Problem Set 1 - Due Sept. 12, 2013
(Producer and consumer surplus; review)

The purpose of this problem set is to review concepts from Econ 301 and to illustrate how these concepts
can be used to answer “real world” policy questions.

1. Consider a perfectly competitive firm which produces shirts, and let \( y \) denote the firm’s output.
The firm’s total costs are given by:

\[
TC(y) = 2y + \left( \frac{y^2}{6} \right); \quad y \geq 0
\]

Let the price at which the firm sells its shirts be denoted \( p_f \) (superscript \( f \) stands for firm). Then:

a) The firm’s profits are given by:

\[
\pi(y, p_f) = p_f y - TC(y) = \left( p_f - 2y - \left( \frac{y^2}{6} \right) \right) y
\]

i. Find the output level, \( y^* \left( p_f \right) \), which maximizes profits.

ii. Show that the firm’s supply curve (which shows output as a function of price) is the firm’s marginal cost curve.

iii. Using the firm’s supply curve, show graphically the increase in production costs when the firm’s
output increases from \( y = 60 \) to \( y = 78 \). Using this area, calculate how much costs increase with
the increase in output, and then use the cost curve to verify your answer.

iv. Using your answer to part (i), calculate the firm’s maximized profits as a function of price:

\[
\pi^* (p_f) = p_f y^* \left( p_f \right) - 2y^* \left( p_f \right) \left( \frac{y^* \left( p_f \right)}{6} \right)^2
\]

b) The firm currently sells output at a price of 22 but it has found new markets where it can sell all its
output at a price of 28. Assuming competitive profit maximization, use the supply curve to show
(graphically and numerically) how much profits increase due to this price change (this area is called
change in producer surplus). Verify your answer using the profit function calculated in part a(iv).

c) Return to the situation where the firm can sell its output at a price of 22. Now assume a retail store,
Walmart, wants to place a large order. Walmart offers to pay 32 per shirt, but only if the store will sell
it 120 units at that price (i.e., Walmart offers the store a total \( 120 \times 32 = 3840 \) for 120 shirts). Using the
supply curve, show how much the firm’s profits change if it takes this offer from Walmart.

2. Consider a consumer who consumes two goods, food and shirts. Let \( f \) denote his consumption of
food and \( \hat{y} \) his consumption of shirts. Let \( (p_f, p_s) \) denote the prices of food and shirts, and let \( I \) denote
his income. The person has the following (quasi-linear) utility function:

\[
U(f, \hat{y}) = f + 52\hat{y} - \left( \frac{\hat{y}^2}{4} \right)
\]

The individual’s budget constraint is:

\[
\{ p_f f + p_s \hat{y} \leq I \}
\]

a) Set up the utility maximization problem and derive the individual’s demand functions for both goods
(assume a solution where both goods are consumed).
i. Find the individual’s **maximized utility** by substituting the demand solutions back into the utility function {this is called the person’s *indirect utility function*}.

ii. Discuss how an increase in the price of good \( y \) affects maximized utility and interpret. Take the (partial) derivative of the indirect utility function with respect to \( p_y \); what does this equal?

b) Consider a consumer, with income \( I=5000 \), who can buy goods at the prices \( p_f = 1, p_y = 22 \). A new mall brings a discount store (Costco) to town, which sells good \( y \) at the price \( p_y = 13 \).

i. Find the consumer’s purchases at this price. Will the consumer be better or worse off as a result of the opening of this store?

ii. Suppose the store charges individuals a one-time fee of $F$ to shop in the store. What is the **maximum** amount this individual would be willing to pay to shop in the store (the alternative is to continue to pay the price \( p_y = 22 \) charged by other stores in town)? Give a numerical answer.

iii. Using the demand curve, show graphically how to calculate your answer for part ii. {This area is called the **change in consumer surplus**}.

3. **{Efficiency of markets}**. Use the supply curve from question 1 and the demand curve from question 2 to answer this question. For the demands from question 2, let \( p_f = 1 \). In answering the question, you are essentially assuming there are a large number of identical producers and consumers, but it is simpler to work with just one of each.

a) Using the supply curve from question 1 and the demand curve from question 2, calculate the equilibrium price and quantity (where supply equals demand). Also, calculate producer and consumer surplus at this equilibrium.

b) Next, assume the government gives producers a production **subsidy** of **15 per shirt sold** {thus, the producer price exceeds the consumer price by 15, i.e., \( p_y = (p_y + 15) \)}. Show how this subsidy affects: (i) equilibrium consumer price (\( p_y \)) and producer price (\( p_y' \)); (ii) equilibrium output; (iii) consumer surplus and (iv) producer surplus. Compare the total change in producer and consumer surplus to the cost to the government (and hence taxpayers) of the subsidy, and discuss how the subsidy affects overall efficiency. **Show graphically this change in efficiency**.

c) Suppose the government does not subsidize output, but the country has the chance to trade on world markets. Suppose the world price of shirts is 28, and the country is small, so it can import or export as many shirts as it wants at this price. Show how the movement from no trade (the equilibrium you calculated in 3a) to free trade affects consumer surplus and producer surplus. Does the country “gain” from trade? Does everybody gain? Explain.