Fall 2011 Harvey Lapan
Econ 455

Answers to Problem Set 2

1. Consider a Ricardian model of comparative advantage. There are two countries, the U.S. and India. Each country can produce two goods, shirts (S) and food (F). Assume the US has 1000 workers and India has 4000 workers. Labor productivity in each country is:

Labor Productivity by Country and Good

	United States	India
Shirts	8 shirts/day	4 shirts/day
Food	16 bushels/day	2 bushels/day

(a)The US has the absolute advantage in both goods because the US marginal productivity of labor is higher for both goods.

(i)The opportunity cost of producing food in the U.S. is 1/2 shirt per bushel since one worker can produce 16 bushels or 8 shirts so if we move a worker from shirt production to food production we have: $US: \frac{\Delta S}{\Delta F} = \frac{-MPL_S}{MPL_F} = \frac{-8 shirts}{16 bushels} = -\frac{1}{2} shirt / bushel$. Similarly, the opportunity cost of food production in India is: $India: \frac{\Delta S}{\Delta F} = \frac{-MPL_S}{MPL_F} = \frac{-4 shirts}{2 bushels} = -2 shirts / bushel$. Thus, the US has the lower opportunity cost of producing food. Of course, that means India has the lower opportunity cost of producing shirts. The opportunity cost of producing **shirts** in:

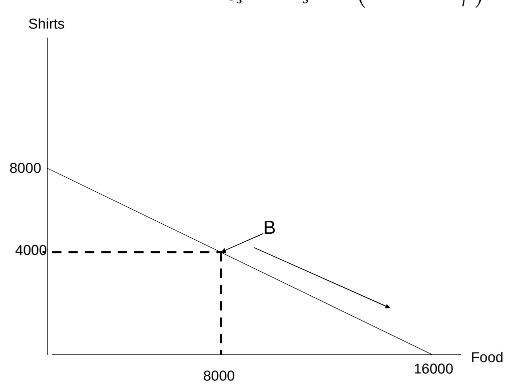
$US: \frac{\Delta F}{\Delta S} = \frac{-MPL_F}{MPL_S} = \frac{-16 bushels}{8 shirts} = -2 bushels / shirt$, whereas for India:

$India: \frac{\Delta F}{\Delta S} = \frac{-MPL_F}{MPL_S} = \frac{-2 bushels}{4 shirts} = -\frac{1}{2} bushel / shirt$.

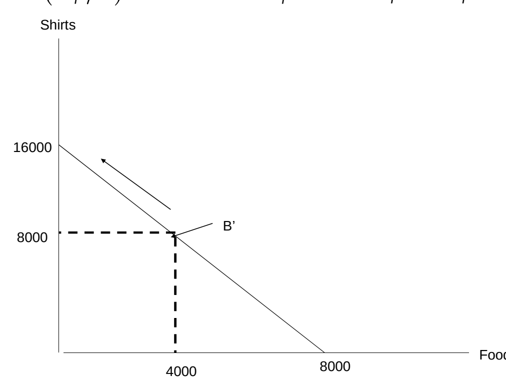
(b)Derive and sketch the production possibility frontier for each country.

For US: $\# Shirts \equiv Q_s = 8L_s = 8(1000 - L_f) = 8,000 - 8(Q_f/16) = 8,000 - (Q_f/2)$ as $L_f = (Q_f/16)$

For India: $\# Shirts \equiv \bar{Q}_s = 4\bar{L}_s = 4(4000 - \bar{L}_f) = 16,000 - 4(\bar{Q}_f/2) = 16,000 - 2\bar{Q}_f$ as $\bar{Q}_f = 2\bar{L}_f$



US Production Possibility Frontier



Indian Production Possibility Frontier

(i)The point B represents the original US production (and consumption) of 4000 shirts and 8000 units of food, while B' represents Indian production (and consumption) of 8000 shirts and 4000 units of food. Let India increase shirt production by "S" units (up to a maximum production of 16000 in India); this will cause Indian food production to fall by (S/2) units {moving in the direction shown by the arrow}. Let the U.S. increase food production by "D" units (up to a maximum of 16000 units of food); this will cause U.S. shirt production to fall by (D/2) {again, moving in the direction shown by the arrow}. Thus, from a world perspective:

Change Food Production = $D-(S/2)>0$ if $D>(S/2)$

Change Shirt Production = $S-(D/2)>0$ if $D<2S$

Thus, for any change in the two countries such that: $2S>D>(S/2)$ (or, equivalently, $2D>S>(D/2)$) world output of both goods increases. {e.g., if India increases shirt production by 100 and U.S. increases food production by 100). Naturally, once one country is fully specialized, it can produce no more of the good in which it has a comparative advantage.

(ii)The world production possibility frontier is such that - at most - only one country produces both goods. For low levels of food output, India should specialize in shirts, while the US produces both goods; while for large levels of food output, the US specializes in food and India produces both goods. Formally:

$$Q_f^T \le 16000; \quad Q_f^{India}=0, Q_s^{India}=16000; \quad Q_s^{us}=8000-\left(Q_f^{us}/2\right); \to Q_s^T = 24000-\left(Q_f^T/2\right)$$

$$24,000 \ge Q_f^T \ge 16,000; Q_f^{us}=16,000, Q_s^{us}=0; Q_s^{India}=16,000-2Q_f^{India}; \to Q_s^T=16,000-2\left(Q_f^T-16,000\right)=48,000-2Q_f^T$$

In the above, $Q_f^T=Q_f^{us}+Q_f^{India}$ is world food output and $Q_s^T=Q_s^{us}+Q_s^{India}$ is world shirt output.

(c)In the absence of trade, autarky (no trade) relative prices would be: $\left(P_s/P_f\right)^{us}=2(\,food/shirt\,)$, and $\left(P_s/P_f\right)^{India}=(1/2)(\,food/shirt\,)$, where P_f is the price of food, P_s is the price of shirts, and the superscript indicates the country.

US Real Wages: $\left(W/P_f\right)^{us}=16\ units\ \ food/day$; $\left(W/P_s\right)^{us}=8 shirts/day$

Indian Real Wages: $\left(W/P_f\right)^{India}=2\ units\ \ food/day$; $\left(W/P_s\right)^{India}=4 shirts/day$

(d) If trade is allowed, each country will export the good in which it has a comparative advantage – thus, the US exports food, India exports shirts. Assuming each country winds up fully specialized (which will happen if country sizes are not *too* different), then post-trade $(1/2)<\left(P_s/P_f\right)^w<2$. In the US, the relative price of shirts falls, and thus the real wage in terms of shirts (the import good) rises while the real wage in terms of food is unchanged. In India, the relative price of food falls (relative price of shirts rises) and hence the real wage in terms of food rises, while in terms of shirts it is unchanged. Thus, the real wage in each country rises in terms of the import good.

(e)Suppose a third country, China is allowed to trade with the US and India. Assume the productivity of labor in each country is given by:

	United States	India	China
Shirts	8 shirts/day	4 shirts/day	9 shirts/day
Food	16 bushels/day	2 bushels/day	3 bushels/day

i How will the addition of China to this agreement affect which good the US exports and which good India exports? Can you tell which good China will export? (a verbal, not a quantitative, answer is expected).

If you look at autarky prices, using the same logic as earlier, you have:

$$\left(P_s/P_f\right)^{US}=2;\quad \left(P_s/P_f\right)^{India}=(1/2);\quad \left(P_s/P_f\right)^{China}=(1/3)$$

Now China is the lowest cost supplier of shirts (highest cost supplier of food), and the U.S. is the low cost supplier of food, whereas *India's* costs are intermediate between the two. If trade occurs among all 3 countries, China will export shirts, the U.S. will export food but India's trade pattern is uncertain. In terms of comparative advantage, India has a comparative advantage in shirts when compared to the U.S., but India has a comparative advantage in food when compared to China. Without more information, we cannot be sure which good India would export.

ii How will the addition of China to the US-India trade agreement affect the (free trade) equilibrium relative price of shirts (in terms of food)? A verbal answer suffices.

Since the world relative price of shirts – before China joined the trading group – was above (1/2), China will want to specialize in shirts. Thus, allowing China to trade will increase the supply of shirts and the demand for foods. Thus, the world relative price of food will rise (i.e., the world relative price of shirts falls) due to China's joining the trade group.

iii How will the addition of China to the US-India trade agreement affect welfare (real wages) in the US and in India? Will both countries necessarily benefit by allowing China to join? Will China benefit by joining this free trade zone? Explain carefully.

Since China's addition to the free trade zone will lower the price of shirts – a U.S. import – U.S. will gain (compared to the situation in which India just traded with the US). China, of course, gains – as compared to autarky. On the other hand, if India continues to export shirts, it will lose as the world price of shirts falls. It is possible India could change from a shirt exporter to a food exporter (if China's shirt production were sufficiently large), in which case India may gain or lose. The main point is that – while the world as a whole potentially gains in that it is possible to increase world output of all goods – **not every country necessarily gains unless it is compensated.** Thus, it would be possible for all countries to be better off after China joins the free trade zone but, without compensation, not every country must gain.

2. To illustrate how the model can be extended to more than two goods, consider the following example

Labor Productivity by Country and Good

	Cell Phones	Food	Motorbikes	Shirts	TVs
United States	4 phones/day	16 bushels/day	1 bike/day	8 shirts/day	3 TVs/day
India	1 phone/day	2 bushels/day	1 bike/day	4 shirts/day	6 TVs/day
US/India productivity	4	8	1	2	1/2

(a) In which good(s) does the U.S. have an absolute advantage? In which good(s) does India have an absolute advantage?

The U.S. has an absolute advantage in cell phones, food and shirts, India has an absolute advantage in TVs, and neither has an absolute advantage in motorbikes.

(b) In which country is the opportunity cost of producing shirts lower? If the answer is ambiguous, explain why.

The question is: the opportunity cost of producing shirts's in terms of WHICH other good? If we look at the last row of the Table above (not given in the problem set itself), we see US relative productivity (compared to India) in producing shirts is higher than the relative productivity for TVs or motorbikes, but lower than the relative productivity in producing food or cell phones. Thus, the opportunity cost of producing shirts – as compared to TVs or motorbikes – is **lower** in the US; BUT the opportunity cost of producing shirts – as *compared to Food or motorbikes*, is **higher** in the US.

Not asked but from the Table we see that the US has the lower opportunity cost of producing food, measured in terms of ANY of the other goods, and India has the lower opportunity cost of producing TVs, measured in terms of ANY of the other goods.

(c) Find autarky *relative* prices in each country (for simplicty, express the price of each good in terms of food).

Autarky relative prices in terms of food. Letting food be the item in which values are measured (what is called the *numeraire*), we have for each country, if the good is produced, $W = P_x MPL_x$ for every good *x*, where MPL_x is the labor productivity; hence: $\frac{P_x}{P_f} = \frac{MPL_f}{MPL_x}$, or $P_x = P_f\left(\frac{MPL_f}{MPL_x}\right)$ and thus:

$$US: \quad \frac{P_{cellphones}}{P_{food}} = \frac{MPL_{food}}{MPL_{cellphones}} = 4\ food\ /\ cellphones; \quad \frac{P_{bikes}}{P_{food}} = 16\ food\ /\ bike; \quad \frac{P_{shirts}}{P_{food}} = 2\ food\ /\ shirt \quad \frac{P_{TVs}}{P_{food}} = \frac{16}{3}\ food\ /\ TV$$

$$India: \quad \frac{P_{phones}}{P_{food}} = \frac{MPL_{food}}{MPL_{phones}} = 2\ food\ /\ phone; \quad \frac{P_{bikes}}{P_{food}} = 2\ food\ /\ bike; \quad \frac{P_{shirts}}{P_{food}} = \frac{1}{2}\ food\ /\ shirt \quad \frac{P_{TVs}}{P_{food}} = \frac{1}{3}\ food\ /\ TV$$

(d) If trade were allowed, what can you predict about the pattern of trade? (*i.e.*, which goods the U.S. exports and which it imports)? Explain.

If free trade were allowed, we would predict that the US would definitely export food, and that India would definitely export TVs, because each country has a comparative advantage in that good as compared to *any* other good. What other goods the countries exported would depend on demand and country size. However, if the US only exports two goods, we know it would be food and phones (because the US has a comparative advantage in phoness, as compared to TVs, shirts or bikes); if the US exported 3 goods, it would be food, phones and shirts; and if the US exported 4 goods, it would be all goods except TVs.

Similar logic applies to India (by just looking above at what the US is not exporting).

(e) Let W denote the wage in the U.S., and $\bar{W}$ denote the Indian wage. Show where each good is produced based upon the ratio of wages $\left(W/\bar{W}\right)$ between the two countries.

The ratio of marginal costs for each good looks like this:

$$\frac{MC_x^{us}}{MC_x^{India}} = \frac{\left(W/MPL_x^{us}\right)}{\left(\bar{W}/MPL_x^{India}\right)} = \omega \cdot \left(\frac{MPL_x^{India}}{MPL_x^{us}}\right) \quad \text{where: } \omega \equiv \left(W/\bar{W}\right). \text{ Thus, for all goods:}$$

$$\frac{MC_{phones}^{us}}{MC_{phones}^{India}} = \omega\left(\frac{1}{4}\right) = \frac{\omega}{4}; \quad \frac{MC_{food}^{us}}{MC_{food}^{India}} = \omega\left(\frac{2}{16}\right) = \frac{\omega}{8}; \quad \frac{MC_{bikes}^{us}}{MC_{bikes}^{India}} = \omega; \quad \frac{MC_{shirts}^{us}}{MC_{shirts}^{India}} = \omega\left(\frac{4}{8}\right) = \frac{\omega}{2}; \quad \frac{MC_{TVs}^{us}}{MC_{TVs}^{India}} = \omega\left(\frac{6}{3}\right) = 2\omega$$

Goods will be produced where the cost is lowest. Thus, if the ratio of US to Indian MC is less than one for a given good, production of that good occurs in the US; if the ratio is greater than one, production occurs in India and when it equals one, production may occur in both countries. Hence:

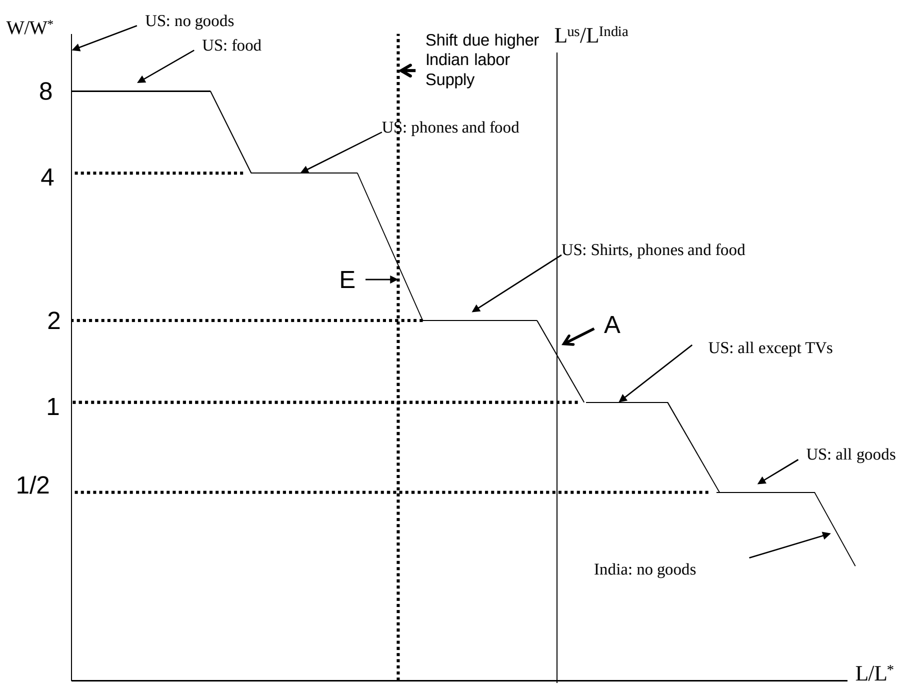
- If $\omega < \left(1/2\right)$, all goods will be produced in the US and none in India (this cannot be an equilibrium);
- If $\left(1/2\right) < \omega < 1$ then TVs will be produced in India, all other goods in the US;
- If $1 < \omega < 2$ then TVs and bikes will be produced in India, the other three goods in the US;
- If $2 < \omega < 4$ then food and cellphones will be produced in the US, the other 3 goods in India;
- If $4 < \omega < 8$ then only food will be produced in the US, the other four goods in India;
- If $\omega > 8$, no goods will be produced in the US (which cannot be an equilibrium because of unemployment).

(Naturally, if we have an equality, such as $\omega = 2$, then one of the goods (here, shirts) could be produced in both countries.)

(f) Given relative labor supplies for each country, draw a graph to show how the equilibrium relative wage (US wage to Indian wage) is determined. Explain the graph.

The figure looks something like that below. The numbers shown on the vertical scale represent the relative wage at which it is equally costly to produce one of the goods in both countries. At $\omega = 8$, the cost of producing food is the same in the US and India; everything else is cheaper to produce in India. Thus, at $\omega = 8$, the (relative) demand for US labor is horizontal, since the percent of world food production that occurs in the US could be between 0 and 100% at that price. Once the relative wage falls below 8, then definitely all food production is in the US. As ω decreases, the relative cost (price) of food production falls, compared to other goods, thereby increasing demand for food and hence the demand for US labor (to produce food) increases – this explains the downward sloping portion of the labor demand curve. Once the relative wage reaches 4, the costs of cellphone production are the same in both countries, and again labor demand is horizontal (infinitely elastic) over some range. Once all cellphoness are produced in the US, it takes further declines in the relative wage to stimulate demand for US labor.

The relative demand and relative supply curves jointly determine the relative wage. Given the initial relative supply of US labor, the initial equilibrium is at a point like **A**.



(g) Use the graph to *discuss* how an increase in the Indian working population will affect: (1)US real wages; (2)the set of goods the US exports; (3)the relative prices of goods; and (4)Indian real wages. **.**

Since productivites are not shifting, just the Indian population (and work force), what happens is shown in the same graph (above) by shifting the relative labor supply to the **left –** that is, the US work force, *relative* to the Indian work force, decreases in size. This is shown by the dotted vertical line. The new equilibrium relative wage is shown at point E, so the US wage rises relative to the Indian wage. Moreover, since productivities are not changing we know: (1)the US real wage in terms of its original exports is unchanged but the real wage in terms of all imports rises (due to the decrease in the Indian relative wage); (2)the set of goods the US exports (in this graph) decreases – in this case, it now exports just phones and food, whereas shirts – previously exported - become an import good for the US (3)the relative price of goods India exports falls – that is, the relative price of TVs and bikes and now shirts - and the relative price of its imports rise); and (4)since productivities are unchanged, the Indian real wage falls. That is, in terms of its initial exports (TVs, bikes), the real wage is unchanged; but in terms of initial imports – phones and food – the real wage falls. The real wage also falls in terms of shirts, which transition from being an Indian import good to being an export good. Thus, the Indian population increase benefits the U.S. but hurts India.

3. **(Specific Factor Model, Chapter 3)** Labor is the only mobile factor, capital (K) is specific to sector C, land (T) is specific to food production. Technology and resources constraints are:

$$C = \beta(K)^{2/3}(L_c)^{1/3}; \quad F = (T)^{2/3}\left(L_f\right)^{1/3}; \quad \beta > 0; \text{ resource constraint: } L_c + L_f = L$$

a) Derive the production possibility frontier for this economy (that is, express *C* as a function of *F*, and also of the resources available and the technology: *K,T,L* and β **).**

Because there is only one variable input, the production possibility frontier can be derived through substitution:

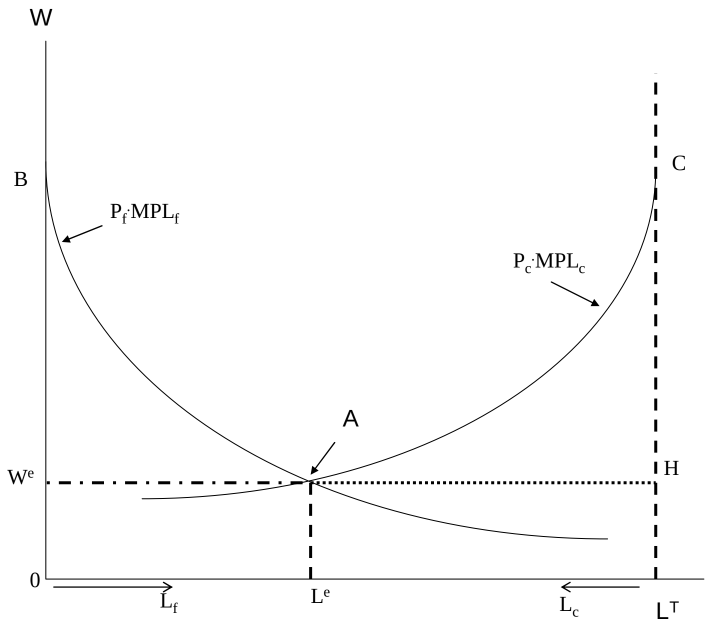
$$C = \beta(K)^{2/3}(L_c)^{1/3} \rightarrow (C/\beta)^3 = \left(K^2\right)(L_c) \rightarrow L_c = \left(C^3/\beta^3K^2\right);$$
$$F = (T)^{2/3}\left(L_f\right)^{1/3} \rightarrow F^3 = \left(T^2\right)\left(L_f\right) \rightarrow L_f = \left(F^3/T^2\right);$$
$$L_c + L_f = L \rightarrow \left(C^3/\beta^3K^2\right) + \left(F^3/T^2\right) = L \quad \text{yields the ppf.}$$

Clearly the ppf is not linear.

b) **Use labor demand to:**

i Given output prices, show **graphically** how the equilibrium wage rate and the allocation of labor between the two sectors is determined.

In the figure below, L^T represents the total supply of labor; labor demand for sector F is measured from the point “0”, in the usual direction – as we move to the right, labor demand in F is increasing. Labor demand for sector C has been “flipped over”, so the demand for labor in sector C is measured from the right vertical line, and as we move to the left, the demand for labor in C is increasing (the arrows at the bottom of the figure also show this). Equilibrium occurs where the two curves intersect – which is where total labor demand equals total labor supply; this is point A, with equilibrium wage W^e, and with (0, L^e) units of labor employed in sector F, and (L^e , L^T) units of labor employed in sector C.



ii Show mathematically how the equilibrium wage rate and the supply curve for each good (*C*, *F*) is determined (as a function of output prices). Also, discuss how the returns to land and capital are determined.

To derive the labor demand curves, set the marginal value product of labor equal to the wage:

$$C = \beta\left(L_c\right)^{1/3}\left(K\right)^{2/3} \to P_c\left(\partial C/\partial L_c\right) = \left(\beta P_c/3\right)\left(L_c\right)^{-2/3}K^{2/3} = W \to L_c^* = K\left(\beta P_c/3W\right)^{3/2}$$

$$F = \left(L_f\right)^{1/3}\left(T\right)^{2/3} \to P_f\left(\partial F/\partial L_f\right) = \left(P_f/3\right)\left(L_f\right)^{-2/3}T^{2/3} = W \to L_f^* = T\left(P_f/3W\right)^{3/2}$$

Adding labor demands and setting it equal to supply yields **the equilibrium wage**:

$$L_f^* + L_c^* = L \to T\left(P_f/3W\right)^{3/2} + K\left(\beta P_c/3W\right)^{3/2} = L \to W^e = \left(1/3\right)\left[\left(T/L\right)\left(P_f\right)^{3/2} + \left(K/L\right)\left(\beta P_c\right)^{3/2}\right]^{2/3}$$

Note that: doubling prices causes wages to double (again, the pure inflation effect); and that doubling all inputs has no effect on wages (because of "constant returns to scale").

As for the equilibrium supply curves: These can be found in one of two ways: (1)using labor demand in each sector, if you plug that back into the production function, and evaluate at the equilibrium wage, you get the ("general equilibrium") supply curve; (2)or, you can get the general equilibrium supply curves from the production possibility frontier, by finding the point where the slope of the frontier equals the relative price ratio. You should use both methods to convince yourself that they give the same result.

Method 1:

$$C=\beta\left(L_c\right)^{1/3}\left(K\right)^{2/3};L_c^*=K\left(\beta P_c/3W\right)^{3/2}\rightarrow C^s=\beta^{3/2}K\left(\frac{P_c}{3W^e}\right)^{1/2}$$

$$=\frac{\beta^{3/2}K}{\left[\left(T/L\right)\left(\left(P_f/P_c\right)\right)^{3/2}+\left(K/L\right)\left(\beta\right)^{3/2}\right]^{1/3}}$$

$$F=\left(L_f\right)^{1/3}\left(T\right)^{2/3};L_f^*=T\left(P_f/3W\right)^{3/2}\rightarrow F^s=T\left(\frac{P_f}{3W^e}\right)^{1/2}$$

$$=\frac{T}{\left[\left(T/L\right)+\left(K/L\right)\left(\beta\left[P_c/P_f\right]\right)^{3/2}\right]^{1/3}}$$

Method 2:

From the ppf, production occurs where (absolute value of) slope = price ratio; that is: $\left|\left(\frac{dQ_c}{dQ_f}\right)\right|=\frac{P_f}{P_c}$

From the ppf:

$$\left(C^3/\beta^3K^2\right)+\left(F^3/T^2\right)=L\rightarrow\left(3C^2/\beta^3K^2\right)dC+\left(3F^2/T^2\right)dF=0\rightarrow\left|\frac{dC}{dF}\right|=\frac{\beta^3F^2K^2}{C^2T^2}=\rho;\quad\rho\equiv\frac{P_f}{P_c};$$

where ρ represents the relative price of food. Simplifying and substituting back into the ppf gives the supply curves:

$$F=\frac{\rho^{1/2}CT}{\beta^{3/2}K};\quad\frac{C^3}{\beta^3K^2}+\left(\frac{F^3}{T^2}\right)=L\rightarrow\frac{C^3}{\beta^3K^2}+\left(\frac{\rho^{1/2}CT}{\beta^{3/2}K}\right)^3\left(\frac{1}{T^2}\right)=\left(\frac{C}{\beta}\right)^3\left\{\frac{\beta^{3/2}K+\rho^{3/2}T}{\beta^{3/2}K^3}\right\}=L\rightarrow$$

$$C^*=\frac{\beta^{3/2}K}{\left\{\beta^{3/2}\left(K/L\right)+\left(\rho^{3/2}T/L\right)\right\}^{1/3}};$$

$$F^*=\frac{\rho^{1/2}CT}{\beta^{3/2}K}=\frac{\rho^{1/2}T}{\left\{\beta^{3/2}\left(K/L\right)+\left(\rho^{3/2}T/L\right)\right\}^{1/3}}\rightarrow F^*=\frac{T}{\left\{\left(\beta/\rho\right)^{3/2}\left(K/L\right)+\left(T/L\right)\right\}^{1/3}}$$

Once you make the substitution $\left(\rho\right)=\left(P_f/P_c\right)$ and thus $\left(\beta/\rho\right)=\left(\beta P_c/P_f\right)$, you can see the supply curves, whether derived by method 1 or 2, are the same. The properties of the supply curves include:

(1)If both output prices double, there is no change in supply of either good – this is "pure inflation";
(2)If all inputs double, the output of both goods doubles (given price) – this is constant returns to scale;
(3)You should be able to see that the supply of C increases as *K* increases but decreases as *T* increases; conversely, for *F*, the supply of *F* decreases as *K* increases, but increases as *T* increases. This is the effect of specific factors.
(4)For the mobile factor, labor, supplies of both goods increase as *L* increases.
(5)Given output prices, an increase in productivity in sector *C* (an increase in β) increases supply of

good *C and* ***decreases*** supply of good *F*.
(6)And, of course, an increase in the price of a good increases the supply of that good and decreases the supply of the other good.

As for the returns to land and capital:
Graphically, since you can think of landowners as "consumers" of labor, the returns to landowners can be measured by the "consumer surplus" area next to the demand for labor in sector F: that is, in terms of the previous figure, the area of {B,W^e,A} would give the total return to land.
Similarly, the return to capital can be measured by the area next to the labor demand curve in sector C: this would be area {C,H,A}

Mathematically, the return to land (per acre) can be measured in either of two ways – they give the same result because of constant returns to scale. Let R_T stand for the return to land. Then:

$$R_T = \frac{P_f F - WL_f}{T} = P_f\left(\frac{\partial F}{\partial T}\right)$$

The first way is the total "profits", per acre, after paying labor costs; the second way is the marginal value product of land. Under constant returns to scale, these two measures are identical. Hence:

$$R_T = P_f\left(\frac{\partial F}{\partial T}\right) = \frac{2P_f}{3}\left(L_f\right)^{1/3} T^{-1/3}.$$

Using the demand for labor in sector *F*,

$$L_f^* = T\left(P_f/3W\right)^{3/2} \rightarrow R_T = \left(2P_f/3\right)\left(L_f\right)^{1/3} T^{-1/3} = \left(2P_f/3\right)\left[\left(P_f/3W\right)^{3/2}\right]^{1/3} = \frac{2P_f^{\,3/2}}{3^{3/2}W^{1/2}}$$

Similarly, the return to capital (R_K) is found from:

$$R_K = \frac{P_c C - WL_c}{K} = P_c\left(\frac{\partial C}{\partial K}\right) = \frac{2\beta P_c}{3}\left(L_c\right)^{1/3} K^{-1/3}.$$

From the labor demand in C: $\frac{L_c^*}{K} = \left(\beta P_c/3W\right)^{3/2} \rightarrow R_K = \frac{2\left(\beta P_c\right)^{3/2}}{3^{3/2}W^{1/2}}$

Finally, using the equilibrium wage calculated above, we get the equilibrium returns to land and capital:

$$R_T^e = \frac{2P_f^{\,3/2}}{3^{3/2}\left(W^e\right)^{1/2}} = \frac{2P_f^{\,3/2}}{3\left[\left(T/L\right)\left(P_f\right)^{3/2} + \left(K/L\right)\left(\beta P_c\right)^{3/2}\right]^{1/3}};$$

$$R_K = \frac{2\left(\beta P_c\right)^{3/2}}{3^{3/2}W^{1/2}} = \frac{2\left(\beta P_c\right)^{3/2}}{3\left[\left(T/L\right)\left(P_f\right)^{3/2} + \left(K/L\right)\left(\beta P_c\right)^{3/2}\right]^{1/3}}$$

iii Using your result in part ii, given output prices, show how an increase in the amount of land (T) available for production affects: (1)the quantity supplied of each good; (2)the real return to land; (3)the real return to capital; and (4)the real wage rate.

(1) Mathematically, from our earlier results:

$$C^* = \frac{\beta^{3/2}K}{\left\{\beta^{3/2}(K/L)+\left(\rho^{3/2}T/L\right)\right\}^{1/3}} \rightarrow \frac{\partial C^*}{\partial T} = \frac{-\beta^{3/2}K\left(\rho^{3/2}/L\right)}{3\left\{\beta^{3/2}(K/L)+\left(\rho^{3/2}T/L\right)\right\}^{4/3}} < 0$$

$$F^* = \frac{T}{\left\{(\beta/\rho)^{3/2}(K/L)+(T/L)\right\}^{1/3}} \rightarrow \frac{\partial F^*}{\partial T} = \frac{1}{\left\{(\beta/\rho)^{3/2}(K/L)+(T/L)\right\}^{1/3}} - \frac{(T/L)}{3\left\{(\beta/\rho)^{3/2}(K/L)+(T/L)\right\}^{4/3}}$$

$$= \frac{3(\beta/\rho)^{3/2}(K/L)+2(T/L)}{3\left\{(\beta/\rho)^{3/2}(K/L)+(T/L)\right\}^{4/3}} > 0$$

That is, an increase in the specific factor used to produce food, given output prices, leads to an increase in the output of food and a decrease in the output of the other good.

In terms of returns to factors, given the output prices, we would expect the increase in the amount of land to reduce the return to land (diminishing marginal productivity), increase the demand for labor and thus increase the wage rate, and hence – through this indirect effect – reduce the return on capital. Mathematically:

$$R_T^e = \frac{2P_f^{\ 3/2}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{1/3}} \rightarrow \frac{\partial R_T^e}{\partial T} = \frac{-2\left(P_f^{\ 3}/L\right)}{9\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{4/3}} < 0$$

$$R_K = \frac{2\left(\beta P_c\right)^{3/2}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{1/3}} \rightarrow \left(\frac{\partial R_K^e}{\partial T}\right) = \frac{-2\left(\beta P_c\right)^{3/2}\left[\left(P_f\right)^{3/2}/L\right]}{9\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{4/3}} < 0$$

$$W^e = (1/3)\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{2/3} \rightarrow$$

$$\frac{\partial W^e}{\partial T} = (2/9)\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{-1/3}\left(\left(P_f\right)^{3/2}/L\right) > 0$$

Since $R_T^e = \frac{2P_f^{3/2}}{3^{3/2}\left(W^e\right)^{1/2}}$ and $R_K^e = \frac{2\left(\beta P_c\right)^{3/2}}{3^{3/2}W^{1/2}}$, it is clear we just need to know how the wage rate changes to know how the returns to the other factors change. Intuitively, the increased abundance of land reduces the return to land (normal supply and demand affect), which means – given technology, that the (food) wage rate must rise. But since wages are the same in both sectors, wages also rise in clothing production, and the higher wages reduce the return to capitalists (owners of capital) in the clothing sector.

iv. Given output prices, show how an increase in productivity (β) affects outputs and the real return to each input.

Intuitively, increased productivity in the clothing sector will – with given inputs – raise output. Also, since it increases the value of workers in that sector, workers will move from food production to clothing production. Hence, the output of clothing increases for two reasons (more labor, better technology) while the output of food decreases. As for factor prices, the productivity increase in clothing should increase the return to capital and the return to labor in clothing. That attracts workers from the food sector, meaning wages rise there – and ultimately since wages are the same in the two sectors, wages must increase overall. But higher wages raise labor costs in food production, meaning the return to land falls. Mathematically, using the earlier relationships:

$$C^* = \frac{\beta^{3/2}K}{\left\{\beta^{3/2}(K/L)+\left(\rho^{3/2}T/L\right)\right\}^{1/3}} \rightarrow \left(\frac{\partial C}{\partial \beta}\right) = \frac{(3/2)\beta^{1/2}K}{\left\{\beta^{3/2}(K/L)+\left(\rho^{3/2}T/L\right)\right\}^{1/3}} - \frac{\left(\beta^{3/2}K\right)(1/3)\left(3\beta^{1/2}K/2L\right)}{\left\{\beta^{3/2}(K/L)+\left(\rho^{3/2}T/L\right)\right\}^{4/3}} =$$

$$\frac{\left(\beta^2K^2/L\right)+(3/2)\beta^{1/2}K\left(\rho^{3/2}T/L\right)}{\left\{\beta^{3/2}(K/L)+\left(\rho^{3/2}T/L\right)\right\}^{4/3}} > 0$$

$$F^* = \frac{T}{\left\{(\beta/\rho)^{3/2}(K/L)+(T/L)\right\}^{1/3}} \rightarrow \left(\frac{\partial F^*}{\partial \beta}\right) = \frac{-T\left(\beta^{1/2}/2\rho^{3/2}\right)(K/L)}{\left\{(\beta/\rho)^{3/2}(K/L)+(T/L)\right\}^{4/3}} < 0$$

$$R_T^e = \frac{2P_f^{3/2}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{1/3}} \rightarrow \frac{\partial R_T^e}{\partial \beta} = \frac{-P_f^{3/2}(K/L)P_c^{3/2}\beta^{1/2}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{4/3}} < 0$$

$$R_K = \frac{2\left(\beta P_c\right)^{3/2}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{1/3}} \rightarrow \left(\frac{\partial R_K^e}{\partial \beta}\right) = \frac{3\beta^{1/2}P_c^{3/2}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{1/3}} -$$

$$\frac{-2\left(\beta P_c\right)^{3/2}(1/3)(K/L)\left(P_c\right)^{3/2}(3/2)\beta^{1/2}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{4/3}} = \frac{\beta^{1/2}P_c^{3/2}\left\{3(T/L)\left(P_f\right)^{3/2}+2(K/L)\left(\beta P_c\right)^{3/2}\right\}}{3\left[(T/L)\left(P_f\right)^{3/2}+(K/L)\left(\beta P_c\right)^{3/2}\right]^{4/3}} > 0$$

$$W^e = (1/3)\left[(T/L)(P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{2/3} \rightarrow$$

$$\frac{\partial W^e}{\partial \beta} = (1/3)\left[(T/L)(P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{-1/3} (K/L)\beta^{1/2}(P_c)^{3/2} > 0$$

c) Assume two countries, Canada and Italy, which are almost identical. They differ only in that Canada has more land than Italy.

i Based upon your results from part (b), what predictions would you make concerning the autarky (no trade) relative price of clothing in Italy as compared to Canada?

By analogy to the exercise above, given output prices, an increase in a specific factor increases output of the good that uses that specific factor and reduces output of the other good (given output prices); thus, an increase in the amount of land, ***given output prices,*** increases the supply of food and decreases the supply of clothing; also, given output prices, the increase in land increases the wage rate, and decreases the return on land and capital.

Now, the greater availability of land – and increased supply of food – will lead food prices to fall (clothing prices to rise) in Canada, as compared to Italy. As food prices fall (clothing prices rise), the return on land decreases, the return on capital increases, and the real wage in terms of clothing falls, but the real wage in terms of food rises.

So, in comparing Canada to Italy, the relative price of food will be lower in Canada; the real return on land will be lower in Canada, both because of the abundance of land and because this abundance leads to a decrease in the relative price of food. The real wage in terms of food will be higher in Canada but we cannot say for sure about the real wage in terms of clothing (since more land increases the wage, but higher clothing prices reduce the real wage in terms of clothing, leading to an ambiguous impact). Similarly, since the greater availability of land depresses the return on capital (due to higher wages) BUT the higher price of clothing raises the return to capital, we cannot be sure how the return on capital compares between the two countries.

ii If trade is allowed between the two countries, what will the pattern of trade be and how will the relative price of clothing change in each country?

If trade is allowed between Canada and Italy, clearly Italy will export clothing and Canada will export food. Hence, trade causes the relative price of clothing to rise in Italy and to fall in Canada (the opposite for the relative price of food).

iii **How does trade affect the real return to each factor in each country? Does the country as a whole *potentially* gain from trade?**

In Canada, the increased relative price of food **due to trade**: raises the real return to land, lowers the real return to capital and has an ambiguous affect on wages: the real return to labor rises in terms of clothing but falls in terms of food. Thus, due to trade, land owners in Canada gain, capitalists lose, and the impact on workers is ambiguous.

In Italy, the opposite happens. The decreased relative price of food (increased relative price of clothing) **due to trade** raises the real return to capital, lowers the real return to land, and has an ambiguous impact on the real wage. The real wage in terms of food rises, the real wage in terms of clothing falls.

Both countries potentially gain from trade in that it is possible for the winners in each country to compensate the losers in that country and have *everybody* better off than without trade. **But** if the compensation does not take place, there will be opposition to trade.