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System Reliability and Ancillary Services

Notes by Daniel Kirschen (with edits by Leigh Tesfatsion) on Chapter 5 in *Power System Economics*, by D. Kirschen and G. Strbac

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Key Definitions: Reliability, Security, & Adequacy

- ***Reliability*** of an electric power system can be viewed as two interrelated elements: **adequacy + security**.

- ***Adequacy*** ➔ Generation can be kept in balance with loads, losses, and intertie outflows (system balance)

- ***Security*** ➔ Electric power system is able to withstand contingencies (sudden changes) on a daily and hourly basis, e.g., loss of a generating unit or transmission line.

- In North America, reliability standards for electric power systems are set by the North American Electric Reliability Council (NERC).

Key Definition: Ancillary Services (A/S)

Ancillary Services (A/S) → Services needed to ensure system reliability

Types of A/S:

- Balancing services (gen = load/losses/outflow)
  - Regulation, load-following, spinning/non-spinning reserves
- Reactive Support
  - Regulate voltage in normal times, provide reactive support in emergency conditions
- Intertrip Schemes (autodisconnect in event of a fault)
- Black-start capability (system restoration)

Introduction

Section 5.1 (pp. 105-106)

- Wholesale electric energy markets rely on transmission grids to support energy trades
- Participants in any given wholesale electric power market have no choice regarding this transmission grid – they must use the given grid
- Grid outages impose high costs on market participants
- Participants value system reliability (preventive and corrective actions to maintain normal grid operations)
- But the cost of reliability should not exceed its benefit

System Reliability = Adequacy + Security

- System must be able to operate continuously under normal operating conditions (adequacy)
- System must stabilize after contingencies (security)
  - Fault on a transmission line or other component
  - Sudden failure of a generating unit
  - Rapid change in load
- To maintain system reliability, operator must use both:
  - ***Preventive actions (ex ante)***
  - ***Corrective actions (ex post)***

Preventive Actions

- Put the system in a state such that it will remain stable whether or not a contingency occurs

- ***Example:*** Operate the system at less than full available generation and/or line capacity (so operations can be maintained even if a generator or line outage occurs)

  - Implies some feasible energy trades are not allowed

  - Opportunity cost of lost energy trades (e.g., decrease in GenCo net earnings) can outweigh gain in system reliability

Corrective Action

- Taken only if a disturbance does occur
- Limits the consequences of this disturbance
- Can require system operator to purchase various types of A/S from market participants
- Some A/S entail the ***delivery*** of energy whereas other A/S entail the ***curtailment*** of energy usage
- The important factor for the system operator is ensuring in advance the *capacity* to deliver/curtail energy as needed to limit consequences of a disturbance.
- Value of A/S is its availability to deliver/curtail energy as reliability needs arise, not simply actual energy delivery or curtailment

Outline for Remaining Chapter 5 Notes

- **What are the needs for A/S (Section 5.2)**
  - Keeping the generation and load in balance
  - Ensuring the security of the transmission network
- **How should A/S be procured (Section 5.3)**
  - How much A/S should be bought? Who should determine this?
  - Through what processes should A/S be obtained? Markets?
  - Who should pay for A/S? GenCos? LSEs? Retail consumers?
- **Buying and Selling A/S (Sections 5.4 - 5.5)**
  - Minimize cost of A/S purchases (traditional system operator goal)
  - Maximize net earnings from energy & A/S sales (goal of energy and A/S providers)

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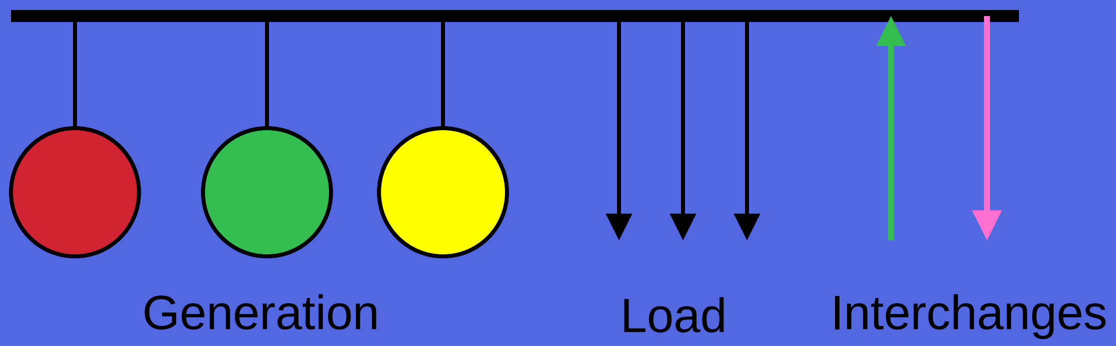
Needs for Ancillary Services

(Section 5.2, pp. 107-117)

Balancing Generation and Load

- Assume that all generators, loads and tie-lines are connected to the same bus
- Only system variables are total generation, total load and net interchange with other systems



Balancing Generation and Load…Continued

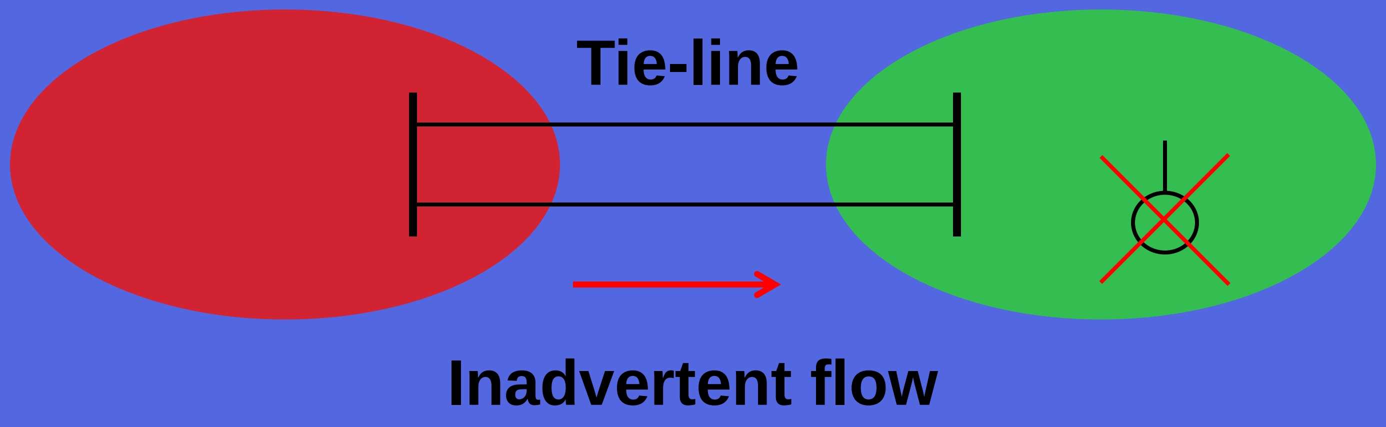
- ***System balance*** means power injections (generation) = power withdrawals (load+losses+net interchange outflow)
- If the system is balanced, frequency remains constant
- In practice, system imbalance arises due to:
  - Constant fluctuations in the load
  - Inaccurate control of the generation
  - Sudden outages of generators or transmission lines
  - Sudden outages of tie-lines between systems
- **In an isolated system (no tie-lines):**
  - ***Excess generation*** causes an ***increase*** in frequency
  - ***Excess load*** causes a ***drop*** in frequency

Balancing Generation and Load … Continued

- Generators can only operate within a narrow range of frequencies
  - Protection devices disconnect generators from rest of the system when the frequency is too low, causing further imbalance between load and generation
- A large sudden regional imbalance between load and generation in a system connected to other systems can cause the disconnection of their tie-lines.
- System operator must maintain frequency within limits

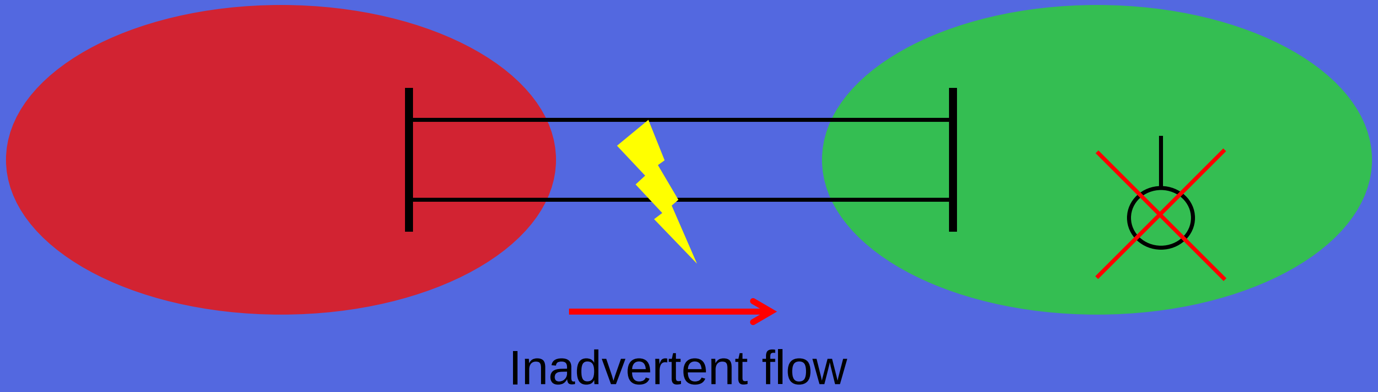
Balancing Generation and Load … Continued

- Rate of change in frequency inversely proportional to total inertia of generators and rotating loads
- Frequency changes much less in large interconnected systems than in small isolated systems
- Local imbalance in an interconnected system causes a change in tie-line flows



Balancing Generation and Load … Continued

- Inadvertent flows can overload the tie-lines
- Protection system may disconnect these lines
- Could lead to further imbalance between load and generation
- Each system must remain in balance



Balancing Generation and Load … Continued

- Minor frequency deviations and inadvertent flows are not an immediate threat
- However, they weaken the system
- Must be corrected quickly so the system can withstand further problems

Balancing Services (One Form of A/S)

- Different phenomena contribute to imbalances
- Each phenomenon has a different time signature
- Different services are required to handle these phenomena
- Exact definition of “balancing services” differs from market to market

Balancing Services…Continued

Difference between power traded and actual load:

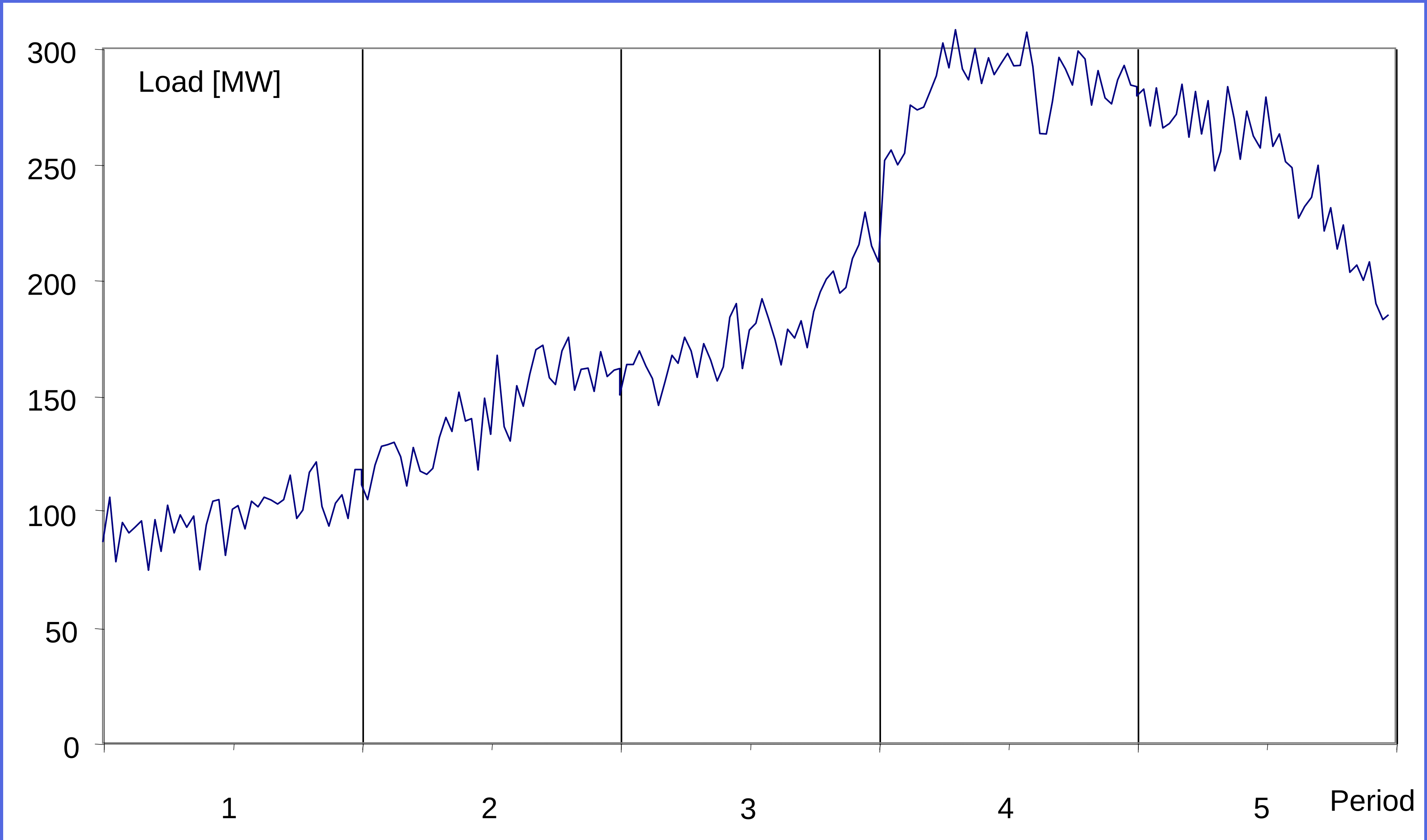
- Markets assume load (MW) remains constant over each trading period (e.g., each hour in day-ahead market)
- Actual load varies over each trading period (retail consumers choose when to use power – typically flat fee)
- Demand bids of LSEs to service retail consumers can differ from actual demands due to load forecast errors
- Power traded in markets cannot track rapid intra-period fluctuations in load

Balancing Services…Continued

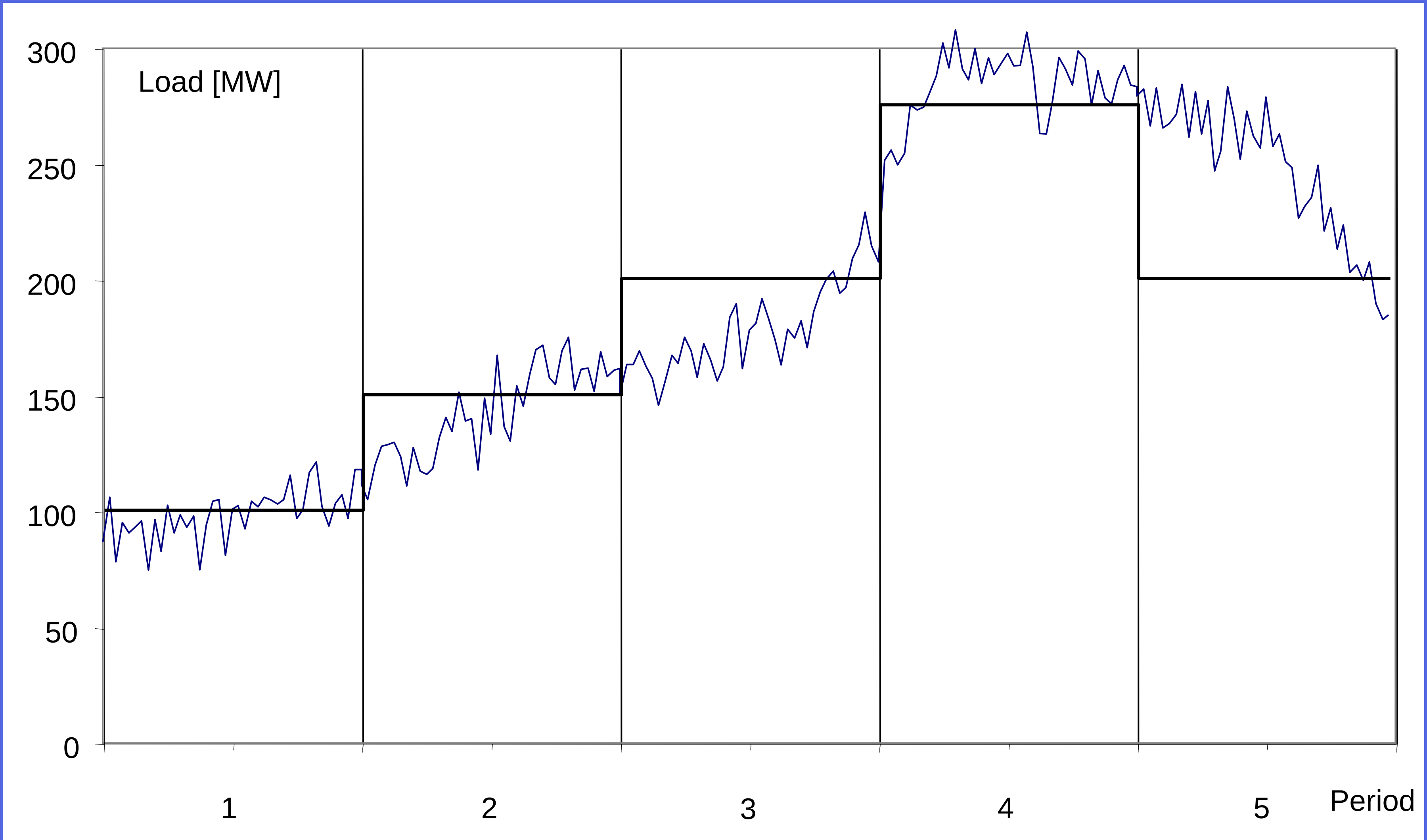
Difference between power traded and power produced:

- Market assumes generation (MW) remains constant over each trading period (e.g., each hour in day-ahead market)
- In fact, generation (MW) varies over each trading period
    - Minor errors in control
    - Unit commitment issues (e.g., ramp constraints at ends of periods)
    - Generating unit outages can create large imbalances
- Power traded in markets cannot track rapid intra-period fluctuations in generation

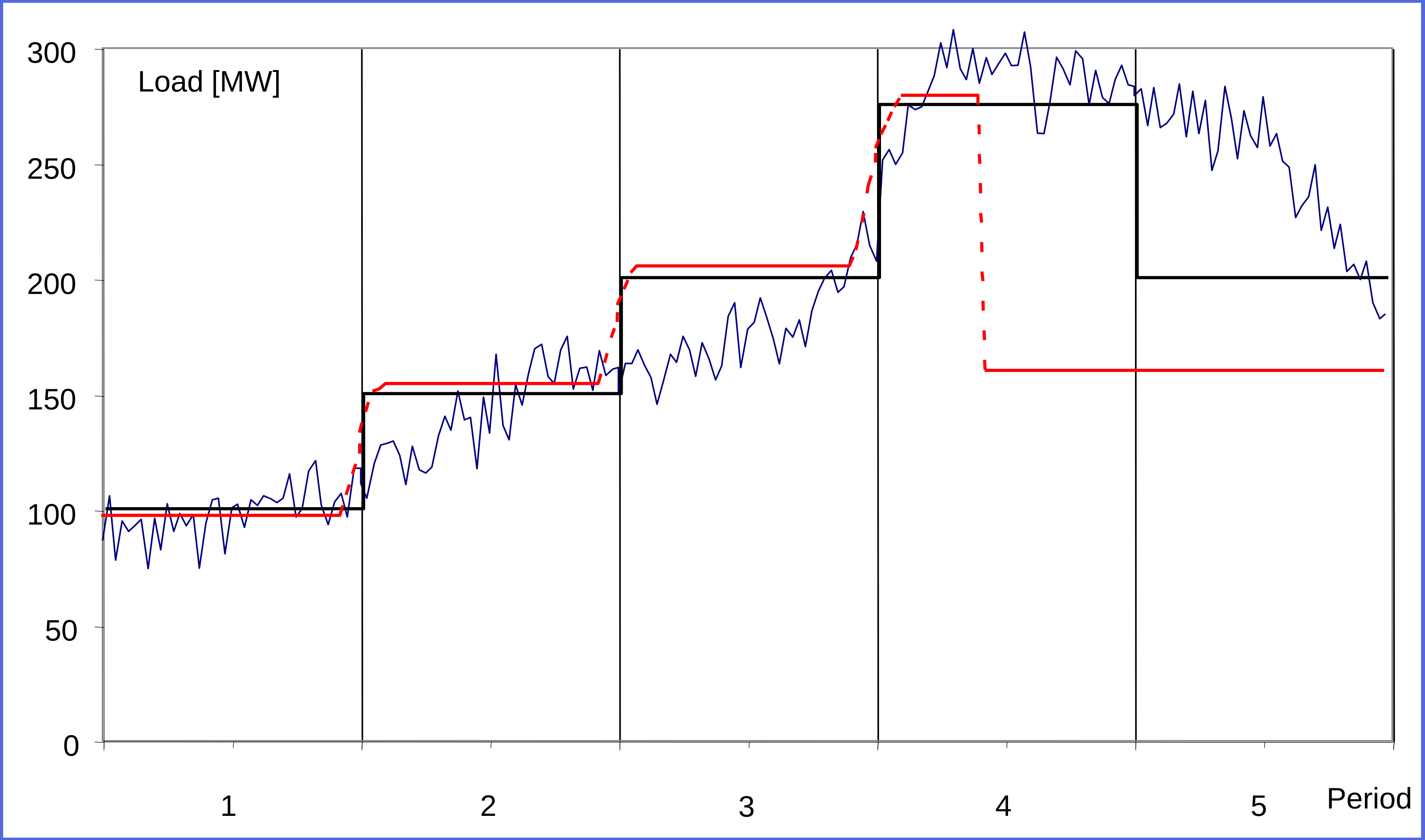
Example: Load (MW) Over 5 Trading Periods K/S Figure 5.2



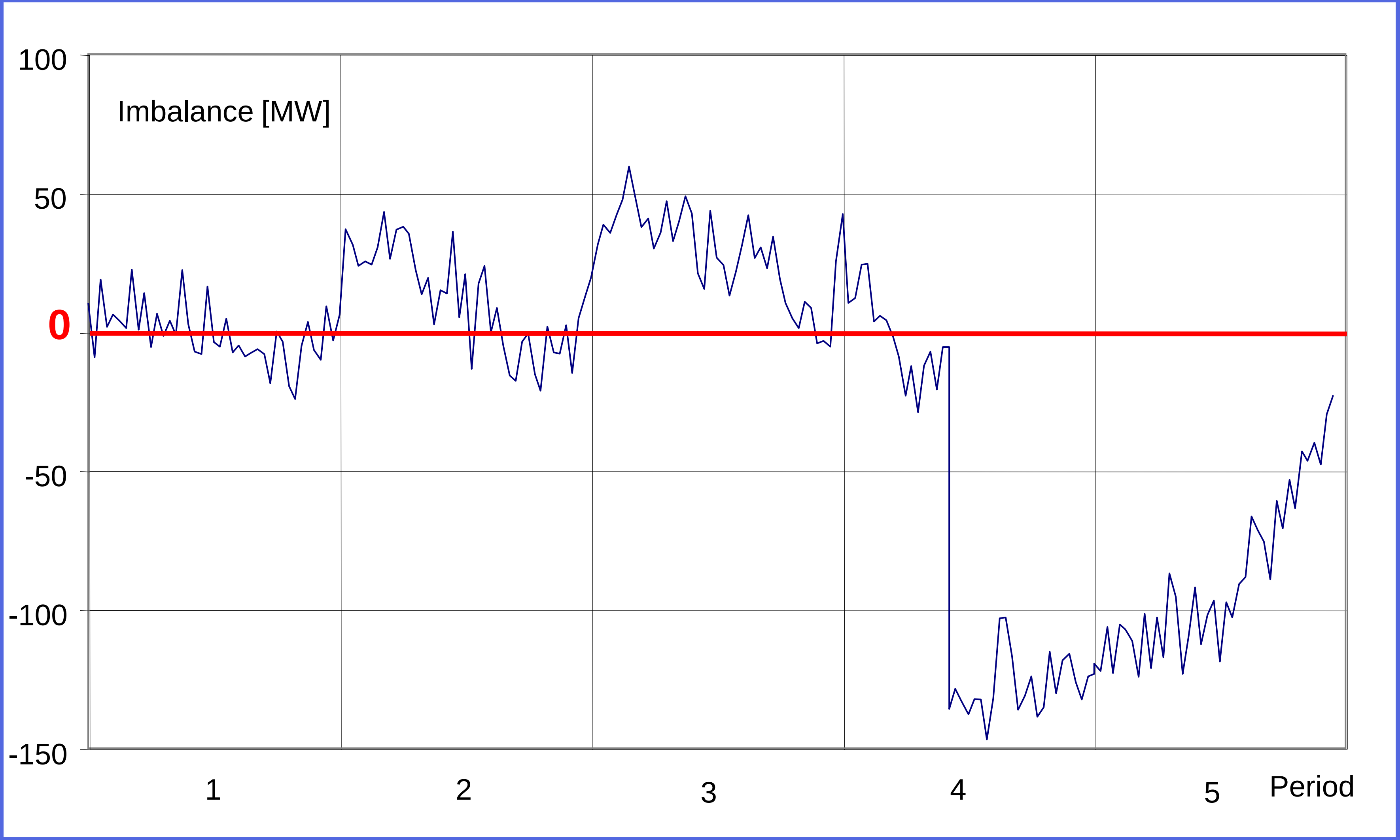
Power (MW) Traded Over the 5 Periods



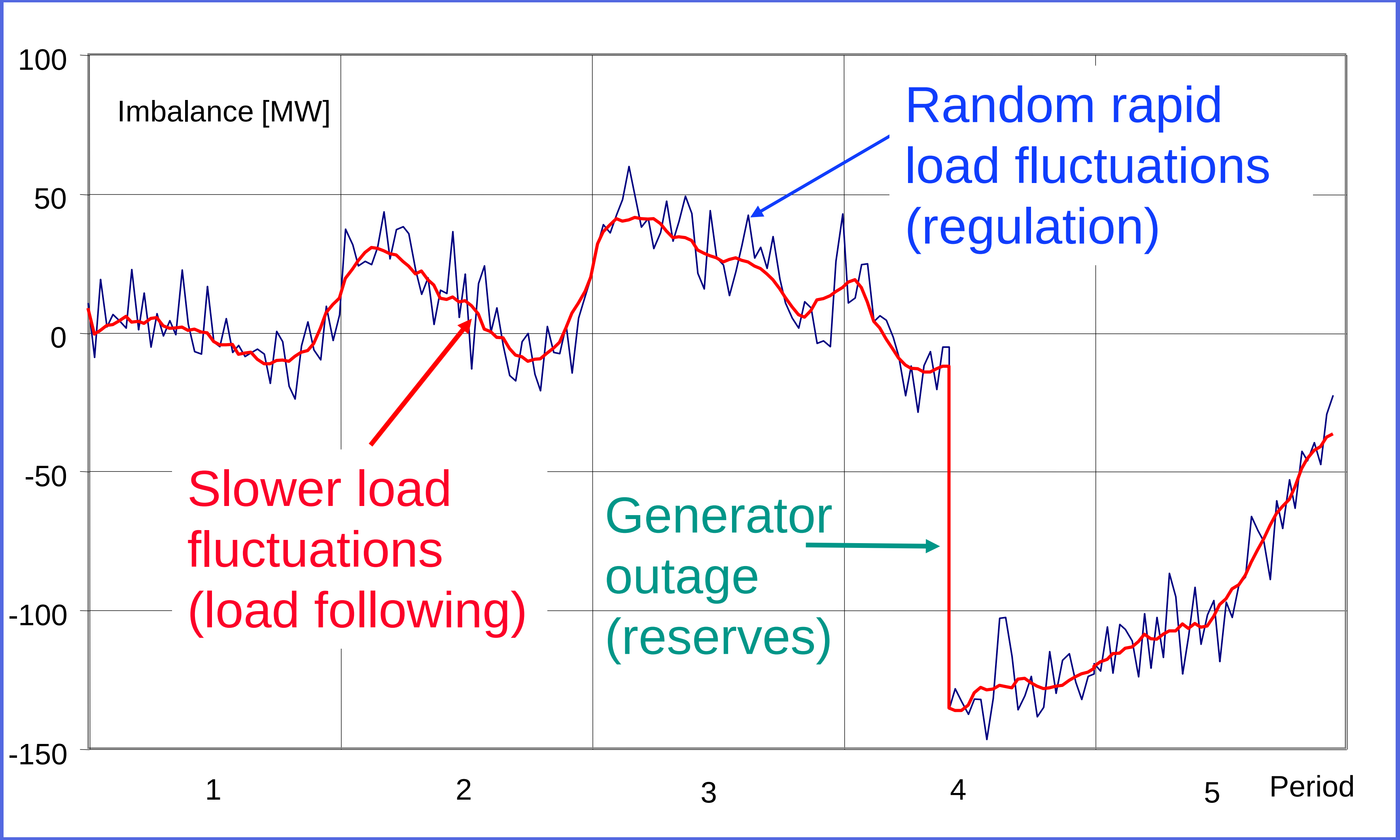
Actual Power Produced Over the 5 Periods



Imbalance [Power – Load] over the 5 Periods



Sources of Imbalance & corresponding A/S



Classification of Balancing Services

- ***Regulation and load following services:***
  - Almost continuous action
  - Relatively small
  - Need for regulation (time, amount) is quite predictable
  - Availability of regulation and load-following services is arranged in advance as a preventive reliability action
- ***Reserve services:***
  - Need for reserves (time, amount) difficult to predict
  - Reserves are called upon for ***corrective*** security actions
  - But advance contracting for reserve provision is a form of ***preventive*** reliability action

Regulation Service in More Detail:

- Designed to handle:
  - Rapid fluctuations in load
  - Small, unintended variations in generation
- Designed to maintain:
  - Frequency close to its nominal value (60 Hz US, 50 Hz most other regions of the world)
  - Interchanges at desired power flows
- Traditionally provided by generating units that:
  - Can adjust output quickly
  - Are connected to the grid
  - Are equipped with a governor with ***Automatic Generation Control (AGC)*** = ability to respond in real time to commands from the central system operator to control power output up/down

Load Following Service in More Detail:

- Designed to handle intra-period load fluctuations
- Designed to maintain:
  - Frequency close to its nominal value
  - Interchanges at desired power flow levels
- Traditionally provided by generating units that can respond at a sufficiently fast rate

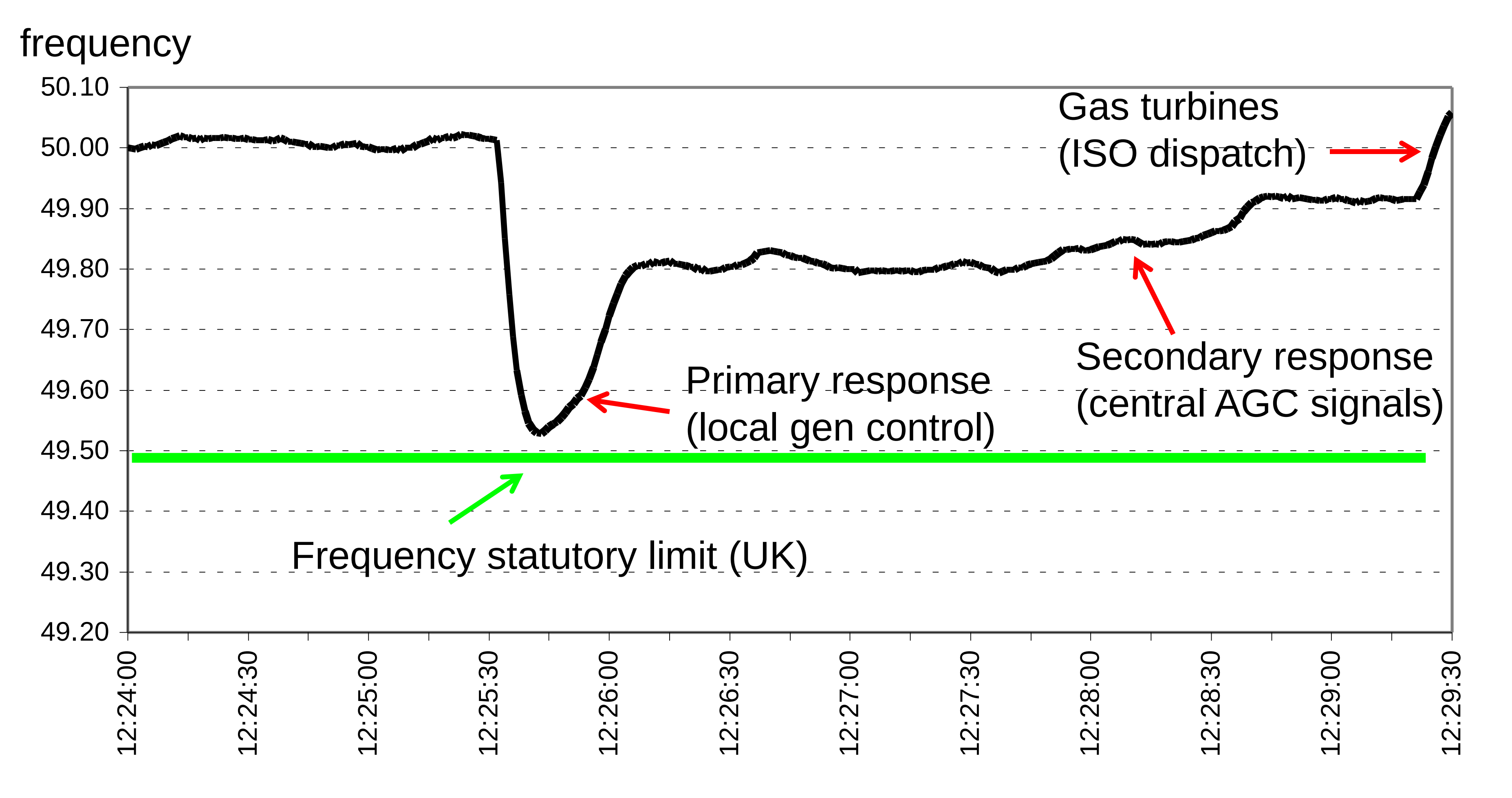
Spinning/Non-Spinning Reserve Services

- Designed to handle large and unpredictable deficits caused by outages of generators and tie-lines

- Two main types:
  - **Spinning reserve**
    - Start immediately (already synchronized to the grid)
    - Full amount available quickly (fast ramping)
  - **Supplemental reserve**
    - Can start more slowly (e.g., in 10 minutes, or in 30 minutes)
    - Designed to replace spinning reserve

- Exact definitions and features differ across regions

Example: Frequency & Reserve Response following major generation outage in the UK on 15 August 1995



K/S Figure 5.3

(Note: 50 Hz = UK Nominal Frequency Value)

Network Issues: Contingency Analysis

- System operator continuously performs contingency analysis
- **Goal**: No credible-threat contingency should be able to destabilize the system
- Modes of destabilization:
  - Thermal overload
  - Transient instability
  - Voltage instability
- If a credible-threat contingency could destabilize the system, the operator should take preventive action

Types of Preventive Actions

- Low-cost preventive actions:
  - ***Examples***
    - Adjust taps of transformers
    - Adjust reference voltage of generators
    - Adjust phase shifters
  - Effective but limited
- High-cost preventive actions:
  - Restrict active power flow on some transmission network branches
  - Requires limiting the power output of some generators
  - Affects the ability of these generators to trade on the market and make money

Example: Thermal Capacity

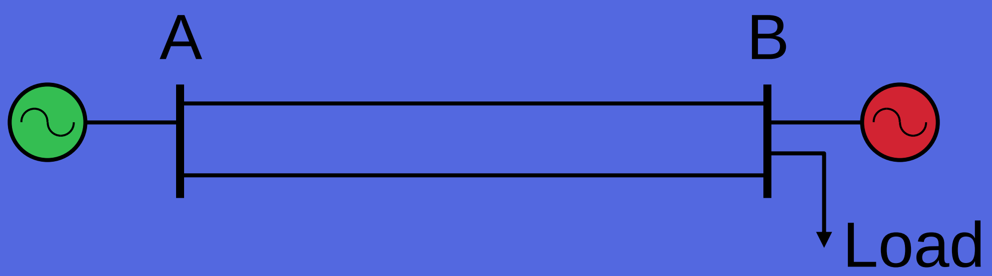


Figure 5.4

- Each line between A and B is rated at 200 MW
- Generator at A can sell only 200 MW to load at B
- Remaining 200 MW of transmission capacity must be kept in reserve in case of outage of one of the lines

Example: Emergency Thermal Capacity

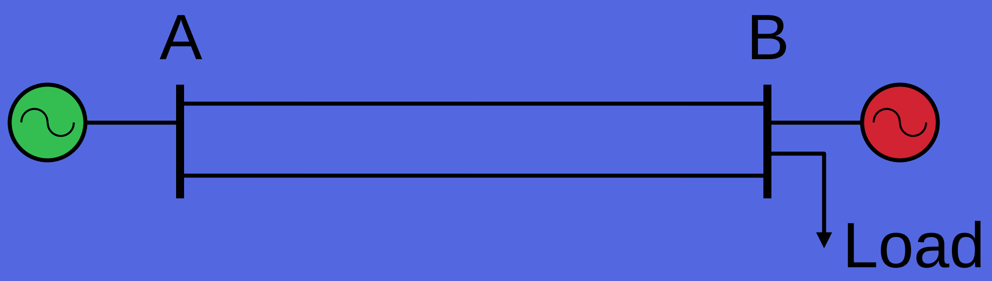
Figure 5.4

- Each line between A and B is rated at 200 MW
- Each line has a 10% emergency rating for 20 minutes, meaning either line can withstand a 10% overload for 20 minutes without equipment damage or line fault problems
- If generator at B can increase its output by 20 MW in 20 minutes if necessary, then generator at A can sell 220 MW to load at B because the operator knows the 220MW load can still be met even if there is a line outage

Example: Transient Stability

- **Assumptions:**
  - B is an infinite bus
  - Transient reactance of A = 0.9 p.u., inertia constant H = 2 s
  - Each line has a reactance of 0.3 p.u.
  - Voltages are at nominal value 1.0 p.u.
  - Line fault can be cleared in 100 ms by tripping affected line
- Maximum power transfer from A to B without endangering transient stability of the system is 108 MW

Example: Voltage Stability



- Case 1: No reactive support at B
  - 198 MW can be transferred from A to B before the voltage at B drops below 0.95 p.u.
  - However, the voltage collapses if a line is tripped when power transfer is larger than 166 MW
  - The maximum power transfer from A to B is thus 166 MW

Example: Voltage Stability … Continued

- Case 2: 25 MVAr of reactive support at B
  - Up to 190 MW can be transferred from A to B before a line outage would cause a voltage collapse

Voltage Control and Reactive Support Services

- Use reactive power resources to maximize active power that can be transferred through the transmission network
- Some of these reactive power resources are under the control of the system operator:
  - Mechanically-switched capacitors and reactors
  - Static VAr compensators
  - Transformer taps
- But the best reactive power resources are generators
- Need to define voltage control services to specify the conditions under which the system operator can obtain and use reactive power resources

Voltage Control & Reactive Support Services…

- Must consider both normal and abnormal conditions
- Normal voltage conditions:
  - 0.95 p.u. $\leq V \leq$ 1.05 p.u.
- Abnormal voltage conditions:
  - Provide enough reactive power to prevent a voltage collapse following an outage
- Requirements for abnormal conditions are much more severe than for normal conditions
- Reactive support is more important than voltage control

Example: Voltage Control under Normal Conditions

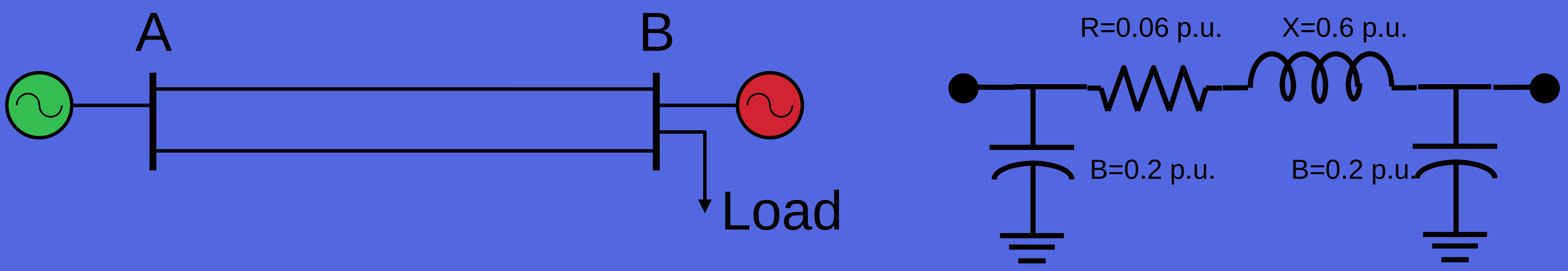


Figure 5.5

- Load at B has a power factor of 1.0 (no reactive support)
- Voltage at A maintained at nominal value 1.0 p.u.
- How might the operator control the voltage at B?

Example: Voltage Control under Normal Conditions

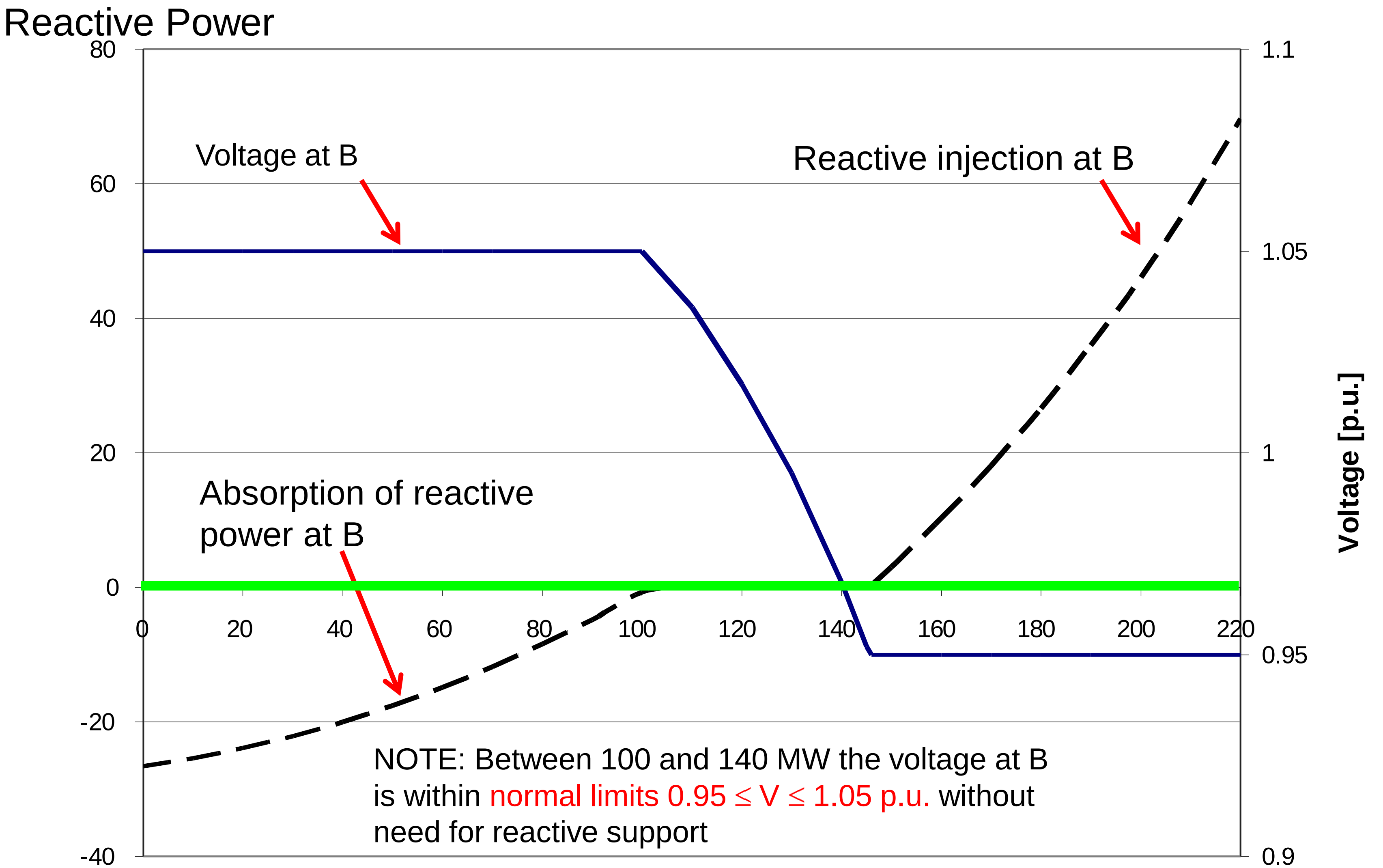


Figure 5.6: **Active Power Transfer [MW]** from A to B

Example: Voltage Control under Normal Conditions

- Controlling the voltage at B using the generator at A?

Power Transfer [MW]	V_B [p.u.]	V_A [p.u.]	Q_A [MVAr]
49.0			-68.3
172.5			21.7

Table 5.1

- Local voltage control is much more effective
- Severe market power issues in reactive support

Example: Reactive support After Line Outage

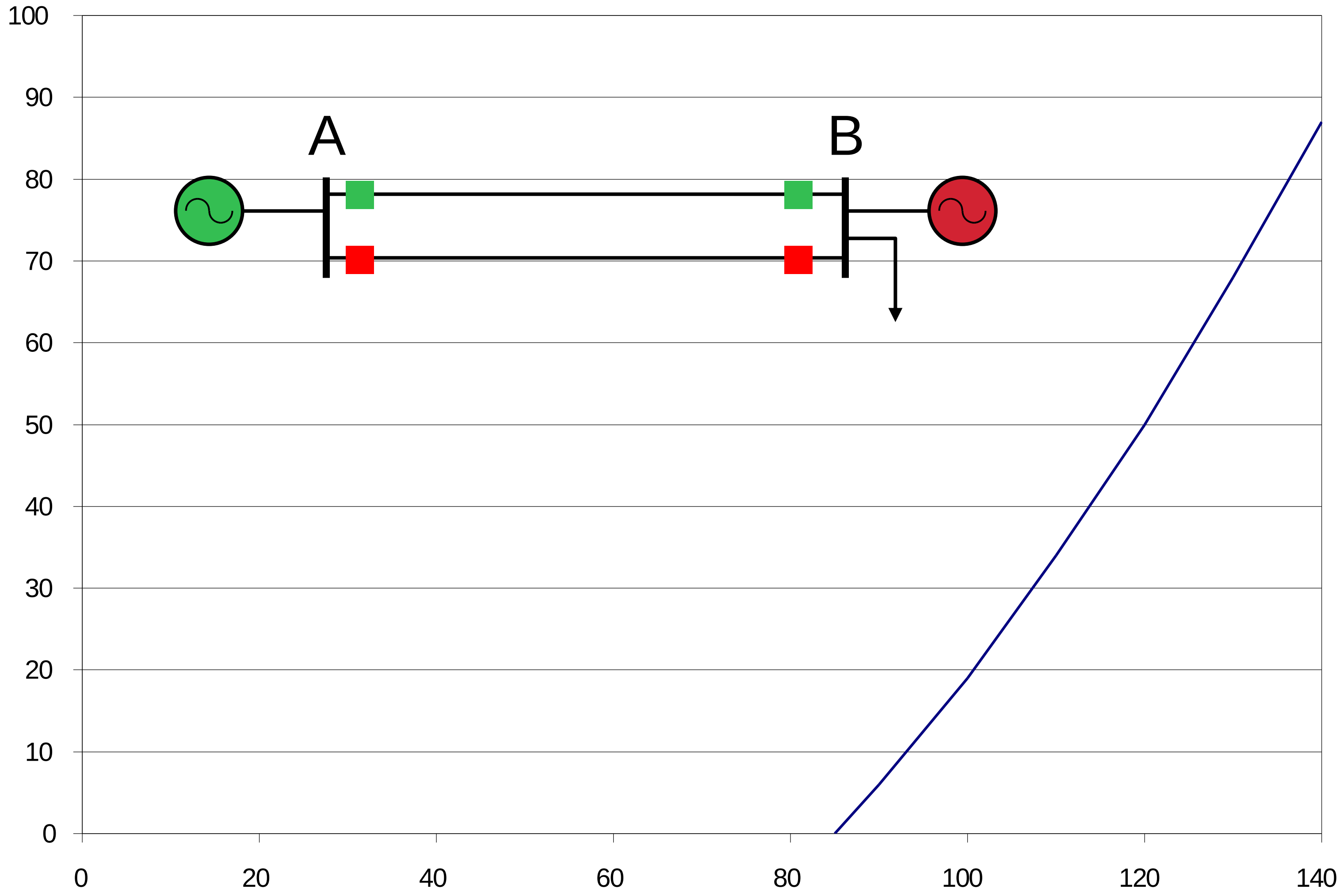
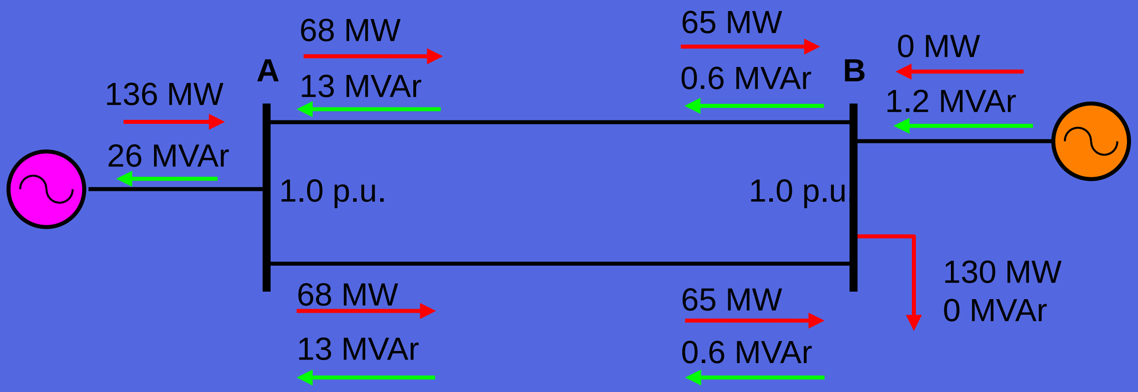


Figure 5.7: **Power Transfer [MW]**

Example: Pre- and post-contingency balance

Pre-contingency:



Post-contingency:

Figure 5.8

Other Forms of Ancillary Services

- ***Stability services***
  - Intertrip schemes
    - Disconnection of generators following faults
  - Power system stabilizers
    - Minute adjustments to generator outputs to dampen oscillations that might develop in the network

- ***Black-start restoration capability service***
  - Restarting of system operations after a total system collapse
  - Requires generators able to restart manually or thru stored energy
  - System operator must ensure enough availability of restoration resources to guarantee a prompt restoration of service at any time

Obtaining Ancillary Services

(Section 5.3, pp. 117-120)

Obtaining Ancillary Services (A/S)

- How ***much*** A/S should be procured?
- How should A/S be procured (compulsory, market) ?
- How much should A/S providers be paid by system operator?
- Who should *ultimately* pay for A/S, i.e., how should system operators allocate the costs of A/S provision?

How much A/S should be procured?

- System Operator is responsible for procuring A/S
  - Works on behalf of the users of the system
- A/S are used mostly for contingencies
  - Availability is more important than actual usage
- What if not enough A/S is procured?
  - Can’t ensure the reliability of the system
  - Can’t maintain the quality of the supply
- What if too much A/S is procured?
  - Life of the system operator is easy
  - A/S costs are passed on to system users
  - But possible wastage of resources (e.g., excess reserves)

How much A/S should be procured … Continued

- System Operator should perform a cost/benefit analysis
  - Should balance benefits of A/S against their cost
  - Optimal balancing can require the solution of complicated probabilistic optimization problems
  - In practice, reliability standards that approximate the optimal solution have often been used instead.

- Benefits of A/S:
  - Improvement in system reliability and the quality of energy supply

- Costs of A/S:
  - A/S procurement payments
  - Typically system operators cover their A/S procurement payments by imposing charges on system loads by load share

How should A/S be procured?

- Two approaches:
  - Compulsory A/S provision
  - Market for A/S
- Each approach has advantages and disadvantages
- Choice influenced by:
  - Type of service
  - Nature of the power system
  - History of the power system

One Possibility: Compulsory A/S Provision

- To be allowed to connect to the system, generators might be ***obliged*** to provide some forms of A/S

- **Examples:**
  - Each generating unit must be equipped with governor with a 4% droop coefficient m, where $\Delta f / f_n = -m \, \Delta P / P_n$ (n = nominal value)
    - Implies all generators contribute to keeping frequency f close to f_n
  - Each generating unit must be able to operate at a power factor P/S = [active power P]/[apparent power S] that ranges from 0.85 lead to 0.9 lag and be equipped with an automatic voltage regulator
    - Implies all generators contribute to voltage control & reactive support

Advantages of Compulsory Provision

- Simplicity
- Minimum deviation from traditional practice by vertically integrated utilities
- Usually ensures system security and quality of supply

Disadvantages of Compulsory Provision

- Not necessarily good economic policy
  - May provide more resources than needed and cause unnecessary investments
    - Not all generating units need to help control frequency
    - Not all generating units need to be equipped with a power system stabilizer
- Discourages technological innovation
  - A/S product definitions based on what generators usually provide
- Generators not always compensated fully for costly services
  - ***Example:*** Providing reactive power "for free" even though it increases losses & can reduce active power generation capacity

Disadvantages of Compulsory Provision …

- Equity
  - How to deal with generators that cannot provide A/S?
    - ***Example:*** Nuclear units can't participate in frequency response because they cannot rapidly change their active power production

- Economic efficiency
  - Not a good idea to force highly efficient units to operate in a partially loaded state in order to provide reserves
  - More efficient to determine centrally how much reserve is needed and commit additional units to meet this reserve requirement

- Compulsory provision is thus not suitable for all A/S

Another Possibility: Have an A/S Market

- Different markets for different types of A/S
- ***Long-term contracts for A/S***
  - Suitable for types of A/S where quantity needed does not change very much over time, and availability depends on equipment characteristics
  - ***Examples:*** Black-start capability, intertrip schemes, frequency regulation
- ***A/S spot market***
  - Needs change over the course of a day
  - Price changes because of interactions with energy market
  - ***Example:*** Reserves
- System operator might reduce its risk by using a combination of spot market and long term contracts for A/S procurement

Advantages of Having a Market for A/S

- More economically efficient than compulsory provision
- System operator buys only the amount of A/S needed
- Only participants that find it profitable provide A/S
- Helps determine the true benefits and costs of A/S
- Opens up opportunities for innovative solutions

Disadvantages of an A/S Market

- More complex
- Probably not applicable to all types of A/S
- Potential for market power abuse (price manipulation)
  - ***Example:***
    - Reactive support by one generator in remote part of the network
    - Generator can charge exorbitant price for reactive power in times of emergency need
    - Any market for reactive power would need to be carefully regulated to prevent this type of local exercise of market power

Demand-Side Provision of A/S

- Unfortunately, A/S product definitions often still follow traditional practice of thinking only generators can provide A/S

- In a truly competitive environment, a system operator should not impose restrictions on who can provide A/S

- Creating a market for A/S opens up an opportunity for the energy demand side (load) to provide A/S as well as the energy supply side (generation)

Advantages of Demand-Side Provision

- Larger number of participants increases competition and lowers cost
- Better utilization of resources
  - ***Example:***
    - Providing reserves with interruptible loads rather than partly loaded thermal generating units
    - Particularly important if the proportion of generation from renewable sources increases
- Demand side may be a more reliable provider
  - Large number of small demand-side providers

Opportunities for Demand-Side Provision

- Different types of reserves
  - Interruptible loads

- Frequency regulation
  - Variable-speed pumping loads

Who Should Pay for A/S?

- Different users value reliability & service quality differently
  - ***Examples:***
    - Generators vs. loads
    - Semi-conductor manufacturing vs. irrigation load
- Ideally, users who get higher benefit from A/S should pay more for this higher benefit
  - Not practically feasible given current technology
  - System operator provides average level of reliability to all users based on centrally determined reliability standards
  - A/S procurement costs are then typically allocated across wholesale energy buyers on the basis of their load shares

Who Should Pay for A/S? … Continued

- Sharing the cost of A/S procurement on the basis of load share is not economically efficient
- Some participants increase the need for A/S more than others
- These participants should pay a larger share of the A/S procurement costs to encourage them to change their behaviour

Example: Allocating Cost of Reserves

- Reserves prevent collapse of the system when there is a large imbalance between load and generation
- Large imbalances usually occur because of a failure of a generating unit or a sudden disconnection of a tie-line
- Owners of large generating units that fail frequently should pay a larger proportion of the cost of reserves
- Would encourage them to improve their unit reliability
- Long-term goals:
  - Reduce need for reserves
  - Reduce overall cost of reserves

Selling Ancillary Services

(Section 5.5, pp. 130-136)

Selling A/S

- Selling A/S is a business opportunity for generators
- Limitations:
  - Technical characteristics of the generating units
    - Maximum ramp rate
    - Reactive capability curve
  - Opportunity cost
    - Can't sell as much energy when selling reserves
- The seven U.S. ISO/RTO-managed energy regions have adopted wholesale power market designs that permit various degrees of ***co-optimization*** for determination of energy and reserves (MISO, PJM, ERCOT, SPP, CAISO, NYISO, ISO-NE)
- Co-optimization has been adopted in recognition of the correlation between energy and reserve supplies

Example: Selling both Energy and Reserves
(Section 5.5, pp. 130-136)

- Consider a generator that is trying to maximize its net earnings from the sale of energy and reserves
- ***Assumptions:***
  - Only one type of reserve service
  - Perfectly competitive energy and reserve markets
    - Generator is a price-taker in both markets
    - Generator can offer to sell any quantity it wants in either market
  - Consider one generating unit over one hour
    - Ignore start-up cost, ramping rates, min up time, min down time
  - Generator does not receive an additional fee when the reserve capacity it provides is actually called upon to provide energy

Notation:

π_1 : Market price for electric energy (£/MWh)

π_2 : Market price for reserves (£/MWh)

x_1 : Quantity of energy offered and sold (MW)

x_2 : Quantity of reserves offered and sold (MW)

P^{min} : Minimum (active) power output (MW)

$P^{\max}$: Maximum (active) power output (MW)

R^{max} : Upper limit on reserves (ramp rate x delivery time)

$C_1(x_1)$: Cost of producing energy in amount x_1

$C_2(x_2)$: Cost of providing reserves x_2 (excludes opportunity cost)

Generator's Optimization Problem:

Objective Function: Net Earnings = [Revenues – Avoidable Costs]

$$f(x_1, x_2) = \pi_1 x_1 + \pi_2 x_2 - C_1(x_1) - C_2(x_2)$$

Constraints:

$x_1 + x_2 \le P^{max}$ = maximum power output of generating unit

$x_1 \ge P^{\min}$ = minimum stable operating level for generating unit

$x_2 \le R^{max}$ (where it is assumed that $R^{max} < P^{max} - P^{\min}$)

Lagrangian function:

$$\ell(x_1, x_2, \mu_1, \mu_2, \mu_3) = \pi_1 x_1 + \pi_2 x_2 - C_1(x_1) - C_2(x_2) + \mu_1(P^{max} - x_1 - x_2) + \mu_2(x_1 - P^{min}) + \mu_3(R^{max} - x_2)$$

First-Order Necessary Conditions for Optimality: The Karush-Kuhn-Tucker (KKT) Conditions

$$\frac{\partial l}{\partial x_1} \equiv \pi_1 - \frac{dC_1}{dx_1} - \mu_1 + \mu_2 = 0$$

$$\frac{\partial l}{\partial x_2} \equiv \pi_2 - \frac{dC_2}{dx_2} - \mu_1 - \mu_3 = 0$$

$$\frac{\partial l}{\partial \mu_1} \equiv P^{max} - x_1 - x_2 \geq 0$$

$$\frac{\partial l}{\partial \mu_2} \equiv x_1 - P^{\min} \geq 0$$

$$\frac{\partial l}{\partial \mu_3} \equiv R^{max} - x_2 \geq 0$$

First-Order Necessary Conditions … continued

$$\mu_1 \cdot (P^{max} - x_1 - x_2) = 0$$

$$\mu_2 \cdot (x_1 - P^{\min}) = 0$$

$$\mu_3 \cdot (R^{max} - x_2) = 0$$

$$\mu_1 \geq 0;\ \mu_2 \geq 0;\ \mu_3 \geq 0$$

"Complementary Slackness Conditions"

Case 1: $\mu_1 = 0; \mu_2 = 0; \mu_3 = 0$

- No binding constraints

$$\frac{\partial l}{\partial x_1} \equiv \pi_1 - \frac{dC_1}{dx_1} - \mu_1 + \mu_2 = 0 \Rightarrow \frac{dC_1}{dx_1} = \pi_1$$

$$\frac{\partial l}{\partial x_2} \equiv \pi_2 - \frac{dC_2}{dx_2} - \mu_1 - \mu_3 = 0 \quad \Rightarrow \quad \frac{dC_2}{dx_2} = \pi_2$$

- Provide energy and reserves up to the point where marginal cost is equal to price

- No interactions between energy and reserves

Case 2: $\mu_1 > 0; \mu_2 = 0; \mu_3 = 0$

- Generation capacity fully utilized by energy and reserves:

$$x_1 + x_2 = P^{max}$$

$$\left.\begin{aligned} \frac{\partial l}{\partial x_1} &\equiv \pi_1 - \frac{dC_1}{dx_1} - \mu_1 + \mu_2 = 0 \\ \frac{\partial l}{\partial x_2} &\equiv \pi_2 - \frac{dC_2}{dx_2} - \mu_1 - \mu_3 = 0 \end{aligned}\right\} \quad \pi_1 - \frac{dC_1}{dx_1} = \pi_2 - \frac{dC_2}{dx_2} = \mu_1 > 0$$

- At the margin, equal positive net earnings attained on “last” energy and reserve units sold

Case 3: $\mu_1 = 0; \mu_2 > 0; \mu_3 = 0$

- Unit operates at minimum stable generation

$$x_1 = P^{\min}$$

Loss!

$$\frac{\partial l}{\partial x_1} \equiv \pi_1 - \frac{dC_1}{dx_1} - \mu_1 + \mu_2 = 0 \quad \Rightarrow \quad \frac{dC_1}{dx_1} - \pi_1 = \mu_2 > \mathbf{0}$$

$$\frac{\partial l}{\partial x_2} \equiv \pi_2 - \frac{dC_2}{dx_2} - \mu_1 - \mu_3 = 0 \quad \Rightarrow \quad \frac{dC_2}{dx_2} = \pi_2$$

- At the margin, marginal cost of reserves x_2 = reserve price $\pi_{\mathbf{2}}$
- At the margin, a **loss** on energy x_1, minimized by operating at P^{min}
- Given the generator produces (does not shut down), KKT conditions only guarantee maximum net earnings, not positive net earnings!

Cases 4 & 5: $\mu_1 > 0; \mu_2 > 0; \mu_3 = 0$ $\mu_1 > 0; \mu_2 > 0; \mu_3 > 0$

$$\mu_1 : \; x_1 + x_2 \le P^{max}$$

$$\mu_2 : \; x_1 \ge P^{\min}$$

$$\mu_3 : \; x_2 \le R^{max}$$

Since we assume that $R^{max} < P^{max} - P^{\min}$ these cases are not possible because the upper and lower limits cannot be binding at the same time

Case 6: $\mu_1 = 0; \mu_2 = 0; \mu_3 > 0$

- Reserves limited by ramp rate

$$x_2 = R^{max}$$

$$\frac{\partial l}{\partial x_1} \equiv \pi_1 - \frac{dC_1}{dx_1} - \mu_1 + \mu_2 = 0 \quad \Rightarrow \quad \frac{dC_1}{dx_1} = \pi_1$$

$$\frac{\partial l}{\partial x_2} \equiv \pi_2 - \frac{dC_2}{dx_2} - \mu_1 - \mu_3 = 0 \quad \Rightarrow \quad \pi_2 - \frac{dC_2}{dx_2} = \mu_3 > 0$$

- At the margin, net earnings on reserves x_2 are positive.
- This implies that net earnings on reserves could be increased if ramp rate constraint could be relaxed

Case 7: $\mu_1 > 0; \mu_2 = 0; \mu_3 > 0$

- Maximum capacity and ramp rate constraints are both binding

$$\left.\begin{aligned} x_1 + x_2 &= P^{max} \\ x_2 &= R^{max} \end{aligned}\right\} \Rightarrow x_1 = P^{max} - R^{max}$$

$$\frac{\partial l}{\partial x_1} \equiv \pi_1 - \frac{dC_1}{dx_1} - \mu_1 + \mu_2 = 0 \quad \Rightarrow \quad \pi_1 - \frac{dC_1}{dx_1} = \mu_1 > 0$$

$$\frac{\partial l}{\partial x_2} \equiv \pi_2 - \frac{dC_2}{dx_2} - \mu_1 - \mu_3 = 0 \quad \Rightarrow \quad \pi_2 - \frac{dC_2}{dx_2} = \mu_1 + \mu_3 > 0$$

- At the margin, net earnings on energy and reserves are both positive
- At the margin, the sale of reserves results in higher net earnings than the sale of energy but is limited by the binding ramp rate constraint

Case 8: $\mu_1 = 0; \mu_2 > 0; \mu_3 > 0$

- Generator at minimum output, and reserves limited by ramp rate

$$x_1 = P^{\min}$$

$$x_2 = R^{max}$$

$$\frac{\partial l}{\partial x_1} \equiv \pi_1 - \frac{dC_1}{dx_1} - \mu_1 + \mu_2 = 0 \quad \Rightarrow \quad \pi_1 - \frac{dC_1}{dx_1} = -\mu_2 < 0$$

$$\frac{\partial l}{\partial x_2} \equiv \pi_2 - \frac{dC_2}{dx_2} - \mu_1 - \mu_3 = 0 \quad \Rightarrow \quad \pi_2 - \frac{dC_2}{dx_2} = \mu_3 > 0$$

- At the margin, reserve sale results in ***positive*** net earnings but is limited by the ramp rate constraint
- At the margin, energy sale results in ***negative*** net earnings but cannot be reduced due to binding P^{min} constraint.
- Total net earnings from energy/reserve sales could be pos or neg