1. Consider a simplified version of the Heckscher-Ohlin model with the following technology:

**To produce food:** 1 unit of labor and 3 units of capital are required for each unit of F.

**To produce manufactures (M):** 3 units of labor and 1 unit of capital are required for each unit of M.

$L, K$ represents the total amount of labor and capital available in the economy, $P_F, P_m$ denote the prices of output and $W, R$ denote the prices of labor and capital respectively.

**a) Find production costs and output price in terms of factor prices.**

The hint gives the answer for manufactures; just apply the same reasoning to food.

$$TC(M) = Q_m(3W + R) \to MC(M) = (3W + R); \quad Price(M) = MC \to P_m = (3W + R)$$

For food, each unit of output requires one worker and 3 units of capital; hence:

$$TC(food) = Q_f(W + 3R) \to MC(food) = (W + 3R); \quad Price(food) = MC \to P_f = (W + 3R)$$

i. Find factor prices in terms of output price. Show how an increase in $P_f$ affects $W, R$.

From above: $P_m = (3W + R); \quad P_f = (W + 3R)$.

This is like two linear equations in two unknowns; one can invert this relationship and solve for $R$:

$$P_m = 3W + R \to R = (P_m - 3W); \quad \text{substitute this into the relationship for food:}$$

$$P_f = (W + 3R) = (W + 3(P_m - 3W)) = (3P_m - 8W) \quad \text{or} \quad W = \frac{3P_m - P_f}{8}$$

Use the solution for $W$ to solve for $R$: $R = (P_m - 3W) = P_m - 3\left(\frac{3P_m - P_f}{8}\right) = \frac{3P_f - P_m}{8}$. Summarizing:

$$W = \frac{3P_m - P_f}{8}; \quad R = \frac{3P_f - P_m}{8}$$

How does an increase in $P_f$ affect factor prices?

$$\frac{\partial W}{\partial P_f} = -\frac{1}{3} < 0; \quad \left(\frac{\partial R}{\partial P_f}\right) \left(\frac{P_f}{R}\right) = \frac{3P_f}{3P_f - P_m} > 1; \quad \frac{\partial R}{\partial P_c} = -\frac{1}{6} < 0$$

**Thus**, an increase in the price of food lowers the return to labor and raises the real return to capital (since food is capital-intensive and manufactures are labor-intensive). Moreover, the return on capital increases by more (in % terms) than does the price of food so the real return on capital increases in terms of either food or manufactures.
b) Find the production possibility frontier \((\text{ppf})\).

Let \(Q_m, Q_f\) denote the outputs of good \(M\) and \(F\), respectively. The resource constraints are:

**Labor:** (1) \(3Q_m + Q_f \leq \bar{L}\) since the technology implies: \(L_m = 3Q_m\) and \(L_f = Q_f\)

**Capital:** (2) \(Q_m + 3Q_f \leq \bar{K}\) since the technology implies: \(K_m = Q_m\) and \(K_f = 3Q_f\)

The following figure shows the production possibility frontier for this economy; the points on, or below, the line labeled “labor constraint” insure that labor employed is no larger than available labor (with full employment on that line), while the line labeled capital constraint has the same interpretation. For this simplified economy, the only output level where both inputs are fully employed is where the two lines intersect, at point \(V\), where output is:

\[
Q_m = \frac{3\bar{L} - \bar{K}}{8}; \quad Q_f = \frac{3\bar{K} - \bar{L}}{8}
\]

The feasible production set is the region bounded by: \(\{0,(K/3),V,(L/3)\}\), and the production possibility frontier is the line segments described by: \(\{(K/3),V,(L/3)\}\).

(i) **Show how an increase in the supply of capital shifts the ppf.**

An increase in capital shifts the capital constraint outward, as shown by the dotted line in the figure. The point \(Z\) represents the new output level where both factors are fully employed (in this simple version, there is a unique production point that represents full employment of both inputs).
Note that an increase in $K$ leads to an increase in output of the capital intensive good ($F$) and a decrease in output of the labor intensive good –as described in class and in the text.

c) Assuming the Japan is capital abundant and the US is labor abundant (but they have identical tastes and technology), compare autarky prices, then find the pattern of trade and discuss its consequences.

As shown above, given prices, an increase in the supply of capital (in Japan) increases output of the capital intensive good ($F$) and decreases output of the labor intensive good – as described in class and in the text.

Similarly, an increase in the supply of labor (US) causes the equilibrium autarky relative price of the labor intensive good (manufactures) to fall.

Hence, the autarky relative price of good $M$ will be higher in Japan than in the US. This, from (a), implies that the wage rate rises in the US and falls in Japan, while the return on capital ($R$) falls in the US and rises in Japan.

Finally, if free trade equalizes commodity prices and both goods are produced in both countries, it must equalize factor prices (see equations determining factor prices in (a)), provided technology is the same in the two countries. This is the “factor price equalization theorem”.

d) Modify the above model by assuming Japan’s productivity in both sectors double, while US technology remains unchanged. In Japan:

Food requires: 1/2 unit of labor and 3/2 units of capital for each unit of food produced.

Manufactures requires: 3/2 units of labor and 1/2 unit of capital for each unit of M produced.

i) Show how doubling of productivity in Japan affects autarky output prices and factor prices.

In Ricardian terms, while Japan has an absolute advantage (technologically) in both goods, there is no comparative advantage due to technology. To see this specifically, we can re-derive the production possibility frontier for Japan, with the new technology:

Labor: $(1a)(3/2)Q_m + (1/2)Q_f \leq \bar{L}$ as the technology implies: $L_m = (3/2)Q_m$ and $L_f = (1/2)Q_f$

Capital: $(2a)(1/2)Q_m + (3/2)Q_f \leq \bar{K}$ as the technology implies: $K_m = (1/2)Q_m$ and $K_f = (3/2)Q_f$

This yields the full employment point of: $Q_m = \left( \frac{3\bar{L} - \bar{K}}{4} \right)$; $Q_f = \left( \frac{3\bar{K} - \bar{L}}{4} \right)$

Thus, at full employment, output of both goods double – and thus the relative supply is unchanged. Hence, if demand for both goods also doubles (because income doubles) – so that relative demand is
unchanged, the doubling of productivity in both sectors will not affect autarky relative goods prices and hence will not affect the pattern of trade between the US and Japan.

Turning to input prices, using the logic of part (a) of the answer, for Japan we have:

\[ P_m = MC_m = (3/2)W + (1/2)R; \quad P_f = MC_f = (1/2)W + (3/2)R \]

solving for input prices (in Japan) in terms of output prices we have:

\[ W_{Japan} = \frac{3P_m - P_f}{4}; \quad R_{Japan} = \frac{3P_f - P_m}{4}, \]

whereas for the US (from part a):

\[ W^{us} = \frac{3P_m - P_f}{8}; \quad R^{us} = \frac{3P_f - P_m}{8} \]

Thus, we see that – given output prices – the doubling of productivity in both sectors in Japan leads to a doubling of the real return to both factors.

ii) Will free trade equalize factors prices and remove the pressure for factor migration?

Since the doubling of productivity in both sectors in Japan leaves relative autarky prices unchanged, it follows that the pattern of trade is still determined by factor endowments – so Japan will export food and the US manufactures. Trade still lowers the real return to labor (raises the real return to capital) in Japan since it lowers the price of the labor-intensive good, whereas the opposite happens in the US. However, trade will not lead to factor price equalization because technologies are different. As the above example shows, if trade equalizes goods prices, then the real return to both factors will be twice as high in Japan as in the US. Thus, there will still be pressure for factor movements.

2. (Factor movements) There is a single good (e.g., food), produced using land and labor. The amount of land in a country is fixed; labor may be mobile across countries. US and Mexican technology and resources are:

US: \( Q^{us} = 64(T^{us})^{2/3}(L^{us})^{1/3}; \quad T^{us} = 125; \quad L^{us} = 64 \)

Mexico: \( Q^{mex} = 25(T^{mex})^{2/3}(L^{mex})^{1/3}; \quad T^{mex} = 64; \quad L^{mex} = 64 \)

a) For each country, find and sketch the labor demand curve. Also, calculate the equilibrium wage, return on land and per capita income in each country (all measured in terms of output).

US labor: \[ MPLabor = \frac{\partial Q^{us}}{\partial L^{us}} = \left[ \frac{64}{3} \left( \frac{T^{us}}{L^{us}} \right)^{2/3} \right] \rightarrow P \left[ \frac{64}{3} \left( \frac{T^{us}}{L^{us}} \right)^{2/3} \right] = W \rightarrow \left( L^{us} \right)_{\text{demand}} = \left( \frac{64P}{3W} \right)^{3/2} T^{us} \]

US land rent: \[ R = \frac{PQ^{us} - WL^{us}}{T^{us}} = P \left[ \frac{128}{3} \left( \frac{L^{us}}{T^{us}} \right)^{1/3} \right] = P \frac{\partial Q^{us}}{\partial T^{us}} \]
Since there is only one good, you can set \( P = 1 \), since GNP, wages and rental rates are all measured in terms of this good. Evaluating at \( T^{us} = 125, \; L^{us} = 64 \):

\[
W^{us} = \left[ \frac{64}{3} \left( \frac{125}{64} \right)^{2/3} \right] = \left[ \frac{64}{3} \left( \frac{25}{16} \right) \right] = \frac{100}{3} ; \quad R^{us} = \frac{128}{3} \left( \frac{64}{125} \right)^{1/3} = \frac{512}{15}
\]

Similarly for Mexico:

\[
\text{Mexico: } \left( \frac{\partial Q^{mex}}{\partial L^{mex}} \right) = \left[ \frac{25}{3} \left( \frac{T^{mex}}{L^{mex}} \right)^{2/3} \right] \rightarrow \left[ \frac{25}{3} \left( \frac{T^{mex}}{L^{mex}} \right)^{2/3} \right] = W \rightarrow (L^{mex})^{\text{demand}} = \left( \frac{25}{3W} \right)^{3/2} \; T^{mex}
\]

Mexican land rent: \( R = \frac{\partial Q^{mex}}{\partial T^{mex}} = \frac{50}{3} \left( \frac{L^{mex}}{T^{mex}} \right)^{1/3} \)

Evaluating at \( T^{mex} = 64, \; L^{mex} = 64 \):

\[
W^{mex} = \left[ \frac{25}{3} \left( \frac{64}{64} \right)^{2/3} \right] = \frac{25}{3} ; \quad R^{mex} = \frac{50}{3} \left( \frac{64}{64} \right)^{1/3} = \frac{50}{3}
\]

Below you see the labor demand curve for the US, and how immigration shifts the domestic labor supply. The labor demand curve for Mexico looks similar; though emigration from Mexico (to the US) would reduce the labor supply in Mexico.

The returns to land can be measured, in this diagram, as the “consumer surplus” since we can think of landowners as “renting” labor, and hence a lower labor price benefits landowners. In the diagram land
rents, at the initial wage, would be given by \( \{A,E,\frac{(w/P)^e}{e} \} \); if wages fall, as shown, land rents increase by the area \( \{(w/P)^e, E, G, (w/P)^' \} \).

b) Assume the US allows some Mexican workers to enter. Let \( V \) stand for the maximum number of “guest workers” the US allows (the number of visas), and let \( I \) represent the actual number who choose to come, where \( I \leq V \). The US labor force becomes \( (L^us + I) \) and the Mexican labor force \( (L^mes - I) \).

i. Show how (i) US wages, (ii) the return on US land, (iii) Mexican wages, (iv) the return on Mexican land; and (v) world output are determined as a function of the number of Visas \( (V) \).

Assuming workers will want to come to the US if (and only if) US wages are higher than Mexican wages, then the number who would come if there were no restrictions is determined by choosing the number of immigrants \( (I) \) to equalize wages.

From earlier, wages are determined by:

\[
W^{us} = W^{mex} = \left[ \frac{64}{3} \left( \frac{T^{us}}{L^{us} + I} \right)^{2/3} \right] \quad \text{and} \quad \left[ \frac{25}{3} \left( \frac{T^{mex}}{L^{mex} - I} \right)^{2/3} \right]
\]

Where \( L^{us}, L^{mex} \) is the population, and \( (L^{us} + I), (L^{mex} - I) \) are the number of workers. Setting wages in the two countries equal to each other, and using the values for land, we have

\[
W^{us} = W^{mex} \Rightarrow \left[ \frac{64}{3} \left( \frac{T^{us}}{L^{us} + I} \right)^{2/3} \right] = \left[ \frac{25}{3} \left( \frac{T^{mex}}{L^{mex} - I} \right)^{2/3} \right] \Rightarrow \left( \frac{64}{25} \right) \left( \frac{T^{us}}{L^{us} + I} \right) = \left( \frac{T^{mex}}{L^{mex} - I} \right) \Rightarrow \left( \frac{512}{125} \right) \left( \frac{125}{L^{us} + I} \right) = \left( \frac{64}{L^{mex} - I} \right) \Rightarrow 8(L^{mex} - I) = (L^{us} + I) \Rightarrow 9I = 8L^{mex} - L^{us} = 448;\]

\[ I^* = \frac{448}{9} \approx 49.8 \]

Thus, for \( V < \frac{448}{9} \), US wages will be higher and all the visas will be used. Hence

\[
W^{us} = \left[ \frac{64}{3} \left( \frac{T^{us}}{L^{us} + V} \right)^{2/3} \right]; \quad \left( \frac{\partial W^{us}}{\partial V} \right) = \left[ \frac{-128(T^{us})^{2/3}}{9(L^{us} + V)^{5/3}} \right] < 0;
\]

\[
W^{mex} = \left[ \frac{25}{3} \left( \frac{T^{mex}}{L^{mex} - V} \right)^{2/3} \right]; \quad \left( \frac{\partial W^{mex}}{\partial V} \right) = \left[ \frac{50(T^{mex})^{2/3}}{9(L^{mex} - V)^{5/3}} \right] > 0
\]

US wages fall and Mexican wages rise as the number of visas issued increase, until wages are equalized, beyond which issuing more visas will not matter. For the return on land, opposite results hold:
Finally, total output increases as long as US wages are higher; thus:

\[
Q^T = Q^{us} + Q^{mex} = 64\left(T^{us}\right)^{2/3}\left(L^{us} + V\right)^{1/3} + 25\left(T^{mex}\right)^{2/3}\left(L^{mex} - V\right)^{1/3}
\]

\[
= 64\left(125\right)^{2/3}\left(L^{us} + V\right)^{1/3} + 25\left(64\right)^{2/3}\left(L^{mex} - V\right)^{1/3} = 1600\left(L^{us} + V\right)^{1/3} + 400\left(L^{mex} - V\right)^{1/3}
\]

\[
\left(\frac{\partial Q^T}{\partial V}\right) = \frac{1600}{3}\left(L^{us} + V\right)^{-2/3} - \frac{400}{3}\left(L^{mex} - V\right)^{-2/3} > 0 \Leftrightarrow 8\left(L^{mex} - V\right) > \left(L^{us} + V\right) \Leftrightarrow 9V < 8\left(L^{mex} - L^{us}\right)
\]

Which is the same condition we saw above to insure US wages are higher than Mexican wages.

ii. As above, if the number of visas is set very high, then not all will be used and wages will be equalized.

iii. Calculate how U.S. income, Mexican income, and world output change as \(V\) increases. From the problem set:

\[
Y^{us} = Q^{us} - W^{us}V = 64\left(T^{us}\right)^{2/3}\left(L^{us} + V\right)^{1/3} - W^{us}V; \quad T^{us} = 125; \quad L^{us} = 64; \quad I \leq V
\]

\[
Y^{mex} = Q^{mex} + W^{us}V = 25\left(T^{mex}\right)^{2/3}\left(L^{mex} - V\right)^{1/3} + W^{us}V; \quad T^{mex} = 64; \quad L^{mex} = 64
\]

\[
\left(\frac{\partial Y^{us}}{\partial V}\right) = \frac{64}{3}\left(T^{us}\right)^{2/3} - W^{us}V\left(\frac{\partial W^{us}}{\partial V}\right) = -V\left(\frac{\partial W^{us}}{\partial V}\right) > 0
\]

\[
\left(\frac{\partial Y^{mex}}{\partial V}\right) = -\frac{25}{3}\left(T^{mex}\right)^{2/3} + W^{us}V\left(\frac{\partial W^{us}}{\partial V}\right) = \left(W^{us} - W^{mex}\right) + V\left(\frac{\partial W^{us}}{\partial V}\right) < \left(W^{us} - W^{mex}\right)
\]

\[
\text{as } \left(\frac{\partial W^{us}}{\partial V}\right) < 0
\]

{of course, once wages are equalized, issuing more visas will have no effect}

Thus, since the US pays the Mexican workers their marginal value product in the US (as the wage), having more workers come in lowers the wage they receive and the US net income rises as \(V\) increases.

However, for Mexico, since their workers in the US receive the US wage, as more Mexicans enter the US labor market, that hurts Mexican workers already there (reducing their wage). Mexico is like a monopoly seller of labor and would gain by placing its own restrictions on emigration, provided
Mexican workers in the US receive the US wage. (that is, \( \frac{\partial Y^{\text{mex}}}{\partial V} \) < 0 for \( W^{\text{mex}} = W^{\text{us}} \)).

We calculated in the previous part that world output increases with labor movements as long as the marginal product of labor in the US is higher than in Mexico.

iv. Suppose the U.S. government auctions off the work visas. \( P \) is the price paid at auction for each visa, so Mexicans who work in the U.S. receive net income of \( W^{\text{us}} - P \), while Mexicans who stay in Mexico receive net income of \( W^{\text{mex}} \). Show how the following vary with \( V \): (1) \( P \); (2) U.S. net income and (3) Mexican net income change as \( V \) increases.

This is essentially the reverse of the previous problem; Mexico gains by increasing \( V \) up to the point where marginal products are equalized, while the US has an incentive to restrict immigration since the price of visas is reduced as \( V \) increases.

\[
Y^{\text{us}} = Q^{\text{us}} - (W^{\text{us}} - P)V = 64(T^{\text{us}})^{2/3}(L^{\text{us}} + V)^{1/3} - (W^{\text{us}} - P)V;
\]
\[
Y^{\text{mex}} = Q^{\text{mex}} + (W^{\text{us}} - P)V = 25(T^{\text{mex}})^{2/3}(L^{\text{mex}} - V)^{1/3} + (W^{\text{us}} - P)V;
\]

Where \( P = W^{\text{us}} - W^{\text{mex}} \). Hence:

\[
Y^{\text{us}} = 64(T^{\text{us}})^{2/3}(L^{\text{us}} + V)^{1/3} - W^{\text{mex}}V;
\]
\[
\frac{\partial Y^{\text{us}}}{\partial V} = W^{\text{us}} - W^{\text{mex}} - V \left( \frac{\partial W^{\text{mex}}}{\partial V} \right) < (W^{\text{us}} - W^{\text{mex}}) \quad \text{as} \quad \left( \frac{\partial W^{\text{mex}}}{\partial V} \right) > 0
\]
\[
Y^{\text{mex}} = Q^{\text{mex}} + (W^{\text{us}} - P)V = 25(T^{\text{mex}})^{2/3}(L^{\text{mex}} - V)^{1/3} + W^{\text{mex}}V;
\]
\[
\frac{\partial Y^{\text{mex}}}{\partial V} = -W^{\text{mex}} + W^{\text{mex}} + V \left( \frac{\partial W^{\text{mex}}}{\partial V} \right) > 0
\]

Since Mexican workers in the US essentially receive the Mexican wage, Mexico has no reason to limit out-migration but the US has an incentive to limit immigration. World output is maximized when there is free labor mobility but that does not mean individual countries don’t have an economic incentive to restrict mobility.

v. **Does unrestricted labor mobility maximize US income?**

Essentially discussed above. Also, if the immigrants become permanent residents of the US – so there are no wage payments that leave the country – then immigration raises US income but will lower per capita income (paradoxically, both original residents of the US and the immigrants can both be better off but still per capital income can fall because the entering immigrants had a much lower per capita income in their home country).
c) Suppose each worker in the U.S. receives for free some benefits (e.g., medical care or schooling). Assume the guest worker pays no taxes. If a guest worker program allows workers to freely choose where to work (and if Mexico has no such benefits for workers), will free worker movement between the two countries maximize total output of the two countries?

If the Mexican worker values the (medical) benefits at $B per year, then they will compare:

\[
(W^{us} + B) \quad \text{to} \quad W^{mex}
\]

They will migrate to the US until the total “wage”, including benefits, is equalized across countries. Thus, in equilibrium they will settle for a lower employer wage if they receive free medical benefits from the government. This, in turn, means in equilibrium that the marginal product of labor will be lower in the US, assuming the wage paid by firms equals the (value of the) marginal product of labor. Hence, too much labor movement will occur and world output is not maximized.

3. Consider a small country (Nicaragua) with the following demand and supply curves for sugar:

Supply = \(6P_s\); Demand = \(200 - 4P_s\)

Nicaragua can export sugar at a given world price of: \(P^*_s = 40\).

a) Show how: domestic price, consumption and production change as the export tariff \(t\) increases.

Calculate how consumer surplus, producer surplus, and government tariff revenue, and overall welfare, change as \(t\) increases.

Given the world price, the net revenue from exporting for a firm is \((40 - t)\); this will be the domestic price if trade occurs. Since the autarky price is 20, for \(t \geq 20\) the tariff is prohibitive: there are no exports, and the domestic price equals the autarky price of 20.

For \(t < 20\), we have:

\[P_s^d = (40 - t); \quad D = 200 - 4P_s^d = 40 + 4t; \quad S = 6P_s^d = 240 - 6t; \quad X = S - D = 200 - 10t\]

Consumption increases, production decreases and exports fall as \(t\) increases. From the figure below, one can see that, due to the tariff, producer surplus decreases by area \(\{40,B,B',(40-t)\}\), consumer surplus increases by area \(\{40,A,A',(40-t)\}\). Tariff revenue is area \(\{A',K,J,B'\}\). Thus:

\[\Delta PS = -(1/2)t \cdot (240 + 240 - 6t) = -(240t - 3t^2); \quad \Delta CS = (1/2)t \cdot (40 + 40 + 4t) = 40t + 2t^2\]

\[TR = tX = t \cdot (200 - 10t) = 200t - 10t^2 \quad \text{is tariff revenue.}\]

Producer surplus decreases with the tariff, consumer surplus increases (only \(t < 20\) is relevant), whereas tariff revenue increases with the tariff for \(t < 10\), and then decreases thereafter. Overall:

\[\Delta Welfare = TR + \Delta PS + \Delta CS = -5t^2\]

so that the tariff lowers overall welfare. This loss is the sum of the areas of the two “deadweight loss” triangles \(\{A,A',K\}\) and \(\{B,B',J\}\)
i) Even though the tariff hurts Nicaragua compared to free trade, it is still better off with the export tariff than under autarky. The gain, compared to autarky, is area \{A',B',E\} plus the tariff revenue \{A',K,J,B'\}. Some trade is better than no trade.

ii) If \( t > 20 \), the tariff is prohibitive and no trade occurs.

b) Compare the domestic equilibrium when \( t=10 \) to the case where there is no tariff, but there is an export quota of 100 units.

From part (a), with \( t=10 \), exports \( X = 200 - 10t = 100 \). Thus, a quota of 100 and a tariff of 10 have identical effects on domestic price, consumption, production and exports; and hence they have identical effects on consumer and producer surplus. The only possible difference is the tariff revenue (which is 1,000 under the tariff); under the quota, exporters make 10 on each unit exported (since they can buy at the domestic price, 30, and resell on the world market at a price of 40), and hence they will earn excess profits of 100, unless the quota licenses are auctioned off, in which case the two policies are identical. If, for some reason, the quotas were given to foreign importers, then the “revenue” from the tariff is lost to the country, and so the quota, in that case, would be inferior to the tariff for the exporting country.

c) Suppose the government subsidizes exports at a rate of \( s \) per unit of import. Show how this export subsidy affects the following (see Figure below):

(i) The domestic price – domestic price for both consumers and producers increases to \{40+s\} since an exporter receives 40 from the foreign buyer and “s” from the government. Hence, if the domestic price is less than \{40+s\} exporters will try to export all domestic output, driving up price to \{40+s\}.

(ii) Consumer surplus decreases - due to a higher price - by area next to demand curve between two prices – area \{40,A,A',(40+s)\} in the figure; hence: \( \Delta CS = -s \frac{(40 + 40 - 4s)}{2} = \{-40s + 2s^2\} \)
(iii) **Producer surplus** increases due to the higher price, by the area next to supply between two prices – area \{40,B,B',(40+s)\}. Hence: $$\Delta PS = \frac{s}{2} \{240 + 240 + 6s\} = \{240s + 3s^2\}$$

(iv) **Government expenditures**: Cost of subsidy is $$s \times \text{exports} = \text{area \{A'B'ML\}}$$; exports are $$S-D = 200+10s$$, hence: Cost to government = $$s(200+10s) = \{200s +10s^2\}$$

(v) Impact overall welfare: $$\Delta CS + \Delta PS - \text{Government Expenditures} = -5s^2 < 0, \quad s \neq 0$$
This is area of two deadweight loss triangles, \{A',A,L\} and \{B',M,B\}.

\[ PS = \frac{5}{2} \{240 + 240 + 6s\} = \{240s + 3s^2\} \]

\[ PS = \frac{5}{2} \{240 + 240 + 6s\} = \{240s + 3s^2\} \]

\[ S = 240 + 6s \]

\[ D = 240 + 6s \]

\[ Q = 200 + 10s \]

\[ P = 240 + 6s \]

\[ \Delta CS = 0; \]
\[ \Delta PS (s = 10) = \{240s + 3s^2\} = 2700 \]

**Subsidy Cost** = 10 \cdot (300) = 3000 where output is 300 when subsidy = 10

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**Subsidy Cost** = 10 \cdot (300) = 3000 where output is 300 when subsidy = 10

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i. No export quota can force exports to increase over the free trade level so there is no quantitative policy that has the same impact as an export subsidy.

ii. Unlike the case of a tariff, the export subsidy – by encouraging too much trade – can make Nicaragua worse off than under autarky if the subsidy is high enough.

d) **If the goal is to increase domestic sugar production**, an export subsidy accomplishes this by raising domestic price; but the export subsidy also causes domestic consumption to decrease. With a production subsidy of 10, there is no change in consumer surplus compared to free trade, whereas the change in producer surplus is the same as with a export subsidy of 10. Thus, the welfare consequences of a production subsidy of 10:

\[ \Delta CS = 0; \]
\[ \Delta PS (s = 10) = \{240s + 3s^2\} = 2700 \]

**Subsidy Cost** = 10 \cdot (300) = 3000 where output is 300 when subsidy = 10
Welfare Loss = -300.
For the export subsidy, with s=10, the welfare loss is 500.

Thus, the production subsidy which results in the same level of domestic production as the export subsidy has a lower welfare cost because we do not have the loss associated with the reduction in consumption.

4. (15 points extra credit) (More sophisticated version of H-O model). There are two goods (M and F) and two inputs (K and L). The production functions are:

$$Q_f = K_f^{3/4} L_f^{1/4}; \quad Q_m = K_m^{1/4} L_m^{3/4}$$

where \( \{K_m, L_m\} \) are the inputs (capital, labor) used in sector M and \( \{K_f, L_f\} \) are the inputs used in sector F. Let \( W \) denote the wage rate (price of L) and \( R \) the rental rate (cost of using K, capital). Finally, let \( P_m, P_f \) denote the output prices of goods M and F, respectively.

(a) There are two ways to derive the cost function – one by substitution, and one by using non-linear programming (which involves using the Lagrangian function).

By substitution:

Let \( Q_i = K_i^\beta L_i^\alpha \). Solving for labor yields:

$$L_i = \left( \frac{Q_i}{K_i^{\alpha - \beta}} \right)^{1/\alpha} = \left( \frac{Q_i}{\beta} \right)^{1/\alpha} K_i^{-\beta/\alpha}.$$

Total costs are:

$$TC = WL_i + RK_i = RK_i + W \left( Q_i \right)^{1/\alpha} K_i^{-\beta/\alpha}$$  \hspace{1cm} (1)

Equation (1) expresses total costs as a function of factor prices, output and capital inputs. Cost minimization means choose the capital input that minimizes this expression. Thus:

$$\frac{d(TC)}{dK_i} = R - \left( \frac{\beta}{\alpha} \right) W \left( Q_i \right)^{1/\alpha} K_i^{-\beta/\alpha - 1} = 0 \quad \text{since:} \quad \frac{d(K_i^\alpha)}{dK_i} = \alpha \left( K_i \right)^{a-1}$$  \hspace{1cm} (2)

A sufficient condition for an interior minimum is that the first derivative of the function be zero and that the second derivative be positive; it is readily seen that the second derivative is positive. Solving (2) for \( K_i \) yields:

$$K_i^* = \left( \frac{\beta}{\alpha} \right)^{1/(\alpha + \beta)} \left( \frac{W}{R} \right)^{\alpha/(\alpha + \beta)} Q_i$$  \hspace{1cm} (3)

where \( K_i^* \) denotes the solution. Substituting back for \( L_i \) yields:

$$L_i^* = \left( \frac{\alpha R}{\beta W} \right)^{\beta/(\alpha + \beta)} Q_i$$  \hspace{1cm} (4)

Note that the choice of inputs depends on relative factor prices, not absolute factor prices. Also, note that when \( \alpha + \beta = 1 \), the input use is proportional to output. Finally, substituting back into the cost
function (1) yields minimum costs:

\[ C^* (Q, W, R) = WL_i + RK_i^* = \lambda \left( W^\alpha R^\beta Q_i \right)^{\frac{1}{\alpha+\beta}} \]

where:

\[ \lambda = \frac{\left( \alpha + \beta \right)}{\left( \beta \alpha \right)^{\frac{1}{\alpha+\beta}}} \]

(you need to substitute and then simplify the expression; it is a bit tedious, but you should get the result above):

This result can be checked because the derivative of the cost function with respect to input price should give you back the optimal input use. Hence:

\[ \frac{dC^*}{dR} = \frac{\beta}{\left( \alpha + \beta \right)} \left( \frac{\alpha + \beta}{\beta \alpha} \right)^{\frac{1}{\alpha+\beta}} R^{-\alpha/(\alpha+\beta)} W^{\alpha/(\alpha+\beta)} Q_i^{\alpha/(\alpha+\beta)} = \left( \frac{\beta W}{\alpha R} \right) Q_i^{\alpha/(\alpha+\beta)} \]

which is \( K_i^* \). Similarly, differentiating with respect to \( W \) gives you \( L_i^* \):

\[ \frac{dC^*}{dW} = \frac{\alpha}{\left( \alpha + \beta \right)} \left( \frac{\alpha + \beta}{\beta \alpha} \right)^{\frac{1}{\alpha+\beta}} R^{\beta/(\alpha+\beta)} W^{-\beta/(\alpha+\beta)} Q_i^{\beta/(\alpha+\beta)} = \left( \frac{\alpha R}{\beta W} \right) Q_i^{\beta/(\alpha+\beta)} \]

Use of the Lagrangean function gives the same results, of course. Briefly, the Lagrangean is:

\[ H = WL_i + RK_i^* + \theta \left( Q_i - L_i^* K_i^\beta \right) \]

where \( \theta \) is the Lagrangean multiplier. Partially differentiating yields, for an interior solution:

\[ \frac{\partial H}{\partial K_i} = R - \theta \beta K_i^{\beta-1} L_i^{\alpha} = 0 \quad (1a) \]

\[ \frac{\partial H}{\partial L_i} = W - \theta \alpha K_i^\beta L_i^{-1} = 0 \quad (2a) \]

\[ \frac{\partial H}{\partial \theta} = \left( Q_i - L_i^* K_i^\beta \right) = 0 \quad (3a) \]

Taking the ratio of (1a) to (2a) yields:

\[ \frac{\beta L_i}{\alpha K_i} = \frac{R}{W} \rightarrow \frac{L_i}{K_i} = \left( \frac{\alpha}{\beta} \right) \left( \frac{R}{W} \right) \quad (4a) \]

Hence, the labor intensity depends on factor prices and – in terms of the original production function – is increasing in the parameter on “L” and decreasing in the parameter on “K”.

Using (4a) to solve for \( L_i \) in terms of \( K_i \), and then substituting this into (3a) yields the optimum capital input, which will be the same as above. Then, using this solution for capital, the solution for labor is found from (4a), and the cost curve by plugging back into the objective function.

You do not need to solve for \( \theta \), but if you do you get the following from (1a)
\[ \theta^* = \frac{R \left( K_i \right)^{1-\beta}}{\beta \left( L_i \right)^\alpha} = \left( 
abla^\theta W R^\alpha \right)^{\frac{1}{\beta(\alpha+\beta)}} \left( Q_i \right)^{\frac{1}{\beta(\alpha+\beta)-1}} \]  

(5a)

Looking back at the cost function derived above and comparing to (5a), we see that (5a) represents the marginal cost function. This is no coincidence; the Lagrangean multiplier – in this problem – will always yield the marginal cost function.

Finally, for the specific functions given:

\[ C^* (Q,W,R) = W L_i^\alpha + R K_i^\beta = \lambda \left( W^\alpha R^\beta Q_i \right)^{\frac{1}{\beta(\alpha+\beta)}} \lambda = \frac{(\alpha + \beta)}{(\beta^\beta \alpha^\alpha)^{\frac{1}{\beta(\alpha+\beta)}}} \]

where:

\[ \lambda = \frac{1}{\left( \frac{3}{4} \right)^{\frac{3}{4}} \left( \frac{1}{4} \right)^{\frac{1}{4}}} = \frac{4}{3^{3/4}} \]

Clearly, good M is labor intensive and F is capital intensive as we have from (4a) above:

\[ L_i = \left( \frac{\alpha}{\beta} \right) R \rightarrow \frac{K_m}{L_m} = \frac{\omega}{3} \; ; \; \frac{K_f}{L_f} = 3\omega \; \text{where:} \; \omega = \frac{W}{R} \]

(b) **Given output prices**, show how an increase in the available supply of labor changes output.

From the cost curves above we have:

\[ MC_m = \lambda R^{3/4} W^{3/4} = P_m \; \text{and} \; MC_f = \lambda W^{1/4} R^{3/4} = P_f \]  

(1b)

We can use these two equations to solve for factor prices in terms of output price. Taking the ratio of marginal costs and setting this equal to the price ratio (relative prices) yields:

\[ \frac{MC_m}{MC_f} = \frac{\lambda W^{3/4} R^{3/4}}{\lambda W^{1/4} R^{3/4}} = \left( \frac{W}{R} \right)^{\frac{1}{2}} = \frac{P_m}{P_f} \rightarrow \omega = \rho^2 \; \text{where:} \; \rho = \frac{P_m}{P_f} ; \; \omega = \frac{W}{R} \]  

(2b)

Plugging this back into (1b) and solving gives the level of factor prices:
\[ R = \frac{P_f^{3/2}}{\lambda P_m^{1/2}} \quad W = \frac{P_m^{3/2}}{\lambda P_f^{1/2}} \]  

(3b)

From (3) and (4) in part (a) you have the optimal amount of inputs in each sector:

\[ K_i^* = \left( \frac{\beta W}{\alpha R} \right)^{\frac{1}{1+\omega}} Q_i \quad \rightarrow \quad K_m^* = \left( \frac{\omega}{3} \right)^{3/4} Q_m; \quad K_f^* = (3 \omega)^{1/4} Q_f \]  

(4b)

\[ L_i^* = \left( \frac{\alpha R}{\beta W} \right)^{\frac{1}{1+\omega}} Q_i \quad \rightarrow \quad L_m^* = \left( \frac{3}{\omega} \right)^{1/4} Q_m, \quad L_f^* = \left( \frac{1}{3 \omega} \right)^{3/4} Q_f \]  

(5b)

You can express these input demands in terms of output price by substituting for \( \omega \) in terms of \( \frac{P_m}{P_f} \).

Doing so and writing the resource constraints yields:

\[ L_m + L_f = L \quad \rightarrow \quad 3^{3/4} \rho^{-3/2} Q_m + 3^{3/4} \rho^{-3/2} Q_f = L \]  

(6b)

\[ K_m + K_f = K \quad \rightarrow \quad 3^{-3/4} \rho^{3/2} Q_m + 3^{3/4} \rho^{3/2} Q_f = K \]  

(7b)

Given prices, equations (6b) and (7b) are just like problem #1 (i.e., the labor and capital use per unit output are fixed) and can be solved for output levels. Doing so yields:

\[ Q_m = \left( \frac{3^{3/4}}{8} \right) \left( 3 \rho^{1/2} L - \rho^{-3/2} K \right); \quad Q_f = \left( \frac{3^{3/4}}{8} \right) \left( 3 \rho^{-1/2} K - \rho^{3/2} L \right) \]

Thus, given prices, an increase in \( L \) will increase the output of good M, the labor intensive good, and decrease the output of good F. Also, note that if output prices do not change, input prices do not change since, from equation (3b) above, factor prices can be determined in terms of only output prices.

Thus, given prices, an increase in \( L \) causes the supply of good M to increase and that of good F to decrease. But total income increases as \( L \) increases, so demand for both goods increases. Hence, to restore equilibrium, the price of good F must increase (relative price of M decreases). Thus, the autarky (relative) price of good F (the capital intensive good) is an increasing function of the country’s relative labor abundance. Consequently, given the same demands and technology, the labor abundant country will export the labor intensive good (M) and import the capital intensive good (F).

Further, since the wage rate decreases, and the rental rate increases, as the price of good F increases (i.e., the relative price of good M decreases), this means that the autarky wage rate will be lower, and the autarky rental rate on capital higher, in the labor abundant country. Hence, differences in factor supplies lead to differences in autarky output prices, which lead to the differences in input prices one would expect.

i. As discussed above, **given output prices**, input prices are determined and hence changes if factor supplies will not change factor prices.
(c) To show how factor prices change with output prices, look back at equation (3b).

\[ R = \frac{P_f^{3/2}}{\lambda P_m^{1/2}} \quad W = \frac{P_m^{3/2}}{\lambda P_f^{1/2}} \]. Thus

\[ \left( \frac{\partial R}{\partial P_f} \right) = \frac{(3/2) P_f^{1/2}}{\lambda P_m^{1/2}} \rightarrow \left( \frac{\partial R}{\partial P_f} \right) \left( \frac{P_f}{R} \right) = (3/2) > 1; \quad \left( \frac{\partial R}{\partial P_m} \right) = \frac{-P_f^{3/2}}{2\lambda P_m^{3/2}} < 0; \]

\[ W = \frac{P_m^{3/2}}{\lambda P_f^{1/2}} \rightarrow \left( \frac{\partial W}{\partial P_m} \right) = \frac{(3/2) P_m^{1/2}}{\lambda P_f^{1/2}} \rightarrow \left( \frac{\partial W}{\partial P_m} \right) \left( \frac{P_m}{W} \right) = (3/2) > 1, \quad \left( \frac{\partial W}{\partial P_f} \right) = \frac{-P_m^{3/2}}{2\lambda P_f^{3/2}} < 0 \]

These results imply that an increase in \( P_f \), the capital intensive good, lowers the real return to labor and raises the real return to capital in terms of either good (a 1% increase in \( P_f \), increases the rental rate \( R \) by 1.5%, and hence \( (R/P_f) \) increases as \( P_f \) increases). Similarly, an increase in the price of the labor intensive good \( (M) \) lowers the real return to capital and raises the real return to labor in terms of either good.

The results show that, as \( P_m \) increases, \( W \) increases proportionately more than \( P_m \) (i.e., \( W/P_m \)) increases with \( P_m \)). This is the Stolper-Samuelson result.

(d) Use your answer to parts (b) and (c) to predict the pattern of trade between a labor-abundant country (like China) and a capital-abundant country (like the U.S.). How will trade affect the distribution of income in each country? Will everybody gain from trade? Explain.

Since China is labor-abundant, the autarky relative price of \( M \) will be lower in China than in the U.S., the real wage will be lower in China and the real return on capital will be higher. If trade is allowed, the US will export \( F \) and China will export \( M \). The relative price of \( M \) rises in China (falls in the US), so the real wage rises in China and the return on capital falls. Of course, in the US the real wage falls and the real return on capital rises. Not everybody gains from trade, though the gainers could compensate the losers.

(e) Assume the U.S. is capital-abundant. Which group in the U.S. is likely to favor import tariffs and which group is likely to oppose trade restrictions? Explain.

If the H-O model correctly explains trade, the workers in the U.S. will favor import tariffs and capitalists will oppose them. If imports are labor-intensive, imports raise the domestic price of the labor intensive good, increasing the real wage and reducing the real return on capital.