1. Consider a small country (Belgium) with the following demand and supply curves for corn:

\[ \text{Supply} = 3P_c; \quad \text{Demand} = 175 - 2P_c \]

Assume Belgium can import corn at a given world price of \( P_c = 15 \). Further, assume that Belgium imposes a tariff of \( t \) per unit of import.

a) Show how: domestic price, consumption and production change as \( t \) increases. Also, calculate how consumer surplus, producer surplus, and government tariff revenue change as \( t \) increases.

Given the world price, domestic price in Belgium is given by: \( P^d = 15 + t \), if \( t \leq 20 \). Note that for \( t > 20 \) the tariff is prohibitive, there are no imports, and the domestic price equals the autarky price of 35. Thus, assuming \( t < 20 \), we have:

\[ P^d = (15 + t); \quad D = 175 - 2P^d = 145 - 2t; \quad S = 3P^d = 45 + 3t; \quad M = D - S = 100 - 5t \]

Consumption and imports fall, and production rises, as \( t \) increases. The changes in consumer and producer surplus are seen on the diagram on the next page. The increase in producer surplus is given by area \( \{15, A^*, A, (15+t)\} \), whereas the decrease in consumer surplus is given by: \( \{15, B^*, B, (15+t)\} \). Thus:

\[ \Delta PS = \frac{1}{2}t \cdot (45 + 45 + 3t) = 45t + \frac{3t^2}{2}; \quad \Delta CS = -\frac{1}{2}t \cdot (145 + 145 - 2t) = -145t + t^2 \]

\[ TR = tM = t \cdot (100 - 5t) = 100t - 5t^2 \] is tariff revenue.

Thus, it is easily seen that producer surplus increases with the tariff, and consumer surplus decreases (for \( t \leq 20 \)), whereas tariff revenue increases with the tariff for \( t < 10 \), and then decreases thereafter. Overall: \( \Delta Welfare = TR + \Delta PS + \Delta CS = -(5/2)t^2 \) so that the tariff lowers overall welfare.

(i) If \( t > 20 \), the tariff is prohibitive, no trade occurs and domestic price is 35.

b) Compare the domestic equilibrium when \( t = 10 \) to the case where there is no tariff, but there is an import quota of 50 units.

From part (a), with \( t = 10 \), imports \( M = 100 - 5t = 50 \). Thus, a quota of 50 and a tariff of 10 have identical effects on domestic price, consumption, production and imports. The only possible difference is the tariff revenue (which is 500 under the tariff). Under the quota, importers make 10 on each unit imported and hence will earn excess profits of 500, unless the quota licenses are auctioned off, in which case the two policies are identical.
c) Find the import quota $L$ that corresponds to an import tariff of $t$.

With a tariff, imports are: $M(t) = 100 - 5t$. Thus, for each tariff, $t$, the import quota $L(t)$ that has the same effect is given by: $L(t) = 100 - 5t$. Of course, any shift in the demand or supply curve will change the quota that corresponds to each tariff.

2. Next, consider the case of two large countries:

US: Demand $= 120 - 2P_c$; Supply $= 4P_c$ where $P_c$ is the price of computers in the US;

China: Demand $= 160 - 2P_c$; Supply $= 2P_c$ where $P_c$ is the price of computers in China;

a) Assuming free trade (no tariffs), find the equilibrium price and quantities traded.

The US export supply ($X$) is given by: $X = S^us - D^us = 6P_c - 120$

The Chinese import demand ($M$) is given by: $M = D^c - S^c = 160 - 4P_c$

where $P_c$ is the price in China and $P_c$ is the price in the US. Note that under autarky the US price is 20, and the Chinese price is 40 (see figure 2), since exports (and imports) in each country will be zero under these prices. Under free trade we have: $P_c = P^us = P^f$ ($f$ stands for free trade) and thus:

$M = X \rightarrow 160 - 4P_f = 6P_f - 120 \rightarrow P_f = 28$
Thus, exports (imports) equal 48 under free trade. Domestic consumption and supply is found by substituting back into the demand and supply curves. For the US and China, under free trade:

\[
\begin{align*}
D^{us} &= 120 - 2P^I = 64; & S^{us} &= 4P^I = 112; \\
D^c &= 160 - 2P^I = 104; & S &= 2P^I = 56
\end{align*}
\]

b) Show how a US export tariff of 10 affects the volume of trade, prices in China and the US, and welfare in each country. Who pays for the US tax? Explain.

Since part (c) asks you to answer this for the general case of any tariff, it is easier to start by assuming the tariff is “\( t \)”, rather than “10”, and then substitute in for “\( t=10 \)”. Thus, with any tariff, we have:

\[
P^{us} = P^c - t \quad \text{(which means when } t=10, \text{ the US price is less than the Chinese price by 10 since the tax is on US exports). Equilibrium requires:}
\]

\[
X^{us} = 6\left(P^c - t\right) - 120 = M^c = 160 - 4P^c \to 280 + 6t = 10P^c \to P^c = 28 + .6t, \quad P^{us} = 28 - .4t
\]

So, when \( t=10 \), the Chinese price rises by 6 and the US price falls by 4. Note that, even though the US imposes the tax, China pays some of the tax (because its price rises).

As a result of the tax, the volume of trade falls. Substituting the equilibrium price back yields exports (imports) of:

\[
X^{us} (t) = M^c (t) = 6P^{us} (t) - 120 = 6\left(28 - .4t\right) - 120 = 48 - (2.4t); \quad \text{for } t=10, \quad M^c (t = 10) = 24
\]

Clearly, Chinese welfare falls (they pay higher prices for imports and receive no tariff revenue), whereas the impact on the US is ambiguous. In terms of Figure 2, the loss to China is the area of the trapezoid \( \{28,E,A,[28+.6t]\} \) with \( t=10 \). At \( t=10 \), this trapezoid has height 6, one base is (48), the other is 24, so the area (or loss to China) is 216.

For the US, consumers gain (lower prices), producers lose (lower prices), and the government gains tariff revenue. Overall, the US private sector loses area \( \{28,E,B,[28-.4t]\} = 144 \). However, the US government gains the tariff revenue, measured by area: \( \{[28-.4t], B,A,[28+.6t]\} = 10 \times 24 = 240 \).

So, for \( t=10 \), the US gains 96, China loses 216, and the overall loss is area AEB = 120.

(i) If the tariff were removed, but a quota of 24 replaced it, then the volume of trade would be the same under the two policies. Hence, prices in each country would be the same under the quota as they had been under the tariff. The only possible difference is that the tariff revenue now becomes profits for US exporters, who buy at the US price (24) and sell at the Chinese price (34). If the US auctions off the quota licenses, the two policies are identical.
(c) How does a US export tariff of $t$ affect prices, trade volumes and welfare?

(i) How does the tariff affect China’s welfare (sum of consumer and producer surplus)?

The analysis was discussed in part (b), and the figure above illustrates these changes. China, due to higher import prices (and no tariff revenue) is hurt. The total loss in China is the area $\{28, E, A, [28+0.6t]\}$; the US private sector loses area $\{2, E, B, [28-0.4t]\}$, while the US government gains the tariff revenue, measured by area: $\{[28-0.4t], B, A, [28+0.6t]\}$. Thus:

**China loses:** $(0.6t)(48-1.2t)$

**US private sector loses:** $(0.4t)(48-1.2t)$

**US government tariff revenue** = $(t)(48-2.4t)$

**Net US gain** = $(t)(48-2.4t)-(0.4t)(48-1.2t) = 4.8t(6-0.4t) > 0$ if $t < 15$

**Overall welfare change** = Net US gain plus Chinese loss = $4.8t(6-0.4t)-(0.6t)(48-1.2t) = -1.2t^2$

So, even if the US gains from the tariff, it gains less than China loses, so there is an overall inefficiency due to the tariff. In terms of the figure, the inefficiency (deadweight loss) is the triangle: $\{A, B, E\}$.

Finally, to calculate the changes in consumer and producer surplus for China (and the US) you must go back to the demand and supply curves (not just the export supply or import demand curves). While the figures are not drawn, the method of finding the surplus is the same as above. Hence:
For China:
\[
\Delta CS = -\frac{1}{2} (\Delta P)(D_0 + D_i) = -\frac{1}{2} (0.6t)(104 + [104 - 1.2t]) = -0.6t (104 - 0.6t)
\]
\[
\Delta PS = \frac{1}{2} (\Delta P)(S_0 + S_i) = \frac{1}{2} (0.6t)(56 + [56 + 1.2t]) = 0.6t (56 + 0.6t)
\]

Net Change in Chinese welfare = \(\Delta PS + \Delta CS = -0.6t (48 - 1.2t)\)

(ii) The impact on US welfare, summarized above is given by:
\[
\Delta CS = -\frac{1}{2} (\Delta P)(D_0 + D_i) = -\frac{1}{2} (0.4t)(64 + [64 + 0.8t]) = 0.4t (64 + 0.4t)
\]
\[
\Delta PS = \frac{1}{2} (\Delta P)(S_0 + S_i) = \frac{1}{2} (0.4t)(112 + [112 - 1.6t]) = -0.4t (112 - 0.8t)
\]

Change Private Sector Welfare (US) = \(\Delta CS + \Delta PS = -0.4t (48 - 1.2t)\), as above

Tariff Revenue = \((t)\cdot Exports = t (48 - 2.4t)\)

Net US gain = \(t (48 - 2.4t) - (0.4t)(48 - 1.2t) = (t)(28.8 - 1.92t) = 4.8t (6 - 0.4t) > 0 \) if \( t < 15 \)

The US can gain from this tariff because it affects world price; specifically, by restricting exports the tariff raises the world price of US exports (even though domestic price falls). This change in world price has a positive effect on the US economy (and a negative effect on the Chinese economy), and thus from the US perspective the improved terms of trade may offset the inefficiency the tariff causes. However, overall world welfare (sum of US and Chinese surplus) must fall. Thus:

Net US gain + Net Chinese gain = \(4.8t (6 - 0.4t) - 0.6t (48 - 1.2t) = -1.2t^2 < 0\), as explained earlier.

(iii) Find the tariff that maximizes US welfare.

From above, the gain in US welfare is given by: \(G(t) = 4.8t (6 - 0.4t)\). Taking the derivative,
\[
\frac{dG}{dt} = 4.8t (6 - 0.8t).
\]
Thus, for \( t \) small \( G \) (US welfare) increases with the tariff; when \( t \) becomes large enough \( G \) decreases as \( t \) increases further. The value of \( t \) that maximizes \( G \) is thus found by setting \(\frac{dG}{dt} = 0\), as the second order condition is easily seen to hold. This implies: \( t^* = (6/0.8) = 7.5 \) is the optimal tariff for the US.

(iv) If the U.S. eliminates its import tariff, but China imposes an export tariff of the same magnitude, the private sector losses are as above, but the tariff revenue is transferred to China. Thus, the US would lose area \(\{28,E,B,28-4t\}\), while China would gain \(\{28,F,B,28-4t\}\), while losing \(\{A,F,E\}\) for the same world loss of \(\{A,B,E\}\). Numerically:
Net Gain to China = \(-0.6t(48-1.2t) + t(48-2.4t) = 0.4t(48-4.2t) > 0\) for \(0 < t < (80/7) = 11\frac{3}{7}\)

Net Gain to US = \(-0.4t(48-1.2t) < 0\)

3. The purpose of the question is to show that, if the goal is to increase domestic production, production subsidies are more effective (less costly) than tariffs. To illustrate this, use the model of problem 1, and assume the government’s goal is to increase production to 90 units.

a) To increase domestic production to 90 with a tariff requires a domestic price of 30, hence a tariff of 15.

Using the results from Problem 1, domestic consumption falls to 115. The welfare impact \((t=15)\) are:

\[\Delta PS = 45t + 1.5t^2 = 1012.5\]
\[\Delta CS' = -145t + t^2 = -1950\]
\[TR = 100t - 5t^2 = 375\]

Net Welfare Change = \(-562.5\)

b) With a production subsidy of 15, the consumer price remains at 15 (the world price) and consumers are unaffected. The price received by producers increases to 30, so the change in producer surplus is the same as (a). However, instead of tariff revenue there is the cost to taxpayers of the subsidy. Hence, with a production subsidy:

\[\Delta PS = 45t + 1.5t^2 = 1012.5; \quad Tax Revenue = -15 \cdot (90) = -1350; \quad Net Welfare Change = -337.5\]

c) Clearly, the production subsidy is more efficient since it does not involve a loss to consumers (in terms of Figure 1, the increase in producer surplus is area \(\{15,A\ast,A,[15+t]\}\), and the cost of the subsidy is \(\{15,E,A,[15+t]\}\), so the deadweight loss is area \(\{A\ast,E,A\}\). With the tariff there is the additional deadweight loss of \(\{B,B\ast,F\}\), due to the change in consumption.

d) An import tariff raises the price to both consumers and producers; hence, it acts like a tax on consumers (they pay more) and as a subsidy to producers (they receive more for their output). A direct production subsidy has the same impact on producers, but does not affect consumers; a tax on consumption (regardless of where the good is produced) would not affect the price producers receive (given the world price), but would raise consumer prices. Hence, the tariff has the same impact as both subsidizing domestic production and taxing domestic consumption.

4. Given the production functions:

a) \(Q_c = 2(L_c)^{1/2}K^{1/2}\), \(Q_f = 2L_f\) implies: \(TC_f = WL_f = W\left(\frac{Q_f}{2}\right)\), \(TC_c = WL_c = W\left(\frac{Q_c^2}{4K}\right)\)

Since profit maximization implies price equals marginal cost:

\(P_f = MC_f = (W/2)\) and \(P_c = MC_c = W\left(\frac{Q_c}{2K}\right)\) or: \(Q_c^* = (2P_cK/W)\)

If good \(F\) is to be produced, we must have \(P_f = (W/2)\) (if \(P_f < (W/2)\), then good \(F\) will not be
produced; it is not possible to have \( P_f > \left( \frac{W}{2} \right) \) since there will be excess profits in producing \( F \), and \( F \) producers will want to hire more labor. But this will bid up the wage until \( P_f = \left( \frac{W}{2} \right) \).

Concentrating on an interior solution (where both goods are produced) we have:

**Supply:** \[ Q_f^* = (2P_fK/W) = (P_fK/P_f) = \rho K = 16 \rho \] where: \( \rho \equiv (P_c/P_f) \)

\[ Q_f^* = 2(\bar{L} - L_c) = 2(\bar{L} - (Q_c^2/4K)) = 2(\bar{L} - (\rho^2K^2/4K)) = 2(100 - 4\rho^2) , \]

since \( \bar{L} = 100, K = 16 \)

This solution require \( \rho \leq 5 \); if \( \rho > 5 \) only good \( C \) is produced.

For preferences: \( U = D_f + 20D_c - \left( \frac{D_c^2}{8} \right) \) demands are found by setting MRS= price ratio:

\[ \left( \frac{\partial U/\partial Q_c}{\partial U/\partial Q_f} \right) = \left( \frac{P_c}{P_f} \right) \rightarrow \left( \frac{20 - (D_c/4)}{20} \right) = \rho . \] Hence:

**Demand:** \( D_c = 80 - 4\rho; \) \( D_f = \frac{(I - P_c(80 - 4(P_c/P_f)))}{P_f} \); \( \rho \leq 20 \)

(Note that if the relative price of \( C \) is too high, demand for \( C \) will be zero; and if income is too low, demand for \( F \) will be there. For simplicity, we focus on cases where both goods are consumed).

The autarky equilibrium thus satisfies:

\( Q_c = D_c \rightarrow 16\rho = 80 - 4\rho \), and hence the autarky price: \( \rho^A = 4 \)

Relative price only matters because doubling output prices will double the wage rate. Hence, marginal cost and price both double, so supply is changed. On the demand side, income doubles, but prices also double, so demand is unchanged. The real question is: what does a price of “4” mean. There is no absolute standard for prices: rather it is “4 units of good \( F \) per unit of good \( C \”).

b) As given in the question, the world prices are such that \( P_f^w = 1, \) \( P_c^w = 2 \), and hence \( \rho^w = 2 \).

Thus, under free trade the country would import good \( C \). Under free trade, with domestic prices equal world prices, we have:

\( Q_f^* = 16\rho = 32 \), \( Q_c^* = 2(100 - 4\rho^2) = 168 \), \( D_c = 80 - 4\rho = 72 \) so:

**Imports of good \( C \):** \( M_c = D_c - Q_c = 40 , \)

**Balance of trade equilibrium implies:** \( P_fX_f = P_cM_c \rightarrow X_f = \rho M_c = 2 \cdot 40 = 80 \),

Hence, domestic consumption of \( F \) (production less exports) is: \( 168 - 80 = 88 \)

c) A 20% import tariff results in the following domestic prices:

\( P_c^d = P_c^w (1 + t) = 2.4; \) \( P_f^d = P_f^w = 1 \) hence: \( \rho^d = 2.4 \)
As long as good $F$ is still produced the domestic “nominal” wage is unchanged \( W = 2P^d_f \), so the real wage in terms of good $F$ is unchanged. Since the domestic price of $C$ rises, the real wage in terms of $C$ falls and hence labor loses from this tariff (slightly different from the specific factor model because there is no specific factor in sector $F$).

The real return to capital will rise, of course (as predicted by the specific factor model):

\[
R = P^d_c \left( \frac{\partial Q_c}{\partial K} \right) = P^d_c \left( \frac{L_c}{K} \right)^{1/2}.
\]

From earlier:

\[
L_c = \left( \frac{Q^2_c}{4K} \right) \quad \text{and supply of $C$ is given by:} \quad Q^*_c = \rho^d K; \quad \text{hence:} \quad L'_c = \left( \frac{(\rho^d)^2 K^2}{4K} \right) \quad \text{and:} \quad \left( \frac{L'_c}{K} \right) = \left( \frac{(\rho^d)^2}{4} \right) \quad \text{implying:} \quad R = P^d_c \left( \frac{\rho^d}{2} \right) \quad \text{and:} \quad \left( \frac{R}{P^d_c} \right) = \left( \frac{(\rho^d)^2}{2} \right)
\]

Thus, increasing the domestic price of $C$ (and its relative price) raises the real return to capital, whether measured in terms of the import or export good.

Finally, domestic production and consumption are found by substitution with $\rho^d = 2.4$:

\[
Q^*_c = 16\rho = 38.4; \quad Q^*_f = 2 \left( 100 - 4\rho^2 \right) = 153.92; \quad D_c = 80 - 4\rho = 70.4 \quad \text{and thus:} \quad M_c = 70.4 - 38.4 = 32. \quad \text{From the balance of trade, since world price is unchanged:} \quad X_f = \rho^w M_c = 64; \quad \text{domestic consumption of $F$:} \quad D_f = Q_f - X_f = 89.92
\]

\[d) \] If the import tariff is removed, but an export tariff of 20% is imposed we have:

\[
P^d_f = P^*_f - t = 1 - .2P^d_f \rightarrow 1.2P^d_f = 1 \quad \text{or:} \quad P^d_f = (5/6); \quad P^d_c = P^*_c = 2
\]

\[
\rho^d = \left( \frac{P^d_c}{P^d_f} \right) = (12/5) = 2.4
\]

So, the relative price is the same with the two plans. Naturally, the nominal wage falls to \((5/3)\) but the real wage (in terms of its purchasing power of either $C$ or $F$) is the same as in part (c), with the import tariff. Similarly, for capital, the nominal return $R$ increases since $\rho^d$ increases, as does the real return in terms of either good. Finally, since production and consumption depend only on relative prices, all outputs, demands, and relative prices are the same as in $c$. There is no difference, in terms of the real economic impact, between an import and export tariff of comparable magnitudes.

\[e) \] With an import tariff of 20% and an export subsidy of 20%, both based on the world price, we have:

\[
P^d_c = 2 \left( 1 + t \right) = 2.4; \quad P^*_f = P^*_f + \text{subsidy} = P^*_f (1 + .2) = 1.2 \quad \text{and:} \quad \rho^d = \left( \frac{P^d_c}{P^d_f} \right) = 2
\]

So, these policies raise the nominal prices of all goods by 20%, and hence will increase the nominal wage by 20%, but relative prices will be unaffected and there will be no real effect (on consumption, production, trade) of these policies. In essence, they are like a devaluation of the country’s currency.