1. Consider the “specific factor” model, with two goods (C, F). C is produced using labor (L) and a specific factor (K), whereas F is produced using labor and a specific factor (T). The total amount of labor is given by \( \bar{L} \), and labor is freely mobile between the two sectors. Production functions are given by:

\[
Q_c = \left( L_c \right)^{1/2} \left( K \right)^{1/2} ; \quad Q_f = \left( L_f \right)^{1/2} \left( T \right)^{1/2} ; \quad \left[ L_c + L_f \right] \leq \bar{L}
\]

a) Derive the production possibility frontier (ppf). Explain why the marginal rate of transformation (MRT) increases as more of a good is produced.

b) Derive the supply curve for each good (let \( \rho = \left( P_c / P_f \right) \) denote the relative price of good C).

c) Given output prices, show how the supply curve for each good and the return to each factor are affected by:
   (i) An increase in T.
   (ii) An increase in L.

d) Show how an increase in \( P_c \) affects all 3 factor prices. Explain your result (i.e., relate the impact to whether the factor is sector-specific or mobile).

e) Suppose demands for each good are: \( D_c = \left( Y / 2 P_c \right) \); \( D_f = \left( Y / 2 P_f \right) \) where \( Y \) denotes income, and \( P_i \) denotes the price of good \( i \). Find the equilibrium prices (output prices and input prices) for this economy [SIMPLIFYING HINT: set the relative supply of goods equal to the relative demand].

f) Assume \( \bar{L} = T = 20, K = 10 \). If the country can trade at the world relative price \( \left( P_c / P_f \right) = 2 \), which good will it export? Compared to autarky, how does trade at this price affect domestic production, returns to each factor, and overall welfare (use a graph to answer the last part)? Does everybody gain from trade?

g) Repeat part (e) for the case where world relative price is \( \left( P_c / P_f \right) = 1/2 \). Do your conclusions about who gains from trade change; is trade still beneficial overall?

2. (World Production Efficiency). The Ricardian model demonstrated that world production might be inefficient even if each country produced on its own production possibility frontier. We now explore this concept in the specific factors model.

Consider the specific factor model of problem one with two countries. Assume, for simplicity, the two countries have identical technologies but different endowments of the specific factors:

US: \( Q_c = \left( L_c \right)^{1/2} \left( K \right)^{1/2} ; \quad Q_f = \left( L_f \right)^{1/2} \left( T \right)^{1/2} ; \quad \left[ L_c + L_f \right] \leq \bar{L}_{US} = 20, K = 10, T = 20 \)

UK: \( Q^*_c = \left( L^*_c \right)^{1/2} \left( K^* \right)^{1/2} ; \quad Q^*_f = \left( L^*_f \right)^{1/2} \left( T^* \right)^{1/2} ; \quad \left[ L^*_c + L^*_f \right] \leq \bar{L}_{UK} = 20; \quad K = 20, T = 10 \)
a) Using your results from question 1, find the autarky equilibrium price in each country, and the equilibrium output levels.

b) Starting from the autarky equilibrium, is it possible to change output levels in each country to increase world output of both goods (given that each country still produces on its own production possibility frontier)? If so, illustrate how; if not, why not?

c) What condition on the marginal rate of transformation in each country must hold to guarantee that world output is efficient (that is, that there is no way to change output levels in each country in order to increase world output of both goods)? Demonstrate your answer.

d) In the Ricardian model efficient production required that at least one country specialize (produce only one good). Does efficiency also imply full specialization in this model? Justify your answer.

e) What will the pattern of trade be in this example? Will free trade achieve efficient production from a world perspective? Explain.

f) Suppose the US technology changed to: 
\[
Q_c = 2 \left( L_c \right)^{1/2} \left( K \right)^{1/2} \quad ; \quad Q_f = 2 \left( L_f \right)^{1/2} \left( T \right)^{1/2},
\]
indicating that the US was twice as efficient as the UK in producing each good. Would trade still be mutually beneficial? Would countries specialize in production? Explain.

3. (Simplified version of Heckscher-Ohlin model). Consider a country that can produce two goods: cloth (C) and food (F) using two inputs: labor (L) and capital (K). Production of each good requires inputs to be used in fixed proportions as follows (these are called Leontief technologies):

To produce cloth requires three units of labor and two units of capital per unit output of cloth.
To produce food requires two units of labor and three units of capital per unit output of food.

Let \( L, K \) represent the total amount of labor and capital available in the economy, let \( P_c, P_f \) denote the prices of output and let \( W, R \) denote the prices of labor and capital respectively.

a) Find the production possibility frontier (ppf) for this economy and sketch it (Hint: it is defined by two linear inequalities that restrict total labor, and total capital, usage).

(i) Show how an increase in the supply of capital shifts the ppf and the production point where both factors are fully used (in this simple version, there is a unique production point that represents full employment of both inputs). Compare to the specific factor model.

b) Find input prices \( \{ W, R \} \) in terms of output prices, assuming both goods are produced and both factors fully used (Hint: setting price equal to marginal cost for each good gives you two linear equations in two unknowns. Solve these equations for factor prices). Show how an increase in the price of cloth will affect factor prices.

(i) Given output prices, how will an increase in the supply of capital affect input prices?
Explain and compare to the specific factor model.

e) Finally, assume that two countries (the US and Mexico) have identical tastes and technology, but assume the U.S. has more capital and Mexico has more labor. Assuming the relative demand (ratio of demand for cloth to demand for food) is independent of income, **show how autarky goods prices and factor prices differ between the two countries, then show how trade will affect factor prices in each country.** Will factor prices be equalized between the two countries? Does free trade benefit everybody in each country?

4. **(10 point extra credit) (More sophisticated version of Hecksher-Ohlin model).** As in question #3, suppose there are two goods (C and F) and two inputs (K and L). However, the technology for producing each good now allows smooth substitutability among the inputs. In particular, the production functions are given by:

\[
Q_c = K_c^{1/4} L_c^{3/4}; \quad Q_f = K_f^{3/4} L_f^{1/4}
\]

where \(\{K_c, L_c\}\) are the inputs (capital, labor) used in sector C and \(\{K_f, L_f\}\) are the inputs used in sector F. Let \(W\) denote the wage rate (price of L) and \(R\) the rental rate (cost of using K, capital). Finally, let \(P_c, P_f\) denote the output prices of goods C and F, respectively.

(a) Derive the cost function for each good by minimizing cost for a given output level. (This is similar to question 1, problem set 1). **Which good is capital intensive?** Why?

(b) **Given output prices,** show how an increase in the available supply of labor will change output of each good. **Use your result to predict which good the labor-abundant country will be likely to export.**

(c) Assuming both goods are produced (so that Price=Marginal Cost for each good), show how an increase in \(P_f\) will affect factor prices (\(W, R\)). Does either factor price rise proportionally more than \(P_f\)? If so, explain which one and why.

(d) Use your answer to parts (b) and (c) to explain how trade will affect the distribution of income in a capital-abundant country (like the US). Will everybody gain from trade? Explain.

(e) Assume the U.S. is capital-abundant. Which group in the U.S. is likely to favor import tariffs and which group is likely to impose trade restrictions? Explain.

(f) If the US were capital-abundant (compared to other countries) and also had the superior technology for producing good F, which good would the US export? Would free trade equalize factor prices?