Problem Set 1 - Due Thursday, September 9, 2004
(Producer and consumer surplus)

1. Consider a firm with total cost curve: \( TC(q) = 4 + 2q + \left(\frac{q^2}{4}\right) \) where 4 is fixed cost.

   (a) Find the firm’s marginal cost curve and, given output price, \( p \), the firm’s short run supply curve.

   (b) Find the firm’s long run supply curve. (NOTE: In the short run, the firm must pay the fixed cost even if it does not produce any output. In the long run, the firm can shut down – i.e., set \( q = 0 \) - and avoid the fixed cost).

   (c) Using the firm’s supply curve, show how much costs increase when the firm increases output from \( q = 10 \) to \( q = 20 \).

   (d) The firm currently sells its output at a price of 7. Suppose price increases to 12; assuming (competitive) profit maximization, use the supply curve to show how much profits increase due to this price change. (You can verify your result by using the original total cost curve and the definition of profits).

   (e) Again, assume price is currently 7. Suppose a large retailer (like WalMart) offers the firm the following deal: it will pay a price of 15 for the firm’s output, but it must produce and sell to the retailer 36 units of output. Use the firm’s supply (marginal cost) curve to calculate how accepting this deal would affect the firm’s profit.

2. Consider a consumer who has the following (quasi-linear) utility function:
\[
U(x, y) = x + 30y - \left(\frac{y^2}{4}\right)
\]
where \((x, y)\) denotes the goods consumed by the individual. Let \( I \) denote the individual’s income, and \((P_x, P_y)\) denote the prices the individual pays for goods \(x\) and \(y\), respectively.

   a) Derive the individual’s demand functions for the two goods (that is, maximize utility subject to the budget constraint).

   b) Find the individual’s maximized utility as a function of income and prices (substitute the solution from (a) into the utility function). This result is called the indirect utility function.

   c) Suppose initially \( I = 1,000; \quad P_x = 1; \quad P_y = 5 \); using your result from part (b), calculate the individual’s maximized utility.

   d) Suppose the government imposes a tax of 5 units on good \( y \), which raises the price to \( P_y = 10 \) (the producer price does not change). Calculate:

   i. How this tax affects the person’s utility, consumption of each good and how much tax revenue is raised (call the tax revenue \( T_i \)).
ii. Assume, instead of a tax on good \( y \), the government imposes a “flat tax” of \( F \) (economists call it a lump sum tax) on the individual. Find the flat tax that has the same affect on the person’s utility as does the tax on good \( y \) from part i (i.e., find \( F \) so that the consumer’s maximized utility is the same in the situation \( (I = 1,000; P_x = 1, P_y = 10) \) as it is in the situation \( (I = [1,000 - F]; P_x = 1, P_y = 5) \).

iii. Compare the tax revenue from the flat tax to that from the tax on good \( y \). What is the deadweight loss (inefficiency) due to the tax on good \( y \)?

e) Using the person’s demand curve for good \( y \), calculate the change in consumer surplus associated with an increase in price from \( P_y = 5 \) to \( P_y = 10 \). Compare your answers for parts (d) and (e).

f) Assume initially \( I = 1,000; P_x = 1; P_y = 12 \). The consumer receives in the mail an offer that, if they join the “y Club”, they can buy as many (or as few) units of good \( y \) as they want at a price of \( P_y = 6 \) per unit. However, there is a membership fee to join this club. What is the maximum fee the person would pay to join?

g) Finally, suppose the same situation as in part f except that, if the consumer joins the Club they commit to buy at least 54 units. How does that change your answer to part f? Show how you can answer the question using the consumer’s demand curve.

3. Brazil has the following market supply (\( S \)) and demand (\( D \)) curves for coffee:

\[
S = 60P^d; \quad D = 4,000 - 20P^d
\]

where \( P^d \) denotes the domestic price of coffee (in cents/pound)

a) Assuming there is no trade, find the equilibrium price of coffee and the quantity transacted.

b) Suppose that the world price of coffee is $1/pound (i.e., the world price \( P^w = 100 \)). Assuming Brazil can freely import or export coffee at this price, how does free trade affect domestic price, production and consumption? Find the change in consumer surplus and producer surplus due to this move to free trade (a quantitative answer is required). Overall, does the country gain from free trade? Does everyone gain? Explain.