{dK_i} = \theta\left(K_i\right)^{\theta-1}$$

A sufficient condition for an interior minimum is that the first derivative of the function be zero and that the second derivative be positive; it is readily seen that the second derivative is positive. Solving (2) for $K_i$ yields:

$$K_i^* = \left(\left(\frac{\beta}{\alpha}\right)\left(\frac{W}{R}\right)(Q_i)^{1/\alpha}\right)^{\alpha/(\alpha+\beta)} = \left(\left(\frac{\beta W}{\alpha R}\right)^{\alpha} Q_i\right)^{1/(\alpha+\beta)} \quad (3)$$

where $K_i^*$ denotes the solution. Substituting back for $L_i$ yields:

$$L_i^* = \left(\left(\frac{\alpha R}{\beta W}\right)^{\beta} Q_i\right)^{1/(\alpha+\beta)} \quad (4)$$

Note that the choice of inputs depends on **relative** factor prices, not absolute factor prices. Also, note that when $(\alpha+\beta)=1$, the input use is proportional to output. Finally, substituting back into the cost function (1) yields minimum costs:

$$C^*(Q,W,R) = WL_i^* + RK_i^* = \lambda\left(W^{\alpha}R^{\beta}Q_i\right)^{1/(\alpha+\beta)} \quad \text{where: } \lambda = \frac{(\alpha+\beta)}{\left(\beta^{\beta}\alpha^{\alpha}\right)^{1/(\alpha+\beta)}}$$

(you need to substitute and then simplify the expression; it is a bit tedious, but you should get the result above):

This result can be checked because the derivative of the cost function with respect to input price should give you back the optimal input use. Hence:

$$\frac{dC^*}{dR} = \frac{\beta}{(\alpha+\beta)}\frac{(\alpha+\beta)}{\left(\beta^{\beta}\alpha^{\alpha}\right)^{1/(\alpha+\beta)}}R^{-\alpha/(\alpha+\beta)}W^{\alpha/(\alpha+\beta)}Q_i^{1/(\alpha+\beta)} = \left(\left(\frac{\beta W}{\alpha R}\right)^{\alpha} Q_i\right)^{1/(\alpha+\beta)}$$

which is $K_i^*$. Similarly, differentiating with respect to W gives you $L_i^*$:

$$\frac{dC^*}{dW} = \frac{\alpha}{(\alpha+\beta)}\frac{(\alpha+\beta)}{\left(\beta^{\beta}\alpha^{\alpha}\right)^{1/(\alpha+\beta)}}R^{\beta/(\alpha+\beta)}W^{-\beta/(\alpha+\beta)}Q_i^{1/(\alpha+\beta)} = \left(\left(\frac{\alpha R}{\beta W}\right)^{\beta} Q_i\right)^{1/(\alpha+\beta)}$$

Use of the Lagrangean function gives the same results, of course. Briefly, the Lagrangean is:

$$\mathrm{H} \equiv WL_i + RK_i + \theta\left(Q_i - L_i^{\alpha}K_i^{\beta}\right)$$

where $\theta$ is the Lagrangean multiplier. Partially differentiating yields, for an interior solution:

$$\frac{\partial \mathrm{H}}{\partial K_i} = R - \theta\beta K_i^{\beta-1}L_i^{\alpha} = 0 \quad (1a)$$

$$\frac{\partial \mathrm{H}}{\partial L_i} = W - \theta\alpha K_i^{\beta}L_i^{\alpha-1} = 0 \quad (2a)$$

$$\frac{\partial \mathrm{H}}{\partial \theta} = \left( Q_i - L_i^{\alpha} K_i^{\beta} \right) = 0 \qquad \text{(3a)}$$

Taking the ratio of (1a) to (2a) yields:

$$\frac{\beta L_i}{\alpha K_i} = \frac{R}{W} \rightarrow \frac{L_i}{K_i} = \left(\frac{\alpha}{\beta}\right)\left(\frac{R}{W}\right) \qquad \text{(4a)}$$

Hence, the labor intensity depends on factor prices and – in terms of the original production function – is increasing in the parameter on "L" and decreasing in the parameter on "K".

Using (4a) to solve for $L_i$ in terms of $K_i$, and then substituting this into (3a) yields the optimum capital input, which will be the same as above. Then, using this solution for capital, the solution for labor is found from (4a), and the cost curve by plugging back into the objective function.

You do not need to solve for $\theta$, but if you do you get the following from (1a)

$$\theta^* = \frac{R}{\beta} \frac{\left(K_i\right)^{1-\beta}}{\left(L_i\right)^{\alpha}} = \left( \frac{R^{\beta} W^{\alpha}}{\alpha^{\alpha} \beta^{\beta}} \right)^{1/(\alpha+\beta)} \left(Q_i\right)^{(1/(\alpha+\beta))-1} \qquad \text{(5a)}$$

Looking back at the cost function derived above and comparing to (5a), we see that (5a) represents the marginal cost function. This is no coincidence; the Lagrangean multiplier – in this problem – will always yield the marginal cost function.

Finally, for the specific functions given:

$$Q_c = K_c^{1/3} L_c^{2/3} \rightarrow \alpha = \frac{2}{3}, \beta = \frac{1}{3}, (\alpha + \beta) = 1 \text{ so: } TC_c^*\left(Q_c, W, R\right) = \lambda W^{2/3} R^{1/3} Q_c$$

$$Q_m = K_m^{2/3} L_m^{1/3} \rightarrow \alpha = \frac{1}{3}, \beta = \frac{2}{3}, (\alpha + \beta) = 1 \text{ so: } TC_m^*\left(Q_m, W, R\right) = \lambda W^{1/3} R^{2/3} Q_m$$

where: $\lambda = \dfrac{1}{(2/3)^{(2/3)} (1/3)^{(1/3)}} = \dfrac{3}{2^{(2/3)}}$

Clearly, good C is labor intensive and M is capital intensive as we have from (4a) above:

$$\frac{L_i}{K_i} = \left(\frac{\alpha}{\beta}\right)\left(\frac{R}{W}\right) \rightarrow \frac{K_c}{L_c} = \frac{\omega}{2}; \quad \frac{K_m}{L_m} = 2\omega \quad \text{where: } \omega = \frac{W}{R}$$

(b) **Given output prices**, show how an increase in the available supply of labor changes output.

From the cost curves above we have:

$$MC_c = \lambda R^{1/3} W^{2/3} = P_c \text{ and } MC_m = \lambda W^{1/3} R^{2/3} = P_m \quad \text{(1b)}$$

We can use these two equations to solve for factor prices in terms of output price. Taking the ratio of marginal costs and setting this equal to the price ratio (relative prices) yields:

$$\frac{MC_c}{MC_m} = \frac{\lambda W^{2/3} R^{1/3}}{\lambda W^{1/3} R^{2/3}} = \left(\frac{W}{R}\right)^{1/3} = \frac{P_c}{P_m} \to \ \omega = \rho^3 \quad \text{where: } \rho = \frac{P_c}{P_m}; \quad \omega = \frac{W}{R} \quad \text{(2b)}$$

Plugging this back into (1b) and solving gives the level of factor prices:

$$R = \frac{P_m^2}{\lambda P_c} \qquad W = \frac{P_c^2}{\lambda P_m} \qquad \text{(3b)}$$

From (3) and (4) in part (a) you have the optimal amount of inputs in each sector:

$$K_i^* = \left(\left(\frac{\beta W}{\alpha R}\right)^{\alpha} Q_i\right)^{1/(\alpha+\beta)} \to K_c^* = \left(\frac{\omega}{2}\right)^{2/3} Q_c; \quad K_m^* = (2\omega)^{1/3} Q_m \qquad \text{(4b)}$$

$$L_i^* = \left(\left(\frac{\alpha R}{\beta W}\right)^{\beta} Q_i\right)^{1/(\alpha+\beta)} \to L_c^* = \left(\frac{2}{\omega}\right)^{1/3} Q_c, \quad L_m^* = \left(\frac{1}{2\omega}\right)^{2/3} Q_m \qquad \text{(5b)}$$

You can express these input demands in terms of output price by substituting for $(\omega)$ in terms of $\left(\frac{P_c}{P_m}\right)$.

Doing so and writing the resource constraints yields:

$$L_c + L_f = L \to 2^{-2/3} \rho^{-2} Q_m + 2^{1/3} \rho^{-1} Q_c = L \qquad \text{(6b)}$$

$$K_c + K_f = K \to 2^{1/3} \rho Q_m + 2^{-2/3} \rho^2 Q_c = K \qquad \text{(7b)}$$

Given prices, equations (6b) and (7b) are just like problem #1 (i.e., the labor and capital use per unit output are fixed) and can be solved for output levels. Doing so yields:

$$Q_c = \left(2^{2/3}/3\right)\left(2\rho L - \rho^{-2} K\right); \quad Q_m = \left(2^{2/3}/3\right)\left(2\rho^{-1} K - \rho^2 L\right)$$

Thus, given prices, an increase in *K* will increase the output of good F, the capital intensive good, and decrease the output of good C. *Also, note that if output prices do not change, input prices do not change since, from equation (3b) above, factor prices can be determined in terms of only output prices.*

Thus, given prices, an increase in L causes the supply of good M to decrease and that of good C to increase (notice if both L and K double, output of each good doubles so the ratio of outputs only depends on (K/L)). But total income increases as L increases, so demand for both goods increases. Hence, to restore equilibrium, the price of good M must increase (relative price of C decreases). Thus, the autarky (relative) price of good C (the labor intensive good) is an increasing function of the country's relative capital abundance. Consequently, given the same demands and technology, **the labor abundant country** will export the labor intensive good (C) and import the capital-intensive good (M).

Further, since the wage rate decreases, and the rental rate increases, as the price of good C falls (i.e., the relative price of good M increases), this means that the autarky wage rate will be lower, and the autarky rental rate on capital higher, in the labor abundant country. Hence, differences in factor supplies lead to differences in autarky output prices, which lead to the differences in input prices one would expect.

i. As discussed above, **given output prices**, input prices are determined and hence changes if factor supplies will not change factor prices.

(c) To show how factor prices change with output prices, look back at equation (3b).

$$R = \frac{P_m^2}{\lambda P_c} \quad W = \frac{P_c^2}{\lambda P_m}$$

$$R = \frac{P_m^2}{\lambda P_c} \rightarrow \left(\frac{\partial R}{\partial P_m}\right) = \frac{2P_m}{\lambda P_c} \rightarrow \left(\frac{\partial R}{\partial P_m}\right)\left(\frac{P_m}{R}\right) = 2 > 1; \ \left(\frac{\partial R}{\partial P_c}\right) = \frac{-P_m^2}{\lambda P_c^2} < 0;$$

$$W = \frac{P_c^2}{\lambda P_m} \rightarrow \left(\frac{\partial W}{\partial P_c}\right) = \frac{2P_c}{\lambda P_m} \rightarrow \left(\frac{\partial W}{\partial P_c}\right)\left(\frac{P_c}{W}\right) = 2 > 1, \quad \left(\frac{\partial W}{\partial P_m}\right) = \frac{-P_c^2}{\lambda P_m^2} < 0$$

These results imply that an increase in $P_m$, the capital intensive good, lowers the real return to labor and **raises** the real return to capital in **terms of either good** (a 1% increase in $P_m$, increases the rental rate $R$ by 2%, and hence $(R/P_m)$ increases as $P_m$ increases). Similarly, an increase in the price of the labor intensive good (*C*) lowers the real return to capital and raises the real return to labor in terms of either good.

*The results show that, as* $P_c$ *increases, W increases proportionately more than* $P_c$ *(i.e.,* $(W/P_c)$ *increases with* $P_c$*).* This is the *Stolper-Samuelson result.*

(d) From part (c), it is apparent that the impact of trade on the distribution of income depends upon how trade affects the relative prices of goods.

If a country exports good M (a capital abundant country), then trade causes the relative price of M to increase, and thus trade increases the real return to capital and lowers the real return to labor.

If a country exports good C (a labor abundant country, such as China), then trades raises the relative price of good C, and hence trade increases the real return to labor and lowers the real return to capital.

From part (b), **assuming countries have identical technologies and tastes**, then the autarky relative price of good C will be higher in the capital-abundant country (equivalently, the autarky relative price of good C will be lower in the labor-abundant country). Thus, from part (b), we predict that the capital-abundant country (the country with more capital per worker, such as the US) will export good M and import good C, and that the labor-abundant country (the one with less capital per worker, such as China) will export C and import M.

**Not everybody gains from trade** – and hence there will be groups opposing trade liberalization.

Finally, note that free trade – **assuming technology is the same throughout the world -** will lead to equal post-trade factor prices across the world (i.e., the wage rate will be equalized between the capital-abundant and labor-abundant countries, and the same for the return on labor) provided that (i)free trade equalizes the prices of goods – i.e., ignore transportation costs and tariffs; and (ii)both goods are produced in both countries.

(e) If the U.S. is capital-abundant and the Heckscher-Ohlin model can be applied to the world economy, the U.S. will export the capital-intensive good (M) and import the labor-intensive good (C). Trade restrictions (tariffs or quotas) will raise the domestic price of the import good (C) and thus will increase the real return to labor and decrease the real return to capital. Hence, we would predict that groups representing labor (Unions, for example) would oppose free trade and groups representing "capitalists" – would favor free trade. This is a reasonably accurate description of actual political positions (though the reality is a bit more complicated, of course).