1. Consider a firm that produces bicycles using capital (machines) and labor. Its production function is given by:

\[ Q = AK^{1/2}L^{1/2} \]

where \( Q \) measures output (of bicycles), while \( K \) and \( L \) measure the amount of capital and labor, respectively, used to produce bicycles. The parameter \( A \) is a measure of the firm’s productivity (“know-how”); increases in \( A \) represent improved productivity.

Let \( W \) denote the wage the firm pays for each unit of labor, and let \( R \) denote the per unit cost of capital (machines). The firm’s costs are \( WL + RK \). In the short run the amount of capital the firm can use is fixed, while in the long run both labor and capital are choice variables.

a) Find the firm’s short run cost curve (that is, express costs as a function of output \( Q \), input prices \( (W,R) \), technology \( A \), and the fixed amount of capital \( K \)).

b) How do the following affect the marginal cost curve: (i) an increase in productivity (in \( A \)); (ii) an increase in \( W \); (iii) an increase in \( R \)?

c) Suppose that, in the long run, the amount of capital (as well as labor) can be varied. Find the firm’s long run cost curve (i.e., choose inputs to minimize cost for each output level). Compare its slope (elasticity) to the short run cost curve.

2. Let \( P \) denote the (constant) price at which the firm can sell its bicycles. Thus, the firm’s profits can be written as:

\[ \pi = PQ - C(Q, A, W, K, R) \]

where \( C(Q, A, W, K, R) \) denotes the firm’s short run cost curve (which you derived in problem 1).

**Assuming \( K=2 \) and \( R=5 \), and that the firm chooses its (short run) output level to maximize profits:**

a) Find the firm’s short run supply curve and the maximum profits of the firm (which depend on prices, capital, and productivity).

b) How do increases in productivity affect the firm’s profits and its output level (i.e., how do productivity increases shift the supply curve)? Explain why this happens.

c) Use the expression for maximum profits derived in part (a) to calculate how an increase in price from \( P = 10 \) to \( P = 20 \) affects the firm’s profits. Show that your answer is the same as the increase in “producer surplus” that is calculated using the area next to the supply curve between the two prices.

d) **For this part, let \( A=1, W=1 \) (and \( K=2, R=5 \)).** Use the firm’s short run cost curve to find the increase in costs when the firm increases output from \( Q = 10 \) to \( Q = 20 \). Calculate the area under the firm’s short run marginal cost curve (short run supply curve) between \( Q = 10 \) and \( Q = 20 \). Why is this area the same as the increase in costs you first calculated? Explain.
3. Consider a consumer who has the utility function: \[ U(x, y) = x + (30y - y^2) \]
where \((x, y)\) denotes the goods consumed by the individual. Let \(I\) denote the individual’s income, and \((P_x, P_y)\) denote the prices the individual pays for goods \(x\) and \(y\), respectively.

   a) Derive the individual’s demand functions for the two goods (that is, maximize utility subject to the budget constraint).

   b) Find the individual’s maximized utility as a function of income and prices (substitute the solution from part (a) back into the utility function). This result is called the *indirect utility function*.

   c) Suppose initially \(I = 1,000; P_x = 1; P_y = 4\); using your result from part (b), calculate the individual’s maximized utility.

   d) Next, suppose the individual discovers a store (Sam’s Club) at which she can buy good \(y\) at a price of \(P_y = 2\). However, she must pay a “membership fee” of \(F\) to be allowed to shop in this store. What is the maximum amount that she would pay for the right to shop in this store? Hint: derive your answer by using the indirect utility function from part (b) and choose \(F\) such that the maximized utility from \((I = 1,000; P_x = 1, P_y = 4)\) is the same as the maximized utility from \((I = [1,000 - F]; P_x = 1, P_y = 2)\).

   e) Using the person’s demand curve for good \(y\), calculate the consumer surplus associated with a decrease in price from \(P_y = 4\) to \(P_y = 2\). Compare your answers for parts (d) and (e).

4. Suppose we have the following market supply (S) and demand (D) curves for coffee:

   \[ S = 20P^d; \quad D = 6,000 - 10P^d \]

   where \(P^d\) denotes the domestic price of coffee (in cents/pound)

   a) Assuming there is no trade, find the equilibrium price of coffee and the quantity transacted.

   b) Suppose that the world price of coffee is $1/pound (i.e., the world price \(P^w = 100\)).
   Assuming the country can freely import or export coffee, how does free trade affect domestic price, production and consumption? Find the change in consumer surplus and producer surplus due to this move to free trade (a quantitative answer is required). Overall, does the country gain from free trade? Does everyone gain? Explain.

   c) Finally, assume that the country limits imports of coffee to 1500 (i.e., it imposes an import quota of 1500). How would this quota affect domestic price, output and consumption? Who gains and who loses from this policy?