1. Consider a Ricardian model of comparative advantage. There are two countries, the U.S. and India. Each country can produce two goods, shirts (S) and food (F). Assume the US has 300 workers and India has 1,000 workers. Labor productivity in each country is:

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shirts</td>
<td>10 shirts/day</td>
<td>4 shirts/day</td>
</tr>
<tr>
<td>Food</td>
<td>20 bushels/day</td>
<td>2 bushels/day</td>
</tr>
</tbody>
</table>

(a) The US has the absolute advantage in both goods because the US marginal productivity of labor is higher for both goods.

(i) The opportunity cost of producing food in the U.S. is \( \frac{10}{20} = \frac{1}{2} \) shirt per bushel since one worker can produce 20 bushels or 10 shirts so if we move a worker from shirt production to food production we have: 

\[
\frac{\Delta S}{\Delta F} = \frac{-MPL_S}{MPL_F} = \frac{-10 \text{ shirts}}{20 \text{ bushels}} = -\frac{1}{2} \text{ shirt / bushel}.
\]

Similarly, the opportunity cost of food production in India is:

\[
\frac{\Delta F}{\Delta S} = \frac{-MPL_F}{MPL_S} = \frac{-4 \text{ shirts}}{2 \text{ bushels}} = -2 \text{ shirts / bushel}.
\]

Thus, the US has the lower opportunity cost of producing food. Of course, that means India has the lower opportunity cost of producing shirts. The opportunity cost of producing shirts in:

\[
\frac{\Delta F}{\Delta S} = \frac{-MPL_F}{MPL_S} = \frac{-2 \text{ bushels}}{10 \text{ shirts}} = -\frac{1}{5} \text{ bushel / shirt},
\]

whereas for India:

\[
\frac{\Delta F}{\Delta S} = \frac{-MPL_F}{MPL_S} = \frac{-2 \text{ bushels}}{4 \text{ shirts}} = -\frac{1}{2} \text{ bushel / shirt}.
\]

(b) Derive and sketch the production possibility frontier for each country.

For US: 

\[ #Shirts \equiv Q_s = 10L_s = 10(300 - L_f) = 3000 - 10\left(\frac{Q_f}{20}\right) = 3000 - \left(\frac{Q_f}{20}\right) \text{ as } L_f = \frac{Q_f}{20} \]

For India: 

\[ #Shirts \equiv Q_s = 4L_s = 4(1000 - L_f) = 4000 - 4\left(\frac{Q_f}{20}\right) = 4000 - 2\frac{Q_f}{20} \text{ as } Q_f = 2L_f \]
(i) The point B represents the original US production (and consumption) of 1500 shirts and 3000 units of food, while $B'$ represents Indian production (and consumption) of 2000 shirts and 1000 units of food. Let India increase shirt production by “S” units (up to a maximum total production of 4000 in India); this will cause Indian food production to fall by $(S/2)$ units {moving in the direction shown by the arrow}. Let the U.S. increase food production by “D” units (up to a maximum total production of 6000 units of food); this will cause U.S. shirt production to fall by $(D/2)$ {again, moving in the direction shown by the arrow}. Thus, from a world perspective:

Change Food Production = $D - (S/2) > 0$ if $D > (S/2)$  
Change Shirt Production = $S - (D/2) > 0$ if $D < 2S$

Thus, for any change in the two countries such that: $2S > D > (S/2)$ (or, equivalently, $(2D) > S > (D/2)$) world output of both goods increases. (e.g., if India increases shirt production by 100 and U.S. increases food production by 100). Naturally, once one country is fully specialized, it can produce no more of the good in which it has a comparative advantage.

(ii) The world production possibility frontier is such that - at most - only one country produces both goods. For low levels of food output, India should specialize in shirts, while the US produces both goods; while for large levels of food output, the US specializes in food and India produces both goods. Formally:

$$Q_f^T \leq 6000; \quad Q_f^{India} = 0, Q_s^{India} = 4000; \quad Q_s^{us} = 3000 - \left(\frac{Q_f^{us}}{2}\right); \quad \rightarrow Q_s^T = 7000 - \left(\frac{Q_f^T}{2}\right)$$

$$6000 \geq Q_f^T \geq 4000; \quad Q_f^{us} = 4000, Q_s^{us} = 0; \quad Q_s^{India} = 4000 - 2Q_f^{India}; \quad \rightarrow Q_s^T = 4000 - 2Q_f^T - 4000 = 12,000 - 2Q_f^T$$

In the above, $Q_f^T = Q_f^{us} + Q_f^{India}$ is world food output and $Q_s^T = Q_s^{us} + Q_s^{India}$ is world shirt output.

(c) In the absence of trade, autarky (no trade) relative prices would be: $(P_f/P_s)^{us} = 2(\text{food} / \text{shirt})$, and $(P_f/P_s)^{India} = (1/2)(\text{food} / \text{shirt})$, where $P_f$ is the price of food, $P_s$ is the price of shirts, and the superscript indicates the country.

US Real Wages: $(W/P_f)^{us} = 20 \text{ units food / day}$; $(W/P_s)^{us} = 10 \text{shirts / day}$

Indian Real Wages: $(W/P_f)^{India} = 2 \text{ units food / day}$; $(W/P_s)^{India} = 4 \text{ shirts / day}$

(d) If trade is allowed, each country will export the good in which it has a comparative advantage – thus, the US exports food, India exports shirts. Assuming each country winds up fully specialized (which will happen if country sizes are not too different), then post-trade $(1/2) < (P_f/P_s)^{us} < 2$ . In the US, the relative price of shirts falls, and thus the real wage in terms of shirts (the import good) rises while the real wage in terms of food is unchanged. In India, the relative price of food falls (relative price of shirts rises) and hence the real wage in terms of food rises, while in terms of shirts it is unchanged. Thus, the real wage in each country rises in terms of the import good.
(e) Assume all people have the same preferences. Let $D_f, D_s$ represent demand for goods F (food) and S (shirts), respectively. Assume people spend one-half their income on each good:

$$D_f = \left( \frac{1}{2} I/P_f \right); \quad D_s = \left( \frac{1}{2} I/P_s \right)$$

where $I$ (personal income) is the wage rate, and $P_f, P_s$ are the prices of the two goods. Total income (for each country) is the wage rate times the number of workers.

i) Find the equilibrium world (relative) price of goods under free trade (i.e., set total world supply equal to total world demand and solve for price).

The equilibrium could occur where both countries are completely specialized, or where one country produces both goods (just one of these will be the case). Assume both countries are completely specialized in the good in which they have a comparative advantage; then:

$$W^{us} = 20P_f; \quad I^{us} = W^{us} L^{us} = 20P_f \cdot 300 = 6000P_f; \quad Q_f = 20L^{us} = 6000$$

$$W^{India} = 4P_s; \quad I^{India} = W^{India} L^{India} = 4P_s \cdot 1000 = 4000P_s; \quad Q_c = 4L^{India} = 4000$$

Since all people have the same taste, total demand for food and shirts, respectively, is:

$$D_f = \frac{I^{us} + I^{India}}{2P_f}; \quad D_s = \frac{I^{us} + I^{India}}{2P_s}$$

If demand equals supply in one market, then the other market must also be in equilibrium. Setting the demand and supply for food equal to each other (and assuming specialization) yields:

$$D_f = \frac{I^{us} + I^{India}}{2P_f} = \frac{6000P_f + 4000P_s}{2P_f} = S_f = 6000 \rightarrow P_s = \left( \frac{3}{2} \right) P_f$$

Since the relative price is between (1/2) and 2, the assumption that both countries specialize is valid, and this is our equilibrium. \{As an aside, note that because the US is the larger country - economically - the equilibrium world price is closer to the US autarky price. In fact, if India only had 300 workers (like the US), then the equilibrium world price would be the US autarky price and in this case trade would not affect welfare in the US.\}

ii) Show how the real wage changes in each country due to free trade. Do both countries gain from free trade?

As noted, the real wage in terms of the export good is unchanged in each country. In terms of the import good:

US: Before trade: $W^{us}/P_c = 10$; after trade: $W^{us}/P_s = \frac{20P_f}{P_s} = \frac{20}{(3/2)} = 13.33 > 10$ as $P_s = \frac{3P_f}{2}$

India: Before trade: $W^{India}/P_f = 2$; after trade: $W^{India}/P_s = \frac{4P_s}{P_f} = \frac{4 \cdot \frac{3}{2}}{2} = 6$

Thus, clearly the real wage in terms of the import good increases in both countries.

f) Under free trade, how would a 50% increase in Indian productivity in both sectors affect world prices and the real wage in the US and India? Explain. (a specific answer is required; use the demand curves from part (e) to find the new equilibrium relative price of shirts).

US productivity is unchanged; Indian productivity increases 50% (to 3 in food or 6 in shirts). If both countries remain specialized then we have:
Since shirt supply increases, the world relative price of shirts falls (the relative price of food rises); since relative prices are still in the interval \(\left(\frac{1}{2}, 2\right)\) both countries remain specialized.

**Real wages after Indian productivity increase:**

- **US:** 
  \[
  \frac{W^{\text{US}}}{P_f} = 20; \quad \frac{W^{\text{US}}}{P_s} = \frac{20P_f}{P_s} = 20 > \frac{13}{3}
  \]

- **India:** 
  \[
  \frac{W^{\text{India}}}{P_s} = 6; \quad \frac{W^{\text{India}}}{P_f} = \frac{6P_s}{P_f} = 6
  \]

In the US, the real wage in terms of food is unchanged since productivity is unchanged and the US continues to produce food; the real wage in terms of shirts rises because of the increase in the relative price of food (decrease in the relative price of shirts). The US gains from the increase in Indian productivity because it makes our imports cheaper.

In **India**, the real wage in terms of shirts rises due to the increase in productivity. The impact on the real wage in terms of food is potentially ambiguous; the real wage in terms of shirts rises due to the productivity increase but since the relative price of shirts falls (or, equivalently, the relative price of the import good – food – rises), the overall impact is potentially ambiguous. In this specific case, the real Indian wage in terms of the import good is unchanged.

**Overall,** the “average” real wage rises in both countries and thus, both countries benefit from the productivity increase in India.

**g)** Suppose a third country, Brazil joins the US-India free trade area. Assume the productivity of labor in each country is given by:

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>India</th>
<th>Brazil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shirts</td>
<td>10 shirts/day</td>
<td>4 shirts/day</td>
<td>3 shirts/day</td>
</tr>
<tr>
<td>Food</td>
<td>20 bushels/day</td>
<td>2 bushels/day</td>
<td>9 bushel/day</td>
</tr>
</tbody>
</table>

**i How will the addition of Brazil to this agreement affect which good the US exports and which good India exports? Can you tell which good Brazil will export?** (A verbal, not a quantitative, answer is expected).

If you look at autarky prices, using the same logic as earlier, you have:

\[
\left(\frac{P_s}{P_f}\right)^{\text{US}} = 2; \quad \left(\frac{P_s}{P_f}\right)^{\text{India}} = \left(\frac{1}{2}\right); \quad \left(\frac{P_s}{P_f}\right)^{\text{Brazil}} = 3
\]

So India remains the lowest cost supplier of shirts (highest cost supplier of food), while Brazil becomes the lowest cost supplier of food, whereas U.S. costs are now intermediate between the two. If trade occurs among all 3 countries, India will export shirts, Brazil will export food but the US’s trade pattern is uncertain. In terms of comparative advantage, Brazil has a comparative advantage in food when compared to the U.S. or to India, but now the US has a comparative advantage in shirts when compared to Brazil (thus, if there were just trade between the US and Brazil, the US would export shirts). However, the US – of course – still has a comparative advantage in food when compared to India. Without more information, we cannot be sure which good the US would export.
ii How will the addition of Brazil to the US-India trade agreement affect the (free trade) equilibrium relative price of shirts (in terms of food)? A verbal answer suffices.

Since the world relative price of shirts – before Brazil joined the trading group – was below 3 (i.e., the relative price of food was above 1/3) Brazil will want to specialize in food. Thus, allowing Brazil to join the trade agreement will increase the supply of food and the demand for shirts. Thus, the world relative price of food will fall (i.e., the world relative price of shirts rises) due to Brazil’s joining the trade group.

iii How will the addition of Brazil to the US-India trade agreement affect welfare (real wages) in the US and in India? Will both countries necessarily benefit by allowing India to join? Will Brazil benefit by joining this free trade zone? Explain carefully.

Since Brazil’s addition to the free trade zone will raise the relative price of shirts – an Indian export – India will gain (equivalently, the price of its imports fall). Prior to Brazil’s entry into the agreement, the U.S. exported food (imported clothing), and since the relative price of clothing rises the US will lose if it continues to export food. It is possible – if Brazil were large enough and the price of shirts rose enough, that the US could become a clothing exporter, in which case the welfare impact on the US is ambiguous. However, in a “real world” context this is unlikely (because of the relative sizes of the markets), so the US is likely to lose as a result of the addition of Brazil to the trade agreement. Brazil, of course, gains as compared to autarky. The main point is that – while the world as a whole potentially gains in that it is possible to increase world output of all goods – not every country necessarily gains unless it is compensated. Thus, it would be possible for all countries to be potentially better off after Brazil joins the free trade zone but, without compensation, not every country must gain.

2. (Extension of Ricardian model to many goods) To illustrate how the model can be extended to more than two goods, consider the following example:

<table>
<thead>
<tr>
<th></th>
<th>Aircraft</th>
<th>Cars</th>
<th>Cell Phones</th>
<th>Food</th>
<th>Shoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>3 planes/month</td>
<td>4 cars/month</td>
<td>6 phones/month</td>
<td>60 bushels/month</td>
<td>10 pair shoes/month</td>
</tr>
<tr>
<td>India</td>
<td>0.5 plane/month</td>
<td>1 car/month</td>
<td>2 phones/month</td>
<td>6 bushels/month</td>
<td>5 pair shoes/month</td>
</tr>
<tr>
<td>Ratio US to</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Indian Productivity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) In which good(s) does the U.S. have an absolute advantage? In which good(s) does India have an absolute advantage?

The U.S. has an absolute advantage in all goods.

(b) In which country is the opportunity cost of producing cars lower? If the answer is ambiguous, explain why.

The question is: the opportunity cost of producing cars in terms of WHICH other good? If we look at the last row of the Table above (not given in the problem set itself), we see US relative productivity...
(compared to India) in producing cars is higher than the relative productivity for cell phones or shoes, but lower than the relative productivity in producing food or aircraft. Thus, the opportunity cost of producing cars — as compared to shoes or cell phones — is lower in the US; BUT the opportunity cost of producing cars — as compared to food or aircraft, is higher in the US.

Not asked but from the Table we see that the US has the lower opportunity cost of producing food, measured in terms of ANY of the other goods, and India has the lower opportunity cost of producing shoes, measured in terms of ANY of the other goods.

(c) Find autarky relative prices in each country (for simplicity, express the price of each good in terms of food).

**Autarky relative prices in terms of food.** Letting food be the item in which values are measured (what is called the *numeraire*), we have for each country, if the good is produced, \( W = P_x MPL_x \) for every good \( x \), where \( MPL_x \) is the labor productivity; hence: \( \frac{P_x}{P_f} = \frac{MPL_f}{MPL_x} \), or \( P_x = P_f \left( \frac{MPL_f}{MPL_x} \right) \) and thus:

**US:**

\[
\frac{P_{\text{aircraft}}}{P_{\text{food}}} = \frac{MPL_{\text{food}}}{MPL_{\text{aircraft}}} = \left( \frac{60}{3} \right) \text{ food / aircraft}; \quad \frac{P_{\text{cars}}}{P_{\text{food}}} = \frac{MPL_{\text{food}}}{MPL_{\text{cars}}} = \left( \frac{60}{4} \right) \text{ food / cars};
\]

\[
\frac{P_{\text{cellphones}}}{P_{\text{food}}} = \left( \frac{60}{6} \right) \text{ food / phone}; \quad \frac{P_{\text{shoes}}}{P_{\text{food}}} = \left( \frac{60}{10} \right) \text{ food / shoe};
\]

**India:**

\[
\frac{P_{\text{aircraft}}}{P_{\text{food}}} = \frac{MPL_{\text{food}}}{MPL_{\text{aircraft}}} = \left( \frac{6}{0.5} \right) = 12 \text{ food / aircraft}; \quad \frac{P_{\text{cars}}}{P_{\text{food}}} = \frac{MPL_{\text{food}}}{MPL_{\text{cars}}} = \left( \frac{6}{1} \right) \text{ food / cars};
\]

\[
\frac{P_{\text{cellphones}}}{P_{\text{food}}} = \left( \frac{6}{2} \right) \text{ food / phone}; \quad \frac{P_{\text{shoes}}}{P_{\text{food}}} = \left( \frac{6}{5} \right) \text{ food / shoe};
\]

(d) If trade were allowed, what can you predict about the pattern of trade? (i.e., which goods the U.S. exports and which it imports)? Explain.

If free trade were allowed, we would predict that the US would definitely export food, and that India would definitely export shoes, because each country has a comparative advantage in that good as compared to any other good. What other goods the countries exported would depend on demand and country size. However, if the US only exports two goods, we know it would be food and aircraft (because the US has a comparative advantage in aircraft, as compared to shoes, cars or cell phones); if the US exported 3 goods, it would be food, aircraft and cars; and if the US exported 4 goods, it would be all goods except shoes.

Similar logic applies to India (by just looking above at what the US is not exporting).

(e) Let \( W \) denote the wage in the U.S., and \( \bar{W} \) denote the Indian wage. Show where each good is produced based upon the ratio of wages \( \left( \frac{W}{\bar{W}} \right) \) between the two countries.

The ratio of marginal costs in the US to that in India, for each good is:
\[
\frac{MC_{x}}{MC_{y}} = \frac{(W/MPL_{x})}{(W/MPL_{y})} = \frac{\omega}{\left(\frac{MPL_{x}}{MPL_{y}}\right)}
\]
where: \(\omega \equiv \left(\frac{W}{W}\right)\). Thus, for all goods:

\[
\frac{MC_{aircraft}}{MC_{India}} = \frac{\omega}{6}, \quad \frac{MC_{cars}}{MC_{India}} = \frac{\omega}{4}, \quad \frac{MC_{cellphones}}{MC_{India}} = \frac{\omega}{3}, \quad \frac{MC_{food}}{MC_{India}} = \frac{\omega}{10}, \quad \frac{MC_{shoes}}{MC_{India}} = \frac{\omega}{2}
\]

Goods will be produced where the cost is lowest. Thus, if the ratio of US to Indian MC is less than one for a given good, production of that good occurs in the US; if the ratio is greater than one, production occurs in India and when it equals one, production may occur in both countries. Hence:

- If \(\omega < 2\), all goods will be produced in the US and none in India (this cannot be an equilibrium);
- If \(2 < \omega < 3\) then shoes will be produced in India, all other goods in the US;
- If \(3 < \omega < 4\) then shoes and cell phones will be produced in India, the other 3 goods in the US;
- If \(4 < \omega < 6\) then food and aircraft will be produced in the US, the other 3 goods in India;
- If \(6 < \omega < 10\) then only food will be produced in the US, the other four goods in India;
- If \(\omega > 10\), no goods will be produced in the US (which cannot be an equilibrium because of unemployment).

(Naturally, if we have an equality, such as \(\omega = 3\), then one of the goods (here, cell phones) could be produced in both countries.)

(f) Given relative labor supplies for each country, draw a graph to show how the equilibrium relative wage (US wage to Indian wage) is determined. Explain the graph.

The figure looks something like that below. The numbers shown on the vertical scale represent the relative wage at which it is equally costly to produce one of the goods in both countries. At \(\omega = 10\), the cost of producing food is the same in the US and India; everything else is cheaper to produce in India. Thus, at \(\omega = 10\), the (relative) demand for US labor is horizontal, since the percent of world food production that occurs in the US could be any number between 0 and 100% at that price. Once the relative wage falls below 10, then definitely all food production is in the US. As \(\omega\) decreases, the relative cost (price) of food production falls, compared to other goods, thereby increasing demand for food and hence the demand for US labor (to produce food) increases – this explains the downward sloping portion of the labor demand curve. Once the relative wage reaches 6, the costs of aircraft production are the same in both countries, and again labor demand is horizontal (infinitely elastic) over some range. Once all aircraft are produced in the US, it takes further declines in the relative wage to stimulate demand for US labor.

The relative demand and relative supply curves jointly determine the relative wage. Given the initial relative supply of US labor, the initial equilibrium is at a point like A.
(g) Use the graph to discuss how an increase in the Indian working population will affect: (1) US real wages; (2) the set of goods the US exports; (3) the relative prices of goods; and (4) Indian real wages.

Since productivities are not shifting, just the Indian population (and work force), what happens is shown in the same graph (above) by shifting the relative labor supply to the left — that is, the US work force, relative to the Indian work force, decreases in size. This is shown by the dotted vertical line. The new equilibrium relative wage is shown at point B, so the US wage falls relative to the Indian wage. Moreover, since productivities are not changing we know: (1) the US real wage in terms of its exports is unchanged but the real wage in terms of all imports rises (due to the decrease in the Indian relative wage); (2) the set of goods the US exports (in this graph) decreases — in this case, it now imports cars, instead of exporting them; and the US real wage, in terms of cars, rises; (3) the relative price of goods India exports falls and the relative price of its imports rise; and (4) since productivities are unchanged, the Indian real wage decreases. That is, in terms of its original exports (shoes, cell phones) the real wage is unchanged; but in terms of initial imports — food, planes and cars — the real wage falls. The real wage in terms of cars, which transitions from being an Indian import good to an export good, falls due to the lost gains from cheaper imports. Thus, the Indian population increase helps the U.S. but hurts India (of course, to be precise, we are assuming the proportion of the population that works is unchanged. If, as in reality, the proportion of the Indian population that is employed falls, the analysis is somewhat different).

3. (Specific Factor Model, Chapter 3) Labor is the only mobile factor, capital (K) is specific to sector C, land (T) is specific to food production. Technology and resources constraints are:

\[ C = 3\beta (K)^{2/3} (L_c)^{1/3} \; ; \; F = 3\alpha (T)^{2/3} (L_f)^{1/3} \; ; \; \beta, \alpha > 0 \; ; \text{resource constraint:} \; L_c + L_f = L \]
a) Derive the production possibility frontier for this economy (that is, express $C$ as a function of $F$, and also of the resources available and the technology: $K, T, L$ and productivity variables, $\beta, \alpha$).

Because there is only one variable input, the production possibility frontier (ppf) can be derived through substitution:

$$C = 3\beta (K)^{2/3} (L_c)^{1/3}; \quad F = 3\alpha (T)^{2/3} (L_f)^{1/3}; \quad \beta, \alpha > 0$$

Because there is only one variable input, the production possibility frontier (ppf) can be derived through substitution:

$$C = 3\beta (K)^{2/3} (L_c)^{1/3} \rightarrow (C/3\beta)^{1/3} = (K^2L_c) \rightarrow L_c = \left( \frac{C^3}{3^3 \beta^3 K^2} \right);$$

$$F = 3\alpha (T)^{2/3} (L_f)^{1/3} \rightarrow (F/3\alpha)^{1/3} = (T^2L_f) \rightarrow L_f = \left( \frac{F^3}{3^3 \alpha^3 T^2} \right);$$

$$L_c + L_f = L \rightarrow \left( \frac{C^3}{3^3 \beta^3 K^2} + \frac{F^3}{3^3 \alpha^3 T^2} \right) = L \quad \text{yields the ppf.}$$

Clearly the ppf is not linear.

b) Use the specific sector model to answer the following:

i) Given output prices, show graphically how the equilibrium wage rate and the allocation of labor between the two sectors is determined.

In the figure below, $L^T$ represents the total supply of labor; labor demand for sector F is measured from the point “0”, in the usual direction – as we move to the right, labor demand in F is increasing. Labor demand for sector C has been “flipped over”, so the demand for labor in sector C is measured from the right dotted vertical line, and as we move to the left, the demand for labor in C is increasing (the arrows at the bottom of the figure also show this). Equilibrium occurs where the two curves intersect – which is where total labor demand equals total labor supply; this is point A, with equilibrium wage $W^e$, and with $(0, L^F)$ units of labor employed in sector F, and $(L^C, L^T)$ units of labor employed in sector C.
ii  Show mathematically how the equilibrium wage rate and the supply curve for each good \((C, F)\) is determined (as a function of output prices). Also, discuss how the returns to land and capital are determined.

To derive the labor demand curves, set the marginal value product of labor equal to the wage:
\[
C = 3\beta (K)^{2/3} (L_c)^{1/3} \rightarrow P_c \left( \frac{\partial C}{\partial L_c} \right) = \beta P_c (L_c)^{-2/3} K^{2/3} = W \rightarrow L_c^e = K \left( \beta P_c / W \right)^{3/2}
\]
\[
F = 3\alpha (T)^{2/3} (L_f)^{1/3} \rightarrow P_f \left( \frac{\partial F}{\partial L_f} \right) = \alpha P_f (L_f)^{-2/3} T^{2/3} = W \rightarrow L_f^e = T \left( \alpha P_f / W \right)^{3/2}
\]
Adding labor demands and setting it equal to supply yields the equilibrium wage:
\[
L_f^e + L_c^e = L \rightarrow W = \left( \alpha P_f / W \right)^{3/2} + K \left( \beta P_c / W \right)^{3/2} = \left( T / L \right) \left( \alpha P_f \right)^{3/2} + \left( K / L \right) \left( \beta P_c \right)^{3/2}
\]

Note that: doubling prices causes wages to double (the pure inflation effect); and that doubling all inputs has no effect on wages (because of “constant returns to scale”).

As for the equilibrium supply curves: These can be found in one of two ways: (1) using labor demand in each sector, if you plug that back into the production function, and evaluate at the equilibrium wage, you get the (“general equilibrium”) supply curve; (2) or, you can get the general equilibrium supply curves from the production possibility frontier, by finding the point where the slope of the frontier equals the relative price ratio. You should use both methods to convince yourself that they give the same result.

Method 1:
Method 2:

From the ppf, production occurs where the (absolute value of ) slope = price ratio; that is:

\[
\rho = \frac{P_f}{P_c}
\]

From the ppf:

\[
\left(\frac{C^3}{3^3 \beta^3 K^2}\right) + \left(\frac{F^3}{3^3 \alpha^3 T^2}\right) = L \rightarrow \left(3C^2/27 \beta^3 K^2\right) dC + \left(3F^2/27 \alpha^3 T^2\right) dF = 0 \rightarrow \frac{dC}{dF} = \frac{F^2 \beta^3 K^2}{C^2 \alpha^3 T^2} = \rho; \quad \rho = \frac{P_f}{P_c}
\]

where \(\rho\) represents the relative price of food. Simplifying and substituting back into the ppf gives the supply curves:

\[
F = \rho^{3/2} C \left(\frac{T}{K}\right) \left(\frac{\alpha^{3/2}}{\beta^{3/2}}\right); \quad \left(\frac{C^3}{3^3 \beta^3 K^2}\right) + \left(\frac{F^3}{3^3 \alpha^3 T^2}\right) = L \rightarrow \left(\frac{C^3}{3^3 \beta^3 K^2}\right) + \left[\rho^{3/2} C \left(\frac{T}{K}\right) \left(\frac{\alpha^{3/2}}{\beta^{3/2}}\right)\right]^3 = \left(\frac{C}{3 \beta^{3/2} K}\right)^3 \left(\beta^{3/2} K + \rho^{3/2} \alpha^{3/2} T\right) = L \rightarrow
\]

\[
C^* = \frac{3 \beta^{3/2} K}{\left[\left(T/L\right)(\alpha \rho)^{3/2} + \left(K/L\right)(\beta)^{3/2}\right]^{1/3}}; \quad F^* = \rho^{1/2} C^* \left(\frac{T}{K}\right) \left(\frac{\alpha^{3/2}}{\beta^{3/2}}\right) = \frac{3 \alpha^{3/2} T}{\left[\left(T/L\right)(\alpha)^{3/2} + \left(K/L\right)(\beta^{-1})^{3/2}\right]^{1/3}}
\]

Hence you can see the supply curves, whether derived by method 1 or 2, are the same. The properties of the supply curves include:

1. If both output prices double, there is no change in supply of either good – this is “pure inflation”;
2. If all input supplies double, the output of both goods doubles (given price) – this is constant returns to scale;
(3) You should be able to see that the supply of C increases as K increases but decreases as T increases; conversely, for F, the supply of F decreases as K increases, but increases as T increases. This is the effect of specific factors.

(4) For the mobile factor, labor, supplies of both goods increase as L increases.

(5) Given output prices, an increase in productivity in sector C (an increase in $\beta$) increases supply of good C and decreases supply of good F; the opposite holds for an increase in $\alpha$.

(6) And, of course, an increase in the price of a good increases the supply of that good and decreases the supply of the other good.

**As for the returns to land and capital:**

Graphically, since you can think of landowners as “consumers” of labor, the returns to landowners can be measured by the “consumer surplus” area next to the demand for labor in sector F: that is, in terms of the previous figure, the area of $\{B,W_e,A\}$ would give the total return to land. Similarly, the return to capital can be measured by the area next to the labor demand curve in sector C: this would be area $\{C,H,A\}$.

Mathematically, the return to land (per acre) can be measured in either of two ways – they give the same result because of constant returns to scale. Let $R_T$ stand for the return to land. Then:

$$R_T = \frac{P_f F - W L_f}{T} = P_f \left( \frac{\partial F}{\partial T} \right)$$

The first way is the total “profits”, per acre, after paying labor costs; the second way is the marginal value product of land. Under constant returns to scale, these two measures are identical. Hence:

$$R_T = P_f \left( \frac{\partial F}{\partial T} \right) = 2\alpha P_f \left( \frac{L_f}{T} \right)^{1/3}$$

Using the demand for labor in sector F,

$$L_f^* = T \left( \alpha P_f / W \right)^{3/2} \rightarrow R_T = 2\alpha P_f \left( L_f / T \right)^{1/3} = 2\alpha P_f \left[ \left( \alpha P_f / W \right)^{3/2} \right]^{1/3} = \frac{2(\alpha P_f)^{3/2}}{W^{1/2}}$$

Similarly, the return to capital ($R_K$) is found from:

$$R_K = \frac{P_c C - W L_c}{K} = P_c \left( \frac{\partial C}{\partial K} \right) = 2\beta P_c \left( L_c \right)^{1/3} K^{-1/3}.$$  

From the labor demand in C:

$$\frac{L_c^*}{K} = \left( \beta P_c / W \right)^{3/2} \rightarrow R_K = \frac{2(\beta P_c)^{3/2}}{W^{1/2}}$$

**Finally,** using the equilibrium wage calculated above, we get the equilibrium returns to land and capital:

$$W^e = \left[ \left( T / L \right) \left( \alpha P_f \right)^{3/2} + \left( K / L \right) \left( \beta P_c \right)^{3/2} \right]^{2/3} \rightarrow$$
iii Using your result in part ii, given output prices, show how an increase in the amount of land (T) available for production affects: (1) the quantity supplied of each good; (2) the real return to land; (3) the real return to capital; and (4) the real wage rate.

(1) Mathematically, from our earlier results:

\[ R_K = \frac{2(\beta P_c)^{3/2}}{W^{1/2}} = \frac{2(\beta P_c)^{3/2}}{\left[ (T/L)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2} \right]^{1/3}} \]

\[ R_T = \frac{2(\alpha P_f)^{3/2}}{W^{1/2}} = \frac{2(\alpha P_f)^{3/2}}{\left[ (T/L)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2} \right]^{1/3}} \]

That is, an increase in the specific factor used to produce food, given output prices, leads to an increase in the output of food and a decrease in the output of the other good (clothing).

In terms of returns to factors, given the output prices, we would expect the increase in the amount of land to reduce the return to land (diminishing marginal productivity), increase the demand for labor and thus increase the wage rate, and hence – through this indirect effect – reduce the return on capital.

Mathematically:
These mathematical results confirm the intuition stated above.

iv. **Given output prices, show how an increase in clothing productivity \((\beta)\) affects the supply of each good and the real return to each input.**

Intuitively, increased productivity in the clothing sector will – with given inputs – increase clothing output. Also, since it increases the value of workers in that sector, workers will move from food production to clothing production. Hence, the output of clothing increases for two reasons (more labor, better technology) while the output of food decreases due to the outflow of labor. As for factor prices, the productivity increase in clothing should increase the return to capital and the return to labor in clothing production. That attracts workers from the food sector, meaning wages rise there – and ultimately since wages are the same in the two sectors, wages must increase overall. But higher wages raise labor costs in food production, meaning the return to land falls. Mathematically, using the earlier relationships:

\[
R_K = \frac{2(\beta P_c)^{3/2}}{\left[\frac{T}{L}\left(\alpha P_f\right)^{3/2} + \left(\frac{K}{L}\right)(\beta P_c)^{3/2}\right]^{1/3}} \quad \rightarrow \frac{\partial R_K}{\partial T} = \frac{-2/3(\beta P_c)^{3/2}}{\left[\frac{T}{L}\left(\alpha P_f\right)^{3/2} + \left(\frac{K}{L}\right)(\beta P_c)^{3/2}\right]^{4/3}} < 0
\]

\[
R_T = \frac{2(\alpha P_f)^{3/2}}{\left[\frac{T}{L}\left(\alpha P_f\right)^{3/2} + \left(\frac{K}{L}\right)(\beta P_c)^{3/2}\right]^{1/3}} \quad \rightarrow \frac{\partial R_T}{\partial T} = \frac{-2/3(\alpha P_f)^{3/2}}{\left[\frac{T}{L}\left(\alpha P_f\right)^{3/2} + \left(\frac{K}{L}\right)(\beta P_c)^{3/2}\right]^{4/3}} < 0
\]

\[
W^e = \left[\frac{T}{L}\left(\alpha P_f\right)^{3/2} + \left(\frac{K}{L}\right)(\beta P_c)^{3/2}\right]^{2/3} \quad \rightarrow \frac{\partial W^e}{\partial T} = \frac{(2/3)(1/L)(\alpha P_f)^{3/2}}{\left[\frac{T}{L}\left(\alpha P_f\right)^{3/2} + \left(\frac{K}{L}\right)(\beta P_c)^{3/2}\right]^{4/3}} > 0
\]
\[
R_K = \frac{2(\beta P_c)^{3/2}}{\left[\left(\frac{T}{L}\right)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{1/3}}
\]

\[
\left(\frac{\partial R_K}{\partial \beta}\right) = \frac{3(P_c)^{3/2} \beta^{1/2}}{\left[\left(\frac{T}{L}\right)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{1/3}} - \frac{(\beta P_c)^{3/2} (K/L)(P_c)^{3/2} \beta^{1/2}}{\left[\left(\frac{T}{L}\right)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{4/3}} = 0
\]

\[
R_T = \frac{2(\alpha P_f)^{3/2}}{\left[\left(\frac{T}{L}\right)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{1/3}} \rightarrow \left(\frac{\partial R_T}{\partial \beta}\right) = \frac{-\left(\alpha P_f\right)^{3/2} (K/L)(P_c)^{3/2} \beta^{1/2}}{\left[\left(\frac{T}{L}\right)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{4/3}} < 0
\]

\[
W^e = \left[\left(\frac{T}{L}\right)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{2/3} \rightarrow \left(\frac{\partial W^e}{\partial \beta}\right) = \frac{(K/L)(P_c)^{3/2} \beta^{1/2}}{\left[\left(\frac{T}{L}\right)(\alpha P_f)^{3/2} + (K/L)(\beta P_c)^{3/2}\right]^{1/3}} > 0
\]

v What happens to outputs and factor prices if both $\beta$ and $\alpha$ double?
This type of productivity change is called balanced growth, and by looking at the supply curves and factor price equations you can see that, given output prices, both output supplies double, and all factor prices double. From a trade perspective, this is a bit like the Ricardian model – doubling productivity in all sectors should not affect a country’s pattern of trade though it will, of course, raise the standard of living.

c) Now assume there are two countries, Mexico and Brazil, which are almost identical. They have the same tastes (the same demand curves), the same technology, and the same amount of capital and labor; however, Brazil has more land than Mexico.

i Based upon your results from part (b), what predictions would you make concerning the autarky (no trade) relative price of food in Brazil as compared to Mexico? (a verbal answer suffices)

Using the exercises above, we know that the additional land in Brazil will mean that at the same output prices, Brazil will produce more food and less clothing than Mexico. Hence, given that demands are the same, the autarky relative price of food will be lower in Brazil – which of course means the autarky relative price of clothing will be higher in Mexico.

ii If trade is allowed between the two countries, what will the pattern of trade be and how will the relative price of clothing change in each country?
The pattern of trade is determined by the differences in autarky prices. Since clothing is relatively cheaper (food relatively more expensive) in Mexico, then Mexico will export clothing and import food. Thus, trade will cause the relative price of clothing to rise in Mexico and to fall in Brazil.

iii Finally, discuss how trade affects the real return to each factor (capital, land and labor) in each country. Does each country as a whole potentially gain from trade? Does each interest group (factor owner) in each country also gain? Be as precise as possible (a verbal answer suffices).

We know that each country potentially gains from trade, but this does not imply that everybody within the country gains. In Mexico, the export of clothing (import of food) raises the relative price of clothing, and hence the real return to the input that is specific to clothing production – capital (K) - will increase. The same logic implies that the real return to land will fall in Mexico (imported food reduces domestic food prices and hence the value of land). The impact on the real wage is ambiguous – the real wage increases in terms of the import good (food) but falls in terms of the export good (clothing).

The impact in Brazil is similar – though the roles of the specific inputs are reversed. Since Brazil exports food and imports clothing – trade increases the real return to land – the input specific to the export sector, and lowers the real return to capital, which is the input specific to the import sector. Again, the impact on the real wage is ambiguous. The real wage in terms of the import good (now clothing) rises in Brazil but the real wage in terms of the export good (food for Brazil) falls.