QUIZ 4: ANSWER OUTLINE

EE/Econ 458 **L. Tesfatsion/S09**

Quiz 4: 1 Question (4 Parts, 6 Points Total) **Date: Tues, April 14, 2009**

QUIZ INSTRUCTIONS:

- **(1) Please fill in your complete name in the indicated space at the top of this quiz sheet. BE SURE TO WRITE CLEARLY.**

- **(2) Read each question part carefully before attempting an answer.**

QUESTION: (6 Points Total). Consider an economic dispatch problem for a 5-bus transmission grid with three GenCos $\{G1, G2, G3\}$ and a given real-power total demand (load) of P_D = 975MW. Suppose the cost functions for the three GenCos, in \$/hr, are as follows:

$$C_1(P_{G1}) = 0.004P_{G1}^2 + 5.3P_{G1} \; ; \tag{1}$$

$$\tag{2}$$

$$C_2(P_{G2}) = 0.006P_{G2}^2 + 5.5P_{G2} \; ; \tag{3}$$

$$\tag{4}$$

$$C_3(P_{G3}) = 0.009P_{G3}^2 + 5.8P_{G3} \; , \tag{5}$$

where P_{Gi} denotes the real-power dispatch level of GenCo i. Suppose, also, that G1 is a must-run unit (i.e., it cannot be shut down) that has operating capacity constraints requiring it to generate at a rate no less than 200 MW and no more than 500 MW. Finally, assume there are no congested transmission lines and no line losses.

Part A (2 Points): Carefully express the objective function and constraint(s) for this economic dispatch problem.

Part B (1 Point): What is the Lagrangian function for this economic dispatch problem.

Part C (2 Points): Use the Lagrangean function from Part B to express the full set of first-order necessary conditions (FONC) for a dispatch $(P_{G1}^*, P_{G2}^*, P_{G3}^*)$ to solve this economic dispatch problem.

Part D (1 Point): In what sense, if any, do the FONC solution values in Part C determine a *Locational Marginal Price (LMP)* for power that, for the system at hand, is uniform across the entire grid?

Answer Outline for Part A (2 Points):
Minimize total cost of generation

$$\sum_{i=1}^{3}[C_i(P_{Gi})] \tag{6}$$

with respect to P_{G1}, P_{G2}, and P_{G3} subject to the **Real-Power Balance Constraint:**

$$\sum_{i=1}^{3}[P_{Gi}] \ = \ P_D \tag{7}$$

and the operating capacity limits

$$200 \ \leq \ P_{G1} \ \leq \ 500 \tag{8}$$

Answer Outline for Part B (1 Point): Letting $P_G = (P_{G1}, P_{G2}, P_{G3})$, the Lagrangian function can be expressed as follows:

$$L(P_G, \lambda, \mu_1, \mu_2, P_D) = \sum_{i=1}^{3}[C_i(P_{Gi})] + \lambda\left(P_D - \sum_{i=1}^{3}[P_{Gi}]\right) + \mu_1[200 - P_{G1}] + \mu_2[P_{G1} - 500] \tag{9}$$

The constraint constant P_D is included for use in Part D below.
Answer Outline for Part C (2 Points):

$$\begin{aligned}
0 \quad &= \quad \frac{\partial L(P_G, \lambda, \mu_1, \mu_2, P_D)}{\partial P_{Gi}} \;=\; \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda \ , \quad i = 2, 3 \ \ ; && (10)\\
0 \quad &= \quad \frac{\partial L(P_G, \lambda, \mu_1, \mu_2, P_D)}{\partial P_{G1}} \;=\; \frac{dC_1(P_{G1})}{dP_{G1}} - \lambda \ - \mu_1 + \mu_2 \ \ ; && (11)\\
0 \quad &= \quad \frac{\partial L(P_G, \lambda, \mu_1, \mu_2, P_D)}{\partial \lambda} \;=\; \left(P_D - \sum_{i=1}^{3}[P_{Gi}]\right) && (12)\\
0 \quad &= \quad \mu_1[200 - P_{G1}] && (13)\\
0 \quad &= \quad \mu_2[P_{G1} - 500] && (14)\\
0 \quad &\geq \quad [200 - P_{G1}] && (15)\\
0 \quad &\geq \quad [P_{G1} - 500] && (16)\\
\mu_1 \quad &\geq \quad 0 && (17)\\
\mu_2 \quad &\geq \quad 0 && (18)\\
& && (19)
\end{aligned}$$

Answer Outline for Part D (1 Point):

By definition, a *locational marginal price* at any transmission grid bus is the least cost of servicing one additional MW of demand at this bus. The objective for the economic dispatch problem is the minimization of total cost. The one equality constraint for the economic dispatch problem is a balance constraint requiring the sum of the real-power dispatch levels to equal given total demand, where given total demand is the "constraint constant" for this balance constraint. Hence, the claim follows from the ability to express λ^*, the shadow price for the balance constraint, as the change in minimized total cost with respect to a change in total demand, as shown in the on-line notes on "Optimization Basics for Electric Power Markets." There is only one such price because of the assumption that the system is loss-less with no congested transmission lines.