3. Show how each of the following would affect the U.S. balance of payments. Include a description of the debit and credit items, and in each case identify which specific account is affected (e.g., imports of goods and services, IM; exports of assets, EX; and so on. For this question, you may find it helpful to refer to Appendix 1.).

a. A California computer manufacturer purchases a $50 hard disk from a Malaysian company, paying the funds from a bank account in Malaysia.

   **Answer:**

<table>
<thead>
<tr>
<th>Description</th>
<th>BOP Account</th>
<th>Account (detail)</th>
<th>Credit/Debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard disk imported from Malaysia</td>
<td>CA (↓)</td>
<td>-IM (↑), TB (↓)</td>
<td>-$50</td>
</tr>
<tr>
<td>Decrease in Malaysian deposits owned by U.S. firm</td>
<td>FA (↑)</td>
<td>-IM (↓)</td>
<td>$50</td>
</tr>
</tbody>
</table>

b. A U.S. tourist to Japan sells his iPod to a local resident for yen worth $100.

   **Answer:**

<table>
<thead>
<tr>
<th>Description</th>
<th>BOP Account</th>
<th>Account (detail)</th>
<th>Credit/Debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>iPod exported to Japan</td>
<td>CA (↑)</td>
<td>+EX (↑), TB (↑)</td>
<td>$100</td>
</tr>
<tr>
<td>Increase in Japanese currency owned by U.S. tourist</td>
<td>FA (↓)</td>
<td>-IM (↑)</td>
<td>-$100</td>
</tr>
</tbody>
</table>

c. The U.S. central bank sells $500 million of its holdings of U.S. Treasury bonds to a British financial firm and purchases pound sterling foreign reserves.

   **Answer:**

<table>
<thead>
<tr>
<th>Description</th>
<th>BOP Account</th>
<th>Account (detail)</th>
<th>Credit/Debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. bonds sold to British firm</td>
<td>FA (↑)</td>
<td>+EX (↑)</td>
<td>$500 mil.</td>
</tr>
<tr>
<td>Pound-sterling reserves imported from Britain</td>
<td>FA (↓)</td>
<td>-IM (↑)</td>
<td>-$500 mil.</td>
</tr>
</tbody>
</table>

d. A foreign owner of Apple shares receives $10,000 in dividend payments, which are paid into a New York bank.

   **Answer:**

<table>
<thead>
<tr>
<th>Description</th>
<th>BOP Account</th>
<th>Account (detail)</th>
<th>Credit/Debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Import of factor service (ownership) from ROW</td>
<td>CA (↓)</td>
<td>-IM (↑), NFIA (↑)</td>
<td>-$10,000</td>
</tr>
<tr>
<td>New York bank deposits paid to ROW</td>
<td>FA (↑)</td>
<td>+EX (↑)</td>
<td>$10,000</td>
</tr>
</tbody>
</table>

e. The central bank of China purchases $1 million of export earnings from a firm that has sold $1 million of toys to the United States, and the central bank holds these dollars as reserves.

   **Answer:**

<table>
<thead>
<tr>
<th>Description</th>
<th>BOP Account</th>
<th>Account (detail)</th>
<th>Credit/Debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Import of toys from China</td>
<td>CA (↓)</td>
<td>-IM (↑), TB (↓)</td>
<td>-$1 mil.</td>
</tr>
<tr>
<td>China central bank buys U.S. dollars</td>
<td>FA (↑)</td>
<td>+EX (↑)</td>
<td>$1 mil.</td>
</tr>
</tbody>
</table>
f. The U.S. government forgives a $50 million debt owed by a developing country.

**Answer:**

<table>
<thead>
<tr>
<th>Description</th>
<th>BOP Account</th>
<th>Account (detail)</th>
<th>Credit/Debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt forgiveness (gift)</td>
<td>$\Delta K_A$</td>
<td>$-\Delta K_{out}$</td>
<td>$-50$ mil.</td>
</tr>
<tr>
<td>Decrease in external assets owned by U.S. entities</td>
<td>$\Delta FA$</td>
<td>$-\Delta IM_A$</td>
<td>+$50$ mil.</td>
</tr>
</tbody>
</table>

4. In 2010 the country of Ikonomia has a current account deficit of $1 billion and a nonreserve financial account surplus of $750 million. Ikonomia’s capital account is in a $100 million surplus. In addition, Ikonomian factors located in foreign countries earn $700 million. Ikonomia has a trade deficit of $800 million. Assume Ikonomia neither gives nor receives unilateral transfers. Ikonomia’s GDP is $9 billion.

a. What happened to Ikonomia’s net foreign assets during 2010? Did it acquire or lose foreign assets during the year?

**Answer:** $BOP = CA + FA + KA = 0$

$CA + KA = -FA$

Current account deficit of $1 billion ($1,000 million) and the capital account is in a $100 million surplus.

$-1,000 + 100 = -FA$

$FA = \$900 = EX_A - IM_A$

The financial account records financial flows in to and out of the country. In this case, the FA surplus indicates that on net, foreigners purchased more Ikonomian assets than Ikonomians purchased foreign assets. Therefore, net foreign assets for Ikonomia declined by $900 million.

b. Compute the official settlements balance (OSB). Based on this number, what happened to the central bank’s (foreign) reserves?

**Answer:** The financial account can be split into those transactions conducted by the central bank (official settlements balance) and those conducted by everyone else (nonreserve financial account):

$FA = \text{Official settlements balance} + \text{Nonreserve financial account}$

Nonreserve financial account is a $750 million surplus.

$\$900 = \text{Official settlements balance} + 750$

Official settlements balance = $150$

The official settlements balance is in a $150 million surplus. This means that foreign central banks purchased more Ikonomian assets (paid for with foreign currency) than the Ikonomian central bank purchases of foreign assets (paid for with domestic currency, U.S. dollars in this case). Therefore, Ikonomia’s central bank experienced an increase in its foreign reserve holdings.
c. How much income did foreign factors of production earn in Ikonomia during 2010?

Answer: The current account can be split into three components: the trade balance (final goods and services), the net factor income from abroad (payments to/from factor services), and the net unilateral transfers.

\[ CA = TB + NFIA + NUT \]

\[ -1,000 = -800 + NFIA + 0 \]

In the question, we are given the trade balance (−$800 million) and the current account (−$1,000 million).

\[ NFIA = -200 \] Net factor income from abroad is −$200 million. This implies that foreign factors of production located in Ikonomia earned more than Ikonomian factors abroad.

\[ NFIA = EX_{FS} - IM_{FS} \]

We know that Ikonomian factors abroad earned $700 million.

\[ -200 = 700 - IM_{FS} \]

\[ IM_{FS} = 900 \] Foreign factors located in Ikonomia earned $900 million.

d. Compute net factor income from abroad (NFIA).

Answer: See (c). \( NFIA = -200 \) million.

e. Using the identity \( BOP = CA + FA + KA \), show that \( BOP = 0 \).

Answer: To check our work, we can verify the BOP identity:

\[ BOP = CA + FA + KA \]

\[ BOP = [TB + NFIA + NUT] + FA + KA \]

\[ BOP = [-800 + -200 + 0] + [750 + 150] + 100 = 0 \]

f. Calculate Ikonomia’s gross national expenditure (GNE), gross national income (GNI), and gross national disposable income (GNDI).

Answer: We know that \( GDP = C + I + G + (EX - IM) = GNE + TB \)

\[ GNE = GDP - TB \]

\[ GNE = 9,000 - (-800) \]

\[ GNE = 9,800 \]

\[ GNI = GDP + NFIA \]

\[ GNI = 9,000 + (-200) = 8,800 \]

\[ GNDI = GDP + NFIA + NUT = GNI + NUT. \]

Because \( NUT = 0 \), \( GNDI = GNI = GNDI = 8,800 \)
5. To answer this question, you must obtain data from the Bureau of Economic Analysis (BEA), http://www.bea.gov, on the U.S. balance of payment (BOP) tables. Go to interactive tables to obtain annual data for 2008 (the default setting is for quarterly data). It may take you some time to become familiar with how to navigate the website. You need only refer to Table 1 on the BOP accounts. Using the BOP data, calculate the following for the United States:

(Answers will vary because of data revisions. The figures below are based on those given in the Table 5-3 [16-3]; release date September 19, 2013.)

a. Trade balance (TB), net factor income from abroad (NFIA), net unilateral transfers (NUT), and current account (CA)

Answer: $702 billion (Line 74)

$146 billion (Line 75)

$125 billion (Line 76)

$681 billion (Line 77)

b. Financial account (FA)

Answer: $730 billion (Lines 40 + 55 + 70)

c. Official settlements balance (OSB), referred to as “U.S. official reserve assets” and “Foreign official assets in the U.S.”

Answer: $550 billion (Lines 41 + 56)

d. Nonreserve financial account (NRFA)

Answer: $180 billion (Lines 40 + 55 + 70 – 41 – 56)

e. Balance of payments (BOP). Note that this may not equal zero because of statistical discrepancy. Verify that the discrepancy is the same as the one reported by the BEA.

Answer: $55 billion (Line 71)
7. During the 1980s, the United States experienced “twin deficits” in the current account and government budget. Since 1998, the U.S. current account deficit has grown steadily, along with rising government budget deficits. Do government budget deficits lead to current account deficits? Identify other possible sources of the current account deficits. Do current account deficits necessarily indicate problems in the economy?

**Answer:** “Twin deficits” are possible, but there are other factors that influence the current account. Since 1998, the decline in the current account has been associated with movements in investment and national savings. Note the following expression from the textbook:

\[
CA = S_p + S_G - I
\]

It is not clear that budget deficits cause current account deficits. There are two possibilities besides a budget deficit \( S_G < 0 \):

- Private savings \( S_p \) may change when the government changes taxes (e.g., tax rates). Suppose tax rates decrease, causing a decrease in government saving. According to Ricardian equivalence, households will respond to a tax cut today by increasing savings in anticipation of a future tax increase needed to finance the current budget deficit. This implies private savings will increase, possibly offsetting the effect on national saving.

- The current account may move independently of saving, namely because of changes in investment \( I \). An increase in domestic investment opportunities could lead to current account deficits.

8. Consider the economy of Opulenza. In Opulenza, domestic investment of $400 million earned $20 million in capital gains during 2012. Opulenzans purchased $120 million in new foreign assets during the year; foreigners purchased $160 million in Opulenzan assets. Assume the valuation effects total $1 million in capital gains. Note that we need to assume a value for the capital account. We will assume \( KA = 0 \) in the following transactions.

a. Compute the change in domestic wealth in Opulenza.

**Answer:** The change in domestic wealth is the sum of additions to the capital stock plus capital gains earned on domestic assets:

\[
\text{Change in domestic wealth} = I + \text{Capital gains on } K = 400 + 20 = 420 \text{ million}
\]

b. Compute the change in external wealth for Opulenza.

**Answer:** The change in external wealth is:

\[
\Delta W = \text{Valuation effects} + (-EA) = 1 - (160 - 120) = -39 \text{ million}
\]

c. Compute the total change in wealth for Opulenza.

**Answer:** The change in total wealth is:

\[
\text{Change in total wealth} = \text{Change in domestic wealth} + \text{Change in external wealth} = 420 + (-39) = 381 \text{ million}
\]

d. Compute domestic savings for Opulenza.

**Answer:** To calculate national savings, note that the change in total wealth is:

\[
\text{Change in total wealth} = S + KA + \text{Capital gains on } K + \text{Capital gains on (A - L)}
\]

\[
381 = S + 0 + (20 + 1)
\]

\[
S = 360 \text{ million}
\]
e. Compute Opulenza’s current account. Is the CA in deficit or surplus?

**Answer:** Using the current account identity: \( S = I + CA \):

\[
S = I + CA \\
$360 = $400 + CA \\
CA = -$40 million
\]

Or, we could use the definition of the change in total wealth:

\[\text{Change total wealth} = I + (CA + KA) + \text{Capital gains on } K + \text{Capital gains on } (A - L)\]

\[\$381 = $400 + CA + $0 + $20 + $1\]

\[CA = -$40 million\]

f. Explain the intuition for the CA deficit/surplus in terms of savings in Opulenza, financial flows, and its domestic/external wealth position.

**Answer:** We see that Opulenza experienced a $420 million increase in its domestic wealth while losing $39 million in external wealth. Opulenza’s investment is $400 million of which $360 million was financed through domestic savings and $40 million from foreign borrowings, which equals the current account deficit. The increase in domestic wealth is in addition (to the investment of $400 million) because of capital gains on domestic investment of $20 million. The decrease in external wealth of only $39 million instead of $40 million is due to its capital gains on foreign wealth of $1 million.

g. How would a depreciation in Opulenza’s currency affect its domestic, external, and total wealth? Assume that foreign assets owned by Opulenzans are denominated in foreign currency.

**Answer:** The answer to this question depends on how Opulenzan external assets and external liabilities are denominated. Since all its foreign assets are in foreign currency a depreciation increases the value of its foreign assets and there is a valuation gain. If its liabilities are denominated in its own currency, then a depreciation does not affect its domestic currency value of liabilities. On the other hand, if its liabilities are also in foreign currency, then a depreciation has a negative valuation effect. The overall valuation effect will thus depend on the relative magnitude and currency composition of its liabilities.

9. This question asks you to compute valuation effects for the United States in 2004 using the same methods mentioned in the chapter. Use the bea.gov website to collect the data needed for this question: look under the “International” heading.

Visit the BEA’s balance of payments data page and obtain the U.S. balance of payments for 2004 in billions of dollars. Be sure to get the correct year, and annual data, not quarterly.

Visit the BEA’s net international investment position data page and obtain the U.S. net international investment position for end 2003 to end 2004.

Answers may vary based on data revisions. The data below were obtained in November 2007:

a. What was the U.S. current account for 2004?

**Answer:** \( CA = -$640,148 \) million

b. What was the U.S. financial account for 2004?

**Answer:** \( FA = $556,742 \) million
Balance of Payments I: The Gains from Financial Globalization

1. Using the notation from the text, answer the following questions. You may assume that net labor income from abroad is zero, there are no capital gains on external wealth, and there are no unilateral transfers.

a. Express the change in external wealth ($ΔW_0$) at the end of period 0 as a function of the economy’s trade balance ($TB$), the real interest rate (a constant, $r$), and initial external wealth ($W_{-1}$).

Answer: The total amount added to (or subtracted from) external wealth is equal to the trade balance plus interest earned on external wealth from the previous period: $ΔW_0 = TB_0 + rW_{-1}$.

b. Using (a), write an expression for the stock of external wealth at the end of period 0 ($W_0$). This should be written as a function of the economy’s trade balance ($TB_0$), the real interest rate, and initial external wealth ($W_{-1}$).

Answer: Rewriting the previous expression, solving for $W_0$: $ΔW_0 = W_0 - W_{-1} = TB_0 + rW_{-1}$

$W_0 = TB_0 + (1 + r)W_{-1}$

c. Using (a) and (b), write an expression for the stock of external wealth at the end of period 1 ($W_1$). This should be written as a function of the economy’s trade balance ($TB$) each period, the real interest rate, and initial external wealth ($W_{-1}$).

Answer: From (a), we know that the change in wealth in period 1 is equal to the trade balance in period 1 plus interest earned on external wealth from period 0:

$ΔW_1 = TB_1 + rW_0$

Rewriting this expression to solve for $W_1$:

$W_1 = TB_1 + (1 + r)W_0$

Plugging in the expression from (b) for $W_0$:

$W_1 = TB_1 + (1 + r)[TB_0 + (1 + r)W_{-1}]$

$W_1 = TB_1 + (1 + r)TB_0 + (1 + r)^2W_{-1}$
d. Using your answers from (a), (b), and (c), write an expression for the stock of external wealth at the end of period 2 ($W_2$). This should be written as a function of the economy’s trade balance ($TB$) each period, the real interest rate, and initial external wealth ($W_{-1}$).

Answer: Using the same method from (c) to solve for $W_2$:

$$\Delta W_2 = TB_2 + r*W_1$$
$$W_2 = TB_2 + (1 + r*)W_1$$
$$W_3 = TB_3 + (1 + r*)[TB_1 + (1 + r*)TB_0 + (1 + r*)^2W_{-1}]$$
$$W_2 = TB_2 + (1 + r*)TB_1 + (1 + r*)^2TB_0 + (1 + r*)^3W_{-1}$$

e. Suppose we require that $W_2$ equal 0. Write down the condition that the three trade balances (in periods 0, 1, and 2) must satisfy. Arrange the terms in present value form.

Answer: If $W_2 = 0$, then

$$TB_2 + (1 + r*)TB_1 + (1 + r*)^2TB_0 + (1 + r*)^3W_{-1} = 0$$

In present value terms:

$$-(1 + r*)W_{-1} = TB_0 + \frac{TB_1}{(1 + r*)} + \frac{TB_2}{(1 + r*)^2}$$

2. Using the assumptions and answers from the previous question, complete the following:

a. Write an expression for the future value of the stock of external wealth in period $N$ ($W_N$). This should be written as a function of the economy’s trade balance ($TB$) each period, the real interest rate ($r^*$) and initial external wealth.

Answer: The future value of external wealth in period $N$ is the sum of the trade balance in each period (compounded by the interest earned or paid) plus compounded interest earned on initial external wealth:

$$W_N = TB_N + (1 + r^*)TB_{N-1} + (1 + r^*)^2TB_{N-2} + \cdots + (1 + r^*)^N TB_0 + (1 + r^*)^{N+1}W_{-1}$$

b. Using the answer from (a), write an expression for present value of the stock of external wealth in period $N$ ($W_N$).

Answer: In present value terms, divide both sides of the previous expression by $(1 + r^*)^N$:

$$\frac{W_N}{(1 + r^*)^N} = \frac{TB_0}{(1 + r^*)} + \frac{TB_1}{(1 + r^*)^2} + \cdots + \frac{TB_N}{(1 + r^*)^N} + (1 + r^*)W_{-1}$$

c. The “no Ponzi game” conditions force the present value of $W_N$ to tend to 0 as $N$ gets large. Explain why this implies that the economy’s initial external wealth is equal to the present value of future trade deficits.

Answer: The “no Ponzi game” condition means that the county cannot have negative or positive external wealth as $N$ gets large (in the limit, $N \rightarrow \infty$):

$$\lim_{N \rightarrow \infty} \frac{W_N}{(1 + r^*)^N} = 0$$

Therefore, we can rewrite the expression from (b), imposing this condition:

$$-(1 + r^*)W_{-1} = TB_0 + \frac{TB_1}{(1 + r^*)} + \frac{TB_2}{(1 + r^*)^2} + \cdots + \frac{TB_N}{(1 + r^*)^N}$$
Therefore, the country’s initial external wealth, \( W_0 = (1 + r^*) W_{-1} \), is equal to the present value of the country’s future trade deficits.

d. How would the expressions in (a) and (b) change if the economy had net labor income (positive or negative) to or from abroad or net unilateral transfers? Explain briefly.

**Answer:** Net labor income to or from abroad would affect the stock of external wealth each period through changing the NFIA from \( NFIA = r^* W_0 \) (with only capital) to \( NFIA = r^* K_0 + \) net labor income from abroad, in which \( K \) denotes initial capital held abroad. Assume that labor abroad, \( L_0 \) is paid a world wage, \( w^* \):

\[
\Delta W_0 = TB_0 + r^* K_{-1} + w^* L_0 + NUT
\]

The country can effectively finance trade deficits through both capital income (as in the standard model) and labor income from abroad. Similarly, if the country receives net unilateral transfers, this would also add to the country’s initial wealth.

3. **In this question, assume all dollar units are real dollars in billions, so $150 means $150 billion.** It is year 0. Argentina thinks it can find $150 of domestic investment projects with an MPK of 10% (each $1 invested pays off $0.10 in every later year). Argentina invests $84 in year 0 by borrowing $84 from the rest of the world at a world real interest rate \( r^* \) of 5%. There is no further borrowing or investment after this.

Use the standard assumptions: Assume initial external wealth \( W (W \text{ in year } -1) = 0 \). Assume \( G = 0 \) always; and assume \( I = 0 \) except in year 0. Also, assume \( NUT = KA = 0 \) and that there is no net labor income so that \( NFIA = r^* W \).

The projects start to pay off in year 1 and continue to pay off all years thereafter. Interest is paid in perpetuity, in year 1 and every year thereafter. In addition, assume that if the projects are not done, then GDP = \( Q = C = $200 \) in all years, so that \( PV(Q) = PV(C) = 200 + 200/0.05 = 4,200 \).

a. Should Argentina fund the $84 worth of projects? Explain your answer.

**Answer:** Yes. The criterion for undertaking an investment project is:

\[
\frac{\Delta Q}{\Delta K} \geq r^*
\]

Because MPK = 10% > \( r^* \) (= 5%), the country will benefit from the investment project.

b. Why might Argentina be able to borrow only $84 and not $150?

**Answer:** Argentina may face borrowing limits. Because 150 units of output accounts for three-fourths of the country’s total production, lenders might be unwilling to lend this much, even for a productive investment project.

c. From this point forward, assume the projects totaling $84 are funded and completed in year 0. If the MPK is 10%, what is the total payoff from the projects in future years?

**Answer:** The project will result in an 8.4 increase in \( Q \) each period (= MPK \( \times \Delta K = 0.10 \times 84 \)).

d. Assume this is added to the $200 of GDP in all years starting in year 1. In dollars, what is Argentina’s \( Q = GDP \) in year 0, year 1, and later years?

**Answer:** \( Q_0 = 200, Q = 208.4 \) in subsequent years.
b. A strike in France leads to a reduction in French income.

**Answer:** This is an idiosyncratic shock to France’s output (from a reduction in labor production). France can buffer the effects on income through borrowing from the Czech Republic. France experiences a decline in output (from the labor stoppage), but still continues to enjoy its previous level of consumption and investment by borrowing from the Czech Republic. France will run a trade deficit that can finance France’s GNE. Likewise, the Czech Republic’s output is relatively high (compared with France), so its GDP > GNE, with the difference flowing to France in the form of net exports.

c. Floods destroy a portion of the Czech capital stock, lowering Czech income.

**Answer:** This is a negative idiosyncratic shock to the Czech Republic. The Czech Republic can run a trade deficit and borrow from France, buffering its GNE against the shock. Once the Czech Republic recovers it can run trade surpluses and pay back its loans from France.

10. Assume that a country produces an output $Q$ of 50 every year. The world interest rate is 10%. Consumption $C$ is 50 every year, and $I = G = 0$. There is an unexpected drop in output in year 0, so output falls to 39 and is then expected to return to 50 in every future year. If the country desires to smooth consumption, how much should it borrow in period 0? What will the new level of consumption be from then on?

**Answer:** There is a one-time decrease in output of 11 units. Therefore, the present value of consumption is:

$$PV(G) = PV(Q) - PV(G) = 39 + \frac{50}{0.10} = 539$$

To determine the level of consumption each period, we know that the country wants to maintain a given level of consumption:

$$PV(G) = C + \frac{C}{r^*}$$

$$539 = C + \frac{C}{0.10} \text{ therefore } C = 49$$

Since the output drops to 39 in the year 0, the country must borrow 10 units to consume 49 units. Note that for every 10 units borrowed, consumption is reduced by one unit, as $NFI_A = 0.10 \times 10$ in subsequent periods. Therefore, $C = 49$ in period 0 and thereafter. Alternatively, the change in consumption can be calculated using the following:

$$\Delta C = \frac{r^*}{1 + r^*} \Delta Q = \frac{0.10}{1 + 0.10}(-11) = -1$$

Consumption decreases by one unit, to $C = 49$.

11. Assume that a country produces an output $Q$ of 50 every year. The world interest rate is 10%. Consumption $C$ is 50 every year, and $I = G = 0$. There is an unexpected war in year 0, which costs 11 units and is predicted to last one year. If the country desires to smooth consumption, how much should it borrow in period 0? What will the new level of consumption be from then on?

The country wakes up in year 1 and discovers that the war is still going on and will eat up another 11 units of expenditure in year 1. If the country still desires to smooth consumption looking forward from year 1, how much should it borrow in period 1? What will be the new level of consumption be from then on?

**Answer:** If the war is temporary, the increase in $G$ should be financed through borrowing (e.g., running a current account deficit). To determine how much the country should borrow, we first must calculate the change in the present value of consumption.
The present value of government spending is equal to 11, as this is a one-time increase in government spending. Therefore, the present value of consumption is:

\[ PV(C) = PV(Q) - PV(G) = 50 + \frac{50}{0.10} - 11 = 539 \]

To determine the level of consumption each period, we know that the country wants to maintain a given level of consumption:

\[ PV(C) = C + \frac{C}{\rho^t} \]

\[ 539 = C + \frac{C}{0.10} \text{ therefore } C = 49 \]

To consume 49 units, along with the war expenditure of 11 units, the country needs to borrow 10 units.

If the government needs to borrow again, then we can use the same approach to find consumption each period. Note that for every 10 units borrowed, consumption is reduced by one unit, as \( NFLA = 0.10 \times 10 \) in subsequent periods. Therefore, in period 0, \( C = 49 \). Thereafter, when the government needs another 11 units, beginning in period 1, \( C = 48 \) (as \( NFLA \) increases by one additional unit).

12. Consider a world of two countries, Highland (H) and Lowland (L). Each country has an average output of 9 and desires to smooth consumption. All income takes the form of capital income and is fully consumed each period.

a. Initially, there are two states of the world: Pestilence (P) and Flood (F). Each happens with 50% probability. Pestilence affects Highland and lowers the output there to 8, leaving Lowland unaffected with an output of 10. Flood affects Lowland and lowers the output there to 8, leaving Highland unaffected with an output of 10. Devise a table with two rows corresponding to each state (rows marked P and F). In three columns, show income to three portfolios: the portfolio of 100% Highland capital, the portfolio of 100% Lowland capital, and the portfolio of 50% Highland + 50% Lowland capital.

\[ \begin{array}{ccc}
\text{State} & \text{100% L Capital} & \text{100% H Capital} & \text{50–50 portfolio} \\
\hline
P & 10 & 8 & 9 \\
F & 8 & 10 & 9 \\
\end{array} \]

b. Two more states of world appear: Armageddon (A) and Utopia (U). Each happens with 50% probability but is uncorrelated with the P-F state. Armageddon affects both countries equally and lowers income in each country by a further 4 units, whatever the P-F state. Utopia leaves each country unaffected. Devise a table with four rows corresponding to each state (rows marked PA, PU, FA, FU). In three columns, show income to three portfolios: the portfolio of 100% Highland capital, the portfolio of 100% Lowland capital, and the portfolio of 50% Highland + 50% Lowland capital.

\[ \begin{array}{ccc}
\text{State} & \text{100% L Capital} & \text{100% H Capital} & \text{50–50 portfolio} \\
\hline
PU & 10 & 8 & 9 \\
FU & 8 & 10 & 9 \\
PA & 6 & 4 & 5 \\
FA & 4 & 6 & 5 \\
\end{array} \]