The purpose of this problem set is to review concepts from Econ 301 and to illustrate how these concepts can be used to answer “real world” policy questions. The specific question is:

Consider a country that does not trade with the rest of the world. If the government of this country subsidizes clothing sales, who gains and who loses and how is overall welfare affected by this policy?

To answer this question, we need the demand curve and the supply curve. That is the purpose of the questions that follow.

1. Consider a perfectly competitive firm which produces shirts, and let \( y \) denote the firm’s output. The firm’s total costs are given by:
\[
TC(y) = 5y + \left(\frac{y^2}{4}\right); \quad y \geq 0
\]
Let the price at which the firm sells its shirts be denoted \( p_y^f \) (superscript \( f \) stands for firm). Then:

a) The firm’s profits are given by:
\[
\pi(y, p_y^f) = p_y^f y - TC(y) = \left\{ p_y^f y - 5y - \left(\frac{y^2}{4}\right) \right\}
\]

i. Find the output level, \( \hat{y} \left( p_y^f \right) \), which maximizes profits.

ii. Show that the firm’s supply curve (which shows output as a function of price) is the firm’s marginal cost curve.

iii. Use the firm’s supply curve to calculate how much production costs increase when output increases from \( y = 20 \) to \( y = 30 \). Use the cost curve to verify your answer.

iv. Using your answer to part (i), calculate the firm’s maximized profits as a function of price:
\[
\pi^* \left( p_y^f \right) = p_y^f \hat{y} \left( p_y^f \right) - 5\hat{y} \left( p_y^f \right) - \left( \hat{y} \left( p_y^f \right) \right)^2 / 4
\]

b) The firm currently sells output at a price of \( 15 \) but it has found new markets where it can sell all its output at a price of \( 20 \). Assuming competitive profit maximization, use the supply curve to show (graphically and numerically) how much profits increase due to this price change. Verify your answer using the profit function calculated in part a(iv).

c) Modify part (b) by assuming the firm must continue to sell its original output of 20 units in the old market at a price of \( 15 \), but is allowed to sell additional units in this new (e.g., a foreign) market at a price of \( 20 \). If it sold 10 units there at a price of \( 20 \) (and continued to sell the 20 units in its home market at a price of \( 15 \)), how much would its profits increase? Give a numerical answer and show graphically how to calculate this answer.

2. Consider a consumer who consumes two goods, food and clothing. Let \( f \) denote her consumption of food and \( \hat{y} \) her consumption of clothing. Let \( p_f, p_y \) denote the prices of food and clothing, and let \( I \) denote her income. The person has the following (quasi-linear) utility function:
The individual’s budget constraint is: 
\[ \{ p_f f + p_y \hat{y} \leq I \} \]

a) Set up the utility maximization problem and **derive the individual’s demand functions** for both goods (assume a solution where both goods are consumed).

i. Find the individual’s **maximized utility** by substituting the demand solutions back into the utility function {this is called the person’s indirect utility function}.

ii. Discuss how an increase in the price of good \( y \) affects maximized utility and interpret. Take the (partial) derivative of the indirect utility function with respect to \( p_y \); what does this equal?

b) Consider a consumer, with income \( I=1000 \), who can buy goods at the prices \( p_f = 1, p_y = 15 \). A new mall brings to town a discount store (Costco) which sells good \( y \) at the price \( p_y = 5 \).

i. Find the consumer’s purchases at this price. Will the consumer be better or worse off as a result of the opening of this store?

ii. Suppose the store charges individuals a one-time fee of $F to shop in the store. What is the **maximum** amount this individual would be willing to pay to shop in the store (the alternative is to continue to pay the price \( p_y = 15 \) charged by other stores in town)? Give a numerical answer.

iii. Using the demand curve, show graphically how to calculate your answer for part ii. {This area is called the change in consumer surplus}.

3. **Efficiency of markets**. Use the supply curve from question 1 and the demand curve from question 2 and **find the equilibrium market price and output level** {equilibrium requires supply equal demand, or \( y^* = \hat{y} \)}. In essence, you are assuming there are a large number of identical producers and consumers, but it is simpler to work with just one of each.

a) Suppose you were a dictator and you could choose the output level and price that would maximize the sum of producer and consumer surplus. What output level would you choose? How does it compare to the equilibrium output level you calculated above?

b) Next, assume the government gives producers a production **subsidy** of 15 per shirt sold {thus, the producer price exceeds the consumer price by 15, i.e., \( p_f^f = (p_y + 15) \)}. Show how this subsidy affects: (i) equilibrium consumer price (\( p_y \)) and producer price (\( p_y^f \)); (ii) equilibrium output; (iii) consumer surplus and (iv) producer surplus. Compare the total change in producer and consumer surplus to the cost to the government (and hence taxpayers) of the subsidy, and discuss how the subsidy affects overall efficiency. **Show graphically this change in efficiency**.

c) Suppose there is no subsidy but the government now permits trade with the rest of the world. The world price is 20, and the country – since it is small – can import or export as much of the good as it wants at this world price. Starting from the original equilibrium (with no subsidy), show how international trade, at the world price of 20, affects consumer surplus, producer surplus, and overall efficiency. Show the net gain – or loss – from this trade graphically and give an economic explanation of why this area represents the overall welfare change from trade.