1. Consider the Ricardian model, with two countries, the U.S. and the U.K. The US has 100 worker/hours, the UK 150 worker/hours. Labor productivities in each country are given by:

<table>
<thead>
<tr>
<th>Country</th>
<th>Shirts/hour</th>
<th>Corn/bushel/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) The U.K has the absolute advantage in shirts and the U.S. in corn because the U.K has higher productivity in shirts than the U.S. does and the U.S. has higher productivity in corn than does the U.K.

(i) For this case it is clear the U.S. has the comparative advantage in corn and U.K in shirts. The opportunity cost of producing shirts in U.S. is 2 bushels of corn, the opportunity cost of shirts in U.K. is one-half (1/2) bushel of corn. Conversely, opportunity cost of corn in U.S. is one-half (1/2) shirt, in U.K. it is 2 shirts.

(b) Derive and sketch the production possibility frontier for each country.

(i) Suppose the U.S. originally produces and consumes 100 shirts and 200 bushels of corn, while the UK produces and consumes 300 shirts and 150 bushels of corn. Let the UK increase shirt production by “$S$” units (up to a maximum production of 600 in the UK); this will cause UK corn production to fall by $(S/2)$. Let the U.S. increase corn production by “$D$” units (up to a maximum of 400 bushels); this will cause U.S. shirt production to fall by $(D/2)$. Thus, from a world perspective:

Change Corn Production = $D - \left( \frac{S}{2} \right) > 0$ if $D > \left( \frac{S}{2} \right)$
Change Shirt Production = $S - \left( \frac{D}{2} \right) > 0$ if $D < 2S$

Thus, for any change in the two countries such that: $2S > D > \left( \frac{S}{2} \right)$ world output of both goods increases.

{e.g., if the UK increases shirt production by 100 and U.S. increases corn production by 100). Naturally, once one country is fully specialized, it can produce no more of the good in which it has a comparative advantage.
(ii) The world production possibility frontier is such that at most only one country would produce both goods. Thus, for low levels of corn output, the UK should specialize in shirts, while the US produces both goods; while for large levels of corn output, the US specializes in food. Formally:

\[ C_T \leq 400; \quad C^{uk} = 0, S^{uk} = 600; \quad S^{us} = 200 - \left( \frac{C_T}{2} \right); \quad S^T = 800 - \left( \frac{C_T}{2} \right) \]

\[ 700 \geq C_T \geq 400; \quad C^{us} = 400, S^{us} = 0; \quad S^{uk} = 600 - 2C^{uk}; \quad S^T = 600 - 2\left( \frac{C_T}{2} - 400 \right) \]

(c) In the absence of trade, autarky (no trade) relative prices would be: \( P_s / P_c \) (bushels / shirt), and \( P_s / P_c \) (bushel / shirt), where \( P_c \) is the price of corn, \( P_s \) is the price of shirts, and the superscript indicates which country.

US Real Wages: \( (W / P_c)^{us} = 4 \) bushels / hour; \( (W / P_s)^{us} = 2 \) shirts / hour

UK Real Wages: \( (W / P_c)^{uk} = 2 \) bushels / hour; \( (W / P_s)^{uk} = 4 \) shirts / hour

(d) We know that if the world relative price, \( (1/2) < \left( \frac{P_s}{P_c} \right)^w < 2 \), then each country will specialize. Hence, the method is to assume each country does specialize, calculate demand under that assumption, and see if the resulting price is consistent with the assumption (this will happen if the demand curve looks like D1 in the figure below). If the equilibrium relative price you calculate is less than (1/2), you know the assumption was false, and in equilibrium the UK will produce both goods (the demand curve looks like D2). Similarly, if the equilibrium relative price you calculate was greater than 2, then again the assumption is false and the US will produce both goods (the demand curve looks like D3 in the figure below).

So, assuming each country specializes, let the US wage be \( W^{us} \) and the UK wage \( W^{uk} \). Then, since each country specializes, \( P_c = \left( \frac{W^{us}}{4} \right) \) and \( P_s = \left( \frac{W^{uk}}{4} \right) \). Thus, income in each country is:

\( I^{us} = W^{us} \cdot 100 = 400P_c \) and \( I^{uk} = W^{uk} \cdot 150 = 600P_s \)

Hence, total demand for corn is:

\[ D_c = \left( \frac{I^{us} + I^{uk}}{2P_c} \right) = \left( \frac{400P_c + 600P_s}{2P_c} \right) = 200 + 300 \left( \frac{P_s}{P_c} \right) \]

Setting demand equal to supply implies:

\[ D_c = S_c = 400 \rightarrow 200 + 300 \left( \frac{P_s}{P_c} \right) = 400 \rightarrow \left( \frac{P_s}{P_c} \right) = \frac{2}{3} \]

Since this price is within the assumed ranged (picture D1) the equilibrium is one in which both countries specialize, with the equilibrium prices as given, which implies \( \left( \frac{W^{us}}{W^{uk}} \right) = \left( \frac{4P_c}{4P_s} \right) = \left( \frac{3}{2} \right) \). Note that shifts in demand, by changing the relative price of output, will change the relative wage between countries.
The post trade real wage:
US: \( (W/P_c)^{US} = 4, \ (W/P_s)^{US} = (4P_c/P_s) = 6 > 2 \)
UK: \( (W/P_s)^{UK} = 4, \ (W/P_c)^{UK} = (4P_s/P_c) = (8/3) > 2 \)

So, in both countries, the real wage increases in terms of the import good. In this example, the US gains more than the UK because the UK is larger, and hence the post-trade relative prices are closer to the pre-trade prices in the UK than in the US.

If you wanted to calculate utility, you would just calculate the consumption bundle (pre and post trade) and plug back into the utility function. Since half of income is spent on each good this is pretty easy. I will illustrate for the US, leaving the UK to you:

US, pre-trade: \( D_c = (W/2P_c) = 2, \ D_s = (W/2P_s) = 1 \rightarrow U = D_c \cdot D_s = 2 \) since pre-trade \( W = 4P_c = 2P_s \)
US, post-trade: \( D_c = (W/2P_c) = 2, \ D_s = (W/2P_s) = 3 \rightarrow U = D_c \cdot D_s = 6 \) as post-trade \( W = 4P_c = 6P_s \)

**NEW PRODUCTIVITY TABLE:**

<table>
<thead>
<tr>
<th></th>
<th>Shirts</th>
<th>Corn</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Kingdom</td>
<td>4 shirts/hour</td>
<td>2 bushels/hour</td>
</tr>
<tr>
<td>United States</td>
<td>5 shirts/hour</td>
<td>10 bushels/hour</td>
</tr>
</tbody>
</table>

(e) Now the U.S. has an absolute advantage in both goods.

(i) However, the U.K. still has a comparative advantage in shirts as the opportunity cost of shirts production (in terms of corn) is lower in the UK than in the US.
(ii) This increase in labor productivities in the U.S. will not affect autarky (no trade) relative prices of goods in the U.S. as the opportunity cost is unchanged. The U.S. real wage will increase due to the productivity increase. The impact of this change in labor productivities on post-trade prices is that the relative price of corn will fall since greater U.S. corn production creates excess supply (remember that the US will not produce shirts and the UK will not produce food if both countries specialize). The UK will gain by the increased productivity in the US (due to cheaper food imports); the US is helped by the higher productivity but hurt by cheaper prices for food exports so it is possible (though not likely) that the US is actually worse off due to the increased US productivity.

(f) Same process as earlier; if the UK increases shirt production by “S” units, UK food production falls by (S/2). If the U.S. increases corn production by “D” units, U.S. shirts production falls by (D/2). Total world output of both goods increases if $2S > D > (S/2)$.

Same process as earlier; start by assuming each country specializes and see if the equilibrium relative price you calculate is consistent with that assumption.

Pre-trade, US: $W = 5P_s = 10P_c$; Pre-trade, UK: $W = 4P_s = 2P_c$

Note that the US real wage is higher in terms of either good (because it has an absolute advantage in both).

Post-trade: If the US specializes in corn and the UK in shirts, then:

$Q_{sc}^{us} = 10 \cdot 100 = 1,000$; $Q_{sk}^{uk} = 4 \cdot 150 = 600$; $W^{us} = 10P_c$; $W^{uk} = 4P_s$; $I^{us} = 100W^{us}$; $I^{uk} = 150W^{uk}$

Hence, world demand is: $D_c = \frac{I^{us} + I^{uk}}{2P_c} = \frac{1,000P_c + 600P_s}{2P_c} = 500 + 300\left(\frac{P_s}{P_c}\right)$

Setting supply equal to demand: $S_c = D_c \rightarrow 1,000 = 500 + 300\left(\frac{P_s}{P_c}\right) \rightarrow \left(\frac{P_s}{P_c}\right) = \left(\frac{5}{3}\right) < 2$

Thus, the equilibrium is one in which both specialize. Note that the UK terms of trade improve due to the increased productivity in the US, whereas the US terms of trade fall due to the increased supply of exports.

The post trade real wages:

US: $\left(\frac{W}{P_s}\right)^{us} = 10$, $\left(\frac{W}{P_c}\right)^{us} = \left(\frac{10P_s}{P_c}\right) = 6$ (which is higher than pre-trade, and coincidentally the real wage in terms of imports is the same as with the lower productivity)

UK: $\left(\frac{W}{P_s}\right)^{uk} = 4$, $\left(\frac{W}{P_c}\right)^{uk} = \left(\frac{4P_s}{P_c}\right) = \left(\frac{20}{3}\right) > 2$

Clearly, in this case, both countries gain from the increased productivity in the US.

(g) If labor migration is allowed, then the answer depends upon the labor productivity of the immigrant workers. If the new immigrants who move to the US have the same productivity as the original US workers, then under the first (productivity) table, all shirts will be produced in UK, all food in US, and labor will move according to demands for the two goods (to equalize wages). For the second table, both goods would be produced in the US and all labor would move to US. Of course, if the migration of workers did not affect their productivity (i.e., UK workers who moved to the US had the same productivity as UK workers who stayed at home) then the migration would not have any impact on output.
2. To illustrate how the model can be extended to more than two countries, consider the following example.

<table>
<thead>
<tr>
<th>Country</th>
<th>Shirts/hour</th>
<th>Corn/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Mexico</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>United States</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

(a) The US has an absolute advantage in corn, the UK in shirts, France and Mexico in neither.
(b) Compared to the UK, France has a comparative advantage in corn (to produce 1 corn cost 2/3 shirt in France and 1 shirt in the UK).
(c) Compared to the US, France has a comparative advantage in shirts (in the US it costs $\frac{1}{2}$ shirt to produce 1 corn or 2 corn to produce 1 shirt; in France it costs 1.5 corn to produce 1 shirt).
(d) Autarky relative prices in each country reflect the opportunity cost; thus:

\[
\left(\frac{P_s}{P_c}\right)_{us} = 2; \quad \left(\frac{P_s}{P_c}\right)_{uk} = 1; \quad \left(\frac{P_s}{P_c}\right)_{mex} = \frac{1}{2};
\]

\[
\left(\frac{P_s}{P_c}\right)_{fr} = \left(\frac{3}{2}\right)
\]

where the units (omitted) are: bushels/shirt.

(e) The world supply curve for shirts looks like:

(f) The world production possibility frontier consists of linear segments, each corresponding to the ppf of one of the countries. As in the supply curve above, the first country to produce shirts is Mexico, then UK, then France, then the US.
In the figure. CA is the Mexican p pf, with all other countries producing corn; AB is the UK p pf, with Mexico specialized in shirts, US & France in corn; BD is the French p pf, with Mexico and the UK specialized in shirts, the US in corn; and finally DE is the US p pf, with the other 3 countries specialized in shirts.

(g) Under free trade, the US would export corn and Mexico would export shirts. The trade pattern of the other countries cannot be determined without knowing country sizes and demands. However, we do know if France exports shirts, then the UK will also export shirts; and if the UK exports corn, then France will also export corn.

3. To illustrate how the model can be extended to more than two goods, consider the following example:

<table>
<thead>
<tr>
<th>Hourly Labor Productivity by Country and Good</th>
<th>Corn</th>
<th>Pants</th>
<th>Radios</th>
<th>Shirts</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>8 bushels/hour</td>
<td>2 pants/hour</td>
<td>1 radios/hour</td>
<td>6 shirts/hour</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>2 bushels/hour</td>
<td>2 pants/hour</td>
<td>2 radios/hour</td>
<td>3 shirts/hour</td>
</tr>
</tbody>
</table>

(a) The U.S. has an absolute advantage in corn and shirts; the U.K. has an absolute advantage in radios; they have equal productivity in pants.

(b) The U.S. has a comparative advantage in corn vs any other good because US labor productivity in producing corn is 4 times UK labor productivity in corn production, whereas the US relative productivity (compared to the UK) is lower in every other good. To say the same thing another way, the opportunity cost of producing corn in the US - measured in any of the other 3 goods - is lower than in the UK.

Does the US have a comparative advantage in shirt production compared to the UK? Now it depends upon which good we are comparing to: i.e., the opportunity cost of producing shirts as compared to producing what other good. Since US labor productivity in shirts is twice that of the UK, it is clear that - in comparing corn and shirts - the UK has the comparative advantage in shirts, whereas the US has the comparative advantage in corn. However, in comparing shirt production to either radio production or pants production, the US does have a comparative advantage in shirts, whereas the UK would have a comparative advantage in the other good. Again, in other words, the opportunity cost of producing shirts - as measured in terms of corn - is higher in the US than in the UK, **but** the
opportunity cost of producing shirts - as compared to producing radios (or pants) is lower in the US than in the 
UK.

Clearly, the UK has a comparative advantage in radios - as compared to any other good, in that the 
opportunity cost of producing radios in the UK - measured in terms of any of the other goods - is lower in the UK 
than in the US. Next would come pants - in that the opportunity cost of producing pants in the UK - measured in 
terms of shirts or corn - is lower in the UK than in the US.

(c) **Autarky prices.** Of course, only relative prices matter. Letting, for example, corn be the item in which 
values are measured, we have for each country:

US: \[ P_{\text{us}}^{\text{pant}} = 4 P_{\text{us}}^{\text{corn}}; \quad P_{\text{us}}^{\text{radio}} = 8 P_{\text{us}}^{\text{corn}}; \quad P_{\text{us}}^{\text{shirt}} = \frac{8}{6} P_{\text{us}}^{\text{corn}} = \frac{4}{3} P_{\text{us}}^{\text{corn}} \]

UK: \[ P_{\text{uk}}^{\text{pant}} = \frac{1}{3} P_{\text{uk}}^{\text{corn}}; \quad P_{\text{uk}}^{\text{radio}} = \frac{1}{3} P_{\text{uk}}^{\text{corn}}; \quad P_{\text{uk}}^{\text{shirt}} = \frac{2}{3} P_{\text{uk}}^{\text{corn}} \]

(d) If trade were allowed, we would predict that the US would definitely export corn, and that the UK 
would definitely export radios, because each country has a comparative advantage in that good as compared to any 
other good. What other goods the countries exported would depend on demand and country size. However, if the 
US only exports two goods, we know it would be corn and shirts (because the US has a comparative advantage in 
shirts, as compared to either pants or radios); and if the US exported 3 goods, it would be corn, shirts and pants.

Similar logic applies to the UK (by just looking above at what the US is not exporting).

(e) Let \( W \) be the US wage, and \( W^* \) the UK wage. Then the ratio of marginal costs for each good looks 
like this:

\[
\frac{MC_{\text{us}}^{\text{corn}}}{MC_{\text{us}}^{\text{shir}}} = \frac{W/8}{W^*/2} = \frac{\omega}{4} \quad \text{where:} \quad \omega \equiv \frac{W}{W^*}. \quad \text{Similarly, for the other goods:}
\]

\[
\frac{MC_{\text{us}}^{\text{shirt}}}{MC_{\text{us}}^{\text{shir}}} = \frac{W/6}{W^*/3} = \frac{\omega}{2}; \quad \frac{MC_{\text{us}}^{\text{pant}}}{MC_{\text{us}}^{\text{shir}}} = \frac{W/2}{W^*/2} = \omega; \quad \frac{MC_{\text{us}}^{\text{radio}}}{MC_{\text{us}}^{\text{shir}}} = \frac{W/1}{W^*/2} = 2\omega
\]

Goods will be produced where the cost is lowest. Hence:

If \( \omega < \frac{1}{2} \) all goods will be produced in the US and none in the UK (this cannot be an equilibrium); If \( \frac{1}{2} < \omega < 1 \) then radios will be produced in the UK, all other goods in the US; If \( 1 < \omega < 2 \) then radios and pants will be produced in the UK, the other two goods in the US; If \( 2 < \omega < 4 \) then corn will be produced in the US, the other 3 goods in the UK; If \( \omega > 4 \) all goods will be produced in the UK and no goods in the US (this cannot be an equilibrium).

(Naturally, if we have an equality, such as \( \omega = 2 \), then one of the goods (here, shirts) could be produced in both 
countries.)

(f) The story is roughly like this. The relative labor supplies, plus demands, determine the equilibrium 
relative wage (\( \omega \)). An increase in the US labor force, given relative wages, will increase the supply of US export
goods (but not change which goods are exported); this will lead to a decline in the relative price of these goods compared to US import goods. But this implies that the US wage falls relative to the UK wage; if it falls enough, the US will start exporting another good. Hence, the relative price of US exports falls, and the US real wage falls in terms of import goods. By symmetry, the relative price of UK exports rises, and the UK real wage rises in terms of import goods. The graph would look something like this:

![Graph showing relative labor supply and demand](image)

The figure shows a situation in which the US is initially specialized to corn production, with the relative wage between 2 and 4. The increase in US labor supply (the rightward shift of the vertical line) causes the US relative wage to fall; as shown, the shift is large enough to cause the relative wage to fall to 2 and the US produces corn and shirts (the UK also produces shirts). As the US labor supply increases within this region, the US production of shirts grows, while the UK shifts resources from shirts to pants and radios. However, as long as the relative supply intersects the relative demand on the horizontal segment, the labor supply change has no price effects. However, eventually the UK will be producing no shirts and further US labor supply increases will necessitate a decrease in shirt and corn prices (hence, a decrease in the US relative wage) to induce people to buy the extra output.