1. (Simplified version of Heckscher-Ohlin model). Consider a country which can produce two goods: electronics (E) and food (F) using two inputs: unskilled labor (L) and skilled labor (S). {Think of: Unskilled labor as high school dropouts, Skilled labor as college graduates} Production of each good requires inputs to be used in fixed proportions as follows (these are called Leontief technologies):

To produce food (F): 2 units of unskilled labor and 1 unit of skilled labor are required for each unit of F.

To produce electronics (E): 1 unit of unskilled labor and 3 units of skilled labor are required for each unit of E.

Let \( L, S \) represent the total amount of unskilled and skilled workers available in the economy, let \( P_F, P_E \) denote the output prices and let \( W^U, W^S \) denote the wages of unskilled labor and skilled labor.

a) Find production costs and hence output price (price=marginal cost) for each good in terms of factor prices. {Hint: To produce \( Q_E \) units of \( E \) requires \( 3Q_E \) skilled workers and \( Q_e \) unskilled workers, hence total costs are \( \{W^U L_e + W^S S_e\} = \{W^U + 3W^S\} Q_e \).}

i. Use your answers to solve for input prices in terms of output prices. How will an increase in the price of electronics affect input prices?

b) Find the production possibility frontier (ppf) for this economy and sketch it. (Hint: it is defined by two linear inequalities that state unskilled (skilled) labor used cannot exceed supply. Use the hint from part (a) and apply the same reasoning for factors used in the food sector).

i. Show how an increase in the supply of skilled labor shifts the ppf and the production point where inputs are fully used (in this simple version, there is a unique production point that represents full employment of both inputs).

c) Assume two countries (the US and Mexico) have identical tastes and technology, but the U.S. has more skilled workers per unskilled worker than does Mexico (i.e., \( \frac{S^US}{L^US} > \frac{S^MEX}{L^MEX} \)).

Assuming the relative demand (ratio of demand for food to demand for electronics) is independent of income, discuss how the autarky prices of goods and factors differ between the two countries, then discuss how trade affects factor prices in each country. Will factor prices be equalized between the two countries?

d) Modify the above model by assuming U.S. productivity in both sectors doubles. Thus, in the U.S.:

To produce food: 1 unskilled worker and \( (1/2) \) skilled worker are required for each unit of \( F \).

To produce electronics: \( (1/2) \) unskilled worker and \( (3/2) \) skilled workers are required for each unit of \( E \).

Mexican technology remains unchanged. Viewed in a Ricardian context, the U.S. has an absolute advantage in both goods but a comparative advantage in neither good (due to technology).

i. Find how this U.S. productivity increase affects its autarky output prices and factor prices.

ii. Assuming free trade between the US and Mexico, what will the pattern of trade be? Will free trade equalize factor prices? Will trade eliminate the pressure for factor migration?
2. (Simplified version Labor Migration). Consider a very simple model with two countries (US, Mexico) and only one good (so there is no trade in goods). Output of this good, in each country, depends upon technology, the amount of land, and the amount of labor in each country. Assume the following production technology:

US: \( Q^{us} = 20 \left( T^{us} \right)^{1/2} \left( L^{us} \right)^{1/2} ; \quad T^{us} = 100; \quad L^{us} = 100 \)

Mexico: \( Q^{mex} = 10 \left( T^{mex} \right)^{1/2} \left( L^{mex} \right)^{1/2} ; \quad T^{mex} = 100; \quad L^{mex} = 100 \)

In the above equations, \( T \) and \( L \) represent the (usable) land area in each country, and \( W \) represent the population (= number of workers initially) in each country.

(a) For each country, find and sketch the labor demand curve. Also, calculate the equilibrium wage, return on land and per capita income in each country (all measured in terms of output), assuming no labor movements between the two countries is allowed.

(b) Suppose the U.S. government decides to allow some Mexicans to work in the U.S., but it controls (or limits) this number by requiring that each Mexican worker obtain a work visa. Let \( V \) denote the number of work visas issued by the U.S. government, and let \( I \) denote the number of Mexicans who work in the U.S. Since a visa is required for Mexicans to work in the U.S., \( I \leq V \).

Suppose Mexicans decide where to work solely based upon where their income is higher, so that if visas are given away for free those who get the visa will choose to work in the U.S. if the U.S. wage is higher than the Mexican wage. Also, assume that, by law, any Mexican working in the U.S. must receive the same wage as a U.S. worker.

i. Show how (i) U.S. wages; (ii) the return on land in the U.S.; (iii) Mexican wages; and (iv) the return on land in Mexico are determined as a function of the number of work visas (\( V \)) issued by the U.S. How does total world output change as \( V \) increases? Quantitative answers are required.

ii. If the number of visas is set very high, is it possible that not all are used? Explain.

iii. Calculate how U.S. income and Mexican income change as \( V \) increases. Note that: U.S. income is U.S. output less wage income paid to Mexicans; Mexican income is Mexican output plus wages received by Mexican workers. Thus:

\[ Y^{us} = Q^{us} - W^{us} I = 20 \left( T^{us} \right)^{1/2} \left( L^{us} + I \right)^{1/2} - W^{us} I ; \quad T^{us} = 100; \quad L^{us} = 100; \quad I \leq V \]

\[ Y^{mex} = Q^{mex} + W^{mex} I = 10 \left( T^{mex} \right)^{1/2} \left( L^{mex} - I \right)^{1/2} + W^{mex} I ; \quad T^{mex} = 100; \quad L^{mex} = 100 \]

iv. Suppose that the U.S. government auctions off the work visas, instead of giving them away for free. Let \( P \) be the price paid at auction for each visa, so that Mexicans who purchase a visa and work in the U.S. receive net income of \( W^{us} - P \), while Mexicans who stay in Mexico receive net income of \( W^{mex} \). Assuming Mexicans will work where net income is higher, show how: (1) \( P \); (2) U.S. net income and (3) Mexican net income change as \( V \) increases. Since Mexican workers pay the U.S. government for the visa, U.S. and Mexican total income are now:

(c) Suppose that each worker in the U.S. gets “free medical care”, regardless of whether they are a U.S. citizen or not. Assume, for simplicity, that no worker pays taxes to support this benefit (e.g., a tax on land rents might finance the program). If a guest worker program allows workers to freely choose where to work (and if Mexico does not have free medical care for workers), will free worker movement between the two countries maximize total output of the two countries? Explain and discuss how your answer might change if the guest worker also had to pay taxes on earned income.

3. (Chapter 8, Trade Policy) Consider a small country (e.g., Senegal) with the following demand and supply curves for rice:

\[ \text{Supply} = 3P_r; \quad \text{Demand} = 3000 - P_r \]

Assume Senegal can import rice at a given world price of: \( P_r = 500 \) (per metric ton). Further, assume that Senegal imposes an import tariff of \( t \) per unit of import.

(a) Show how: domestic price, consumption and production change as \( t \) increases. Also, calculate how consumer surplus, producer surplus, and government tariff revenue change as \( t \) increases. What happens to overall welfare in Senegal as \( t \) increases?

i. Can Senegal be worse off with trade and a tariff than in autarky? Explain.

ii. If \( t > 250 \), what happens to Senegal’s rice imports?

(b) Compare the domestic equilibrium when \( t = 100 \) to the case in which there is an import quota of 600 units (but no tariff). How do imports, domestic price, production and consumption compare under the two plans? What happens to the tariff revenue? Which policy is better for the country?

(c) Suppose, instead of an import tariff, the Senegalese government *subsidizes* imports at a rate of \( s \) per unit of import. Thus, for each unit of rice imported, the importer pays the foreign seller \( P_r' = 500 \) but receives a subsidy of \( s \) from the government, so the net cost to the importer, per unit imported, is (500-s). Show how this import subsidy affects: (i)domestic price; (ii)consumer surplus; (iii)producer surplus; (iv)government expenditures on the subsidy; and (iv)overall welfare.

i. Is there any import quota that would have the same effect as the import subsidy?

ii. Could Senegal be worse off with an import subsidy than with no trade? Explain.

(d) Suppose the government’s (political) goal is to help domestic rice consumers, who live in urban areas. It can accomplish this goal with either an import subsidy or a consumption subsidy. Which is the better tool? Demonstrate your answer by comparing the welfare implications of an import subsidy of 100 and a consumption subsidy of 100. Explain your answer.
4. (15 point extra credit – 100 points for first 3 problems, so max score is 115.) Similar to question #1, suppose there are two goods (F and E) and two inputs, S and L. However, the technology for producing each good now allows smooth substitutability between the inputs. The production functions are:

\[ Q_f = S_f^{1/3} L_f^{2/3}; \quad Q_e = 3S_e^{2/3} L_e^{1/3} \]

where \( \{S_f, L_f\} \) are the inputs (skilled and unskilled labor) used in sector F and \( \{S_e, L_e\} \) are the inputs used in sector E. Let \( W^U \) denote the wage for unskilled workers and \( W^S \) the wage for skilled labor. Finally, let \( P_f, P_e \) denote the output prices of goods F and E, respectively.

(a) Derive the cost function for each good by minimizing cost for a given output level. Which good is skilled labor intensive? Why?

\{One way to derive the cost function for good E: Minimize \( \left( W^U L_e + W^S S_e \right) \) subject to the constraint \( Q_e = 3S_e^{2/3} L_e^{1/3} \), which implies \( S_e = \left( \frac{Q_e}{3} \right)^{3/2} \left( L_e \right)^{-1/2} \); thus, minimize:\n\[
\left( W^U L_e + W^S \left( \frac{Q_e}{3} \right)^{3/2} \left( L_e \right)^{-1/2} \right)
\]

with respect to \( L_e \). Use a similar procedure for good F.\}

(b) Given output prices, show how an increase in the supply of skilled labor changes output of each good. Use your result to predict which good the skilled labor-abundant country will export.

In answering, use the resource constraints: \( L_f + L_e = \bar{L}; \quad S_f + S_e = \bar{S} \).

i. Given output prices, does the increase in input supply change factor prices? Explain.

(c) Assuming both goods are produced (so that Price=Marginal Cost for each good), show how an increase in \( P_f \) will affect factor prices \( \left( W^U, W^S \right) \). Does either factor price rise proportionally more than \( P_f \)? If so, explain which one and why.

(d) Use your answer to parts (b) and (c) to predict the pattern of trade between Mexico and the US, assuming the US has a higher ratio of skilled labor to unskilled labor than Mexico. How will trade affect the distribution of income in each country? Will everybody gain from trade? Explain.

(e) Assume the U.S. is skilled labor-abundant. Which group in the U.S. is likely to favor import tariffs and which group is likely to oppose trade restrictions? Explain.