

Concepts from the Theory of the Firm:

K/S Chapter 2.3: Competitive Firm Behavior

Important Acknowledgement:

These notes on Kirschen/Strbac (Chapter 2, Part 2.3) are based on slides prepared by Daniel Kirschen (U Manchester) with substantial edits by Leigh Tesfatsion (Iowa State U).

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Short-Run Cost: Accountant's Perspective

- Some production costs vary with the level of production and some do not:

Variable cost:

- Labor, fuel, transport...

Fixed cost (amortized):

- Equipment, land, overhead

- Some production costs are sunk and some are avoidable

Sunk (Not Avoidable)

- Loan obligations, special assets with no resale or reuse value,...

Avoidable

- GenCo "start-up" costs (if GenCo self commits), labor, fuel, transport, ...



Two Different Decompositions for Total Cost

- Suppose a firm is considering at time 1 what particular production level y it should produce at a future time $t^* > 1$.
- The total production costs associated with each possible **future** production level y at time t^* can be decomposed in two different ways **at time 1**:
 - Total Cost = Fixed Cost + Variable Cost
 - Total Cost = Sunk Cost + Avoidable Cost

Two Decompositions for the Firm's Total Cost at Time 1

- **Fixed Cost:** Does NOT depend on the firm's production choice y for $t^* > 1$
- **Variable Cost:** DOES depend on the firm's production choice y for $t^* > 1$

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- **Sunk Cost:** Payment obligations that the firm has irrevocably committed to before time 1, hence costs that the firm CANNOT AVOID no matter what the firm does over current and future times $t \geq 1$.
- **Avoidable Cost:** Payments that the firm CAN AVOID at time 1 because the firm has not committed to making them, or because they involve costs for purchased assets that can be recovered through asset resale or alternative asset use.

EXAMPLE: A Farmer's Costs for 2012

- The current date is 1/1/12. A corn farmer is considering how much corn y to produce during 2012.
- The corn farmer has already taken out a loan on 12/31/11 to finance a used tractor worth \$138,000. As down payment for this loan, he gave the bank \$28,000. He is now obliged to make a \$1,000 payment on this loan at the end of every month for the next 10 years, i.e., 2012 – 2021, or forfeit the tractor.
- The farmer's labor and other operating expenses incurred if he produces corn in amount y during year 2012 are given by $h(y) = ay + by^2$.
- If the farmer shuts down his business during 2012, the bank keeps the down payment, takes ownership of the tractor, and retires the loan (i.e., the farmer is not obliged to make any additional loan payments).
- **Corn Farmer's Total Production Cost for 2012** calculated on 1/1/12 as a function of possible future corn production levels $y > 0$ for 2012: $\$40,000 + h(y)$

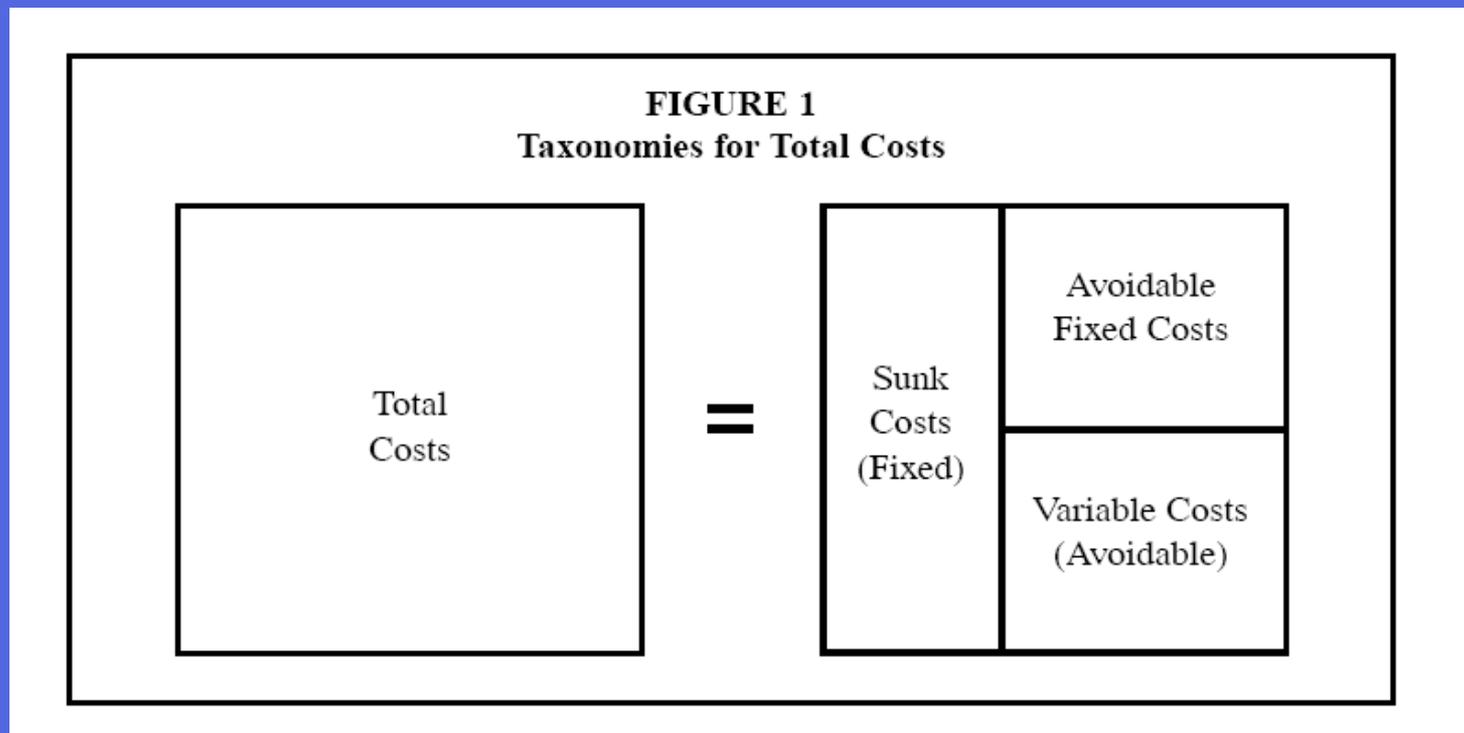
Fixed Cost: $\$40,000 = \$28,000$ (down payment) + $\$12,000$ (loan payments)

Variable Cost: $h(y)$

Sunk Cost: $\$28,000$

Avoidable Cost: $[\$12,000 + h(y)] = [$ Avoidable Fixed Cost + Variable Cost $]$

Total cost decomposition for a planner at the initial time 1 of a production planning period:



Source: X. H. Wang & B. Z. Yang, "Fixed and Sunk Costs Revisited," *Journal of Economic Education*, Spring 2001, pp. 178-185

Total Cost Decomposition ... Continued

We may classify total costs by asking the following two questions sequentially:

Question 1: Is this cost now irrevocably committed over the relevant time period?

Yes → It is a sunk cost. (It is now irrelevant whether the firm intended the costs to be fixed or variable at the time they were irrevocably committed.)

No



It is an avoidable cost. Then, we ask,

Question 2: Does this cost vary with the output level to be produced over that time period?

No → It is an (avoidable) fixed cost.

Yes → It is a variable cost (avoidable).

Source: X. H. Wang & B. Z. Yang, "Fixed and Sunk Costs Revisited," *Journal of Economic Education*, Spring 2001, pp. 178-185

Two Different Decompositions for Total Cost...Continued

- At the initial time 1 for any given production planning period:
 - 1) Total Cost = Fixed Cost + Variable Cost
 - 2) Total Cost = Sunk Cost + Avoidable Cost
- **Second** way of decomposing cost is easier to understand & use.

Example: How to classify at time 1 a “start-up cost” incurred by a GenCo if it starts up its generation unit at time $t^* > 1$ in order to produce a power level $y > 0$ at time $t^* > 1$? The start-up cost changes from 0 to positive at time $t^* > 1$ as the GenCo varies its power level from $y=0$ to $y > 0$ at time $t^* > 1$, but the start-up cost does not depend on the particular magnitude of $y > 0$. This is an example of **Kirschen’s “quasi-fixed cost”**.

Easy answer under 2): It’s a sunk cost at time 1 if the GenCo is *obliged* to start its generation unit at time $t^* > 1$. Otherwise it’s an avoidable cost at time 1.

Costing of Durable-Benefit Expenditures Requires Care

- How should producers cost out expenditures whose benefits last over successive time periods?
 - Purchase of physical assets with long-term durability such as equipment, buildings, and land
 - Start-up costs (once started up, a generation unit can be run over successive hours without incurring additional start-up costs)
- Prior to such expenditures, there must be SOME production planning period T over which full recovery of all avoidable costs is anticipated, else the costs should not be incurred.
- Once incurred, such expenditures become either sunk (e.g., start-up costs) or avoidable fixed costs (e.g., resale value of purchased equipment).
- When deciding whether or not to produce in the presence of avoidable fixed costs, there must be SOME planning period T over which full recovery of all avoidable costs is anticipated, including all avoidable fixed costs, else the firm should shut down and act to reduce its avoidable costs to zero.
- **BOTTOM LINE:** When durable-benefit expenditures are involved, planning periods must be suitably long to permit proper consideration of cost recovery.

Consider again a competitive (price-taking) firm at time $t = 1$ planning its production level y for some future time $t^* > 1$:

- Let π denote the price of y at time t^*
- Let $y = 0$ denote the firm's *shut-down point* at time 1, meaning the firm decides at time 1 to produce 0 at time t^* (no variable costs) **AND** takes actions now, at time 1, to recover all of its avoidable fixed costs.

DEFINE:

Rev(y) = $\pi \cdot y$ = revenues from sale of y at time t^*

AvoidC(y) = avoidable costs associated with production of y at time t^*

NetEarnings(y) = [Rev(y) – AvoidC(y)] at time t^* if $y > 0$ at time t^*

NetEarnings(0) = 0 (Net earnings = 0 at time t^* if firm shuts down at time 1)

Profits(y) = [NetEarnings(y) - SunkCosts] at time t^* for any $y \geq 0$

Determining Opt Production for a Competitive (Price-Taking) Firm...

- **Problem:** At time 1, find production level $y^* \geq 0$ for time $t^* > 1$ that maximizes $\text{NetEarnings}(y)$
- **Step 1:**

Use the calculus to find at time 1 a production plan $y' > 0$ for time t^* that maximizes $\text{NetEarnings}(y)$ over $y > 0$
- **Step 2:**

If net earnings at y' are non-negative, the optimal production plan y^* for time t^* is $y^* = y'$;

If net earnings at y' are negative, **shut down** (i.e., plan to produce $y^* = 0$ at time t^* and take action at time 1 to reduce all avoidable fixed cost to zero), so firm's net earnings at time t^* are zero.

Modern Take on Traditional Economic Distinction:

Long Run versus Short Run

- Some factors of production can be adjusted faster than others.
 - *Example:* Fertilizer vs. planting more trees
- **“Long Run”:** All costs can be avoided
- **“Short Run”:** At least some costs are sunk

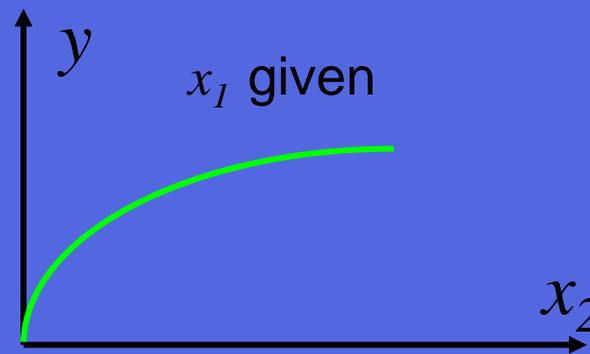
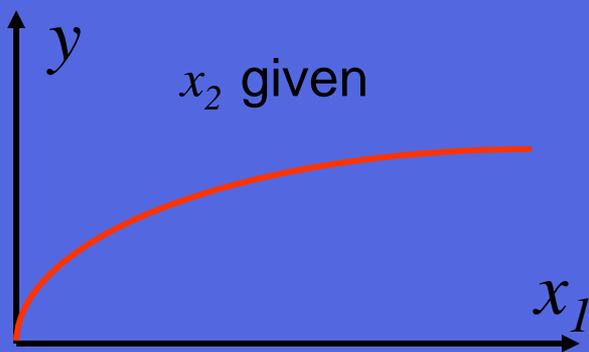
Cautionary Note About Special Assumption Made in K/S Chapter 2:

- It is assumed in K/S Chapter 2, without comment, that ***all fixed costs are sunk costs!***
- Under this special assumption,
 - (a) **Fixed cost = Sunk cost** (i.e., there are **NO** avoidable fixed costs);
 - (b) **Variable cost = Avoidable cost** .
- ◆ In these notes we will treat the general case in which a firm can have avoidable fixed costs as well as sunk costs.

Production function: A function giving maximum possible output for each amount of inputs

$$y = f(x_1, x_2)$$

- y : output
- x_1, x_2 : inputs (or “factors of production”)



**** Law of diminishing marginal product ****

Input-Output Function:

$$y = f(x_1, \overline{x_2}) \quad \overline{x_2} \text{ given}$$

The inverse of this relation (solving for x_1 as a function of y , all else equal) is called the ***input-output function for x_1***

$$x_1 = g(y) \text{ for } x_2 = \overline{x_2}$$

Example: Minimum amount of fuel x_1 required to produce successively higher amounts of electric power $y > 0$, given a particular generation plant x_2

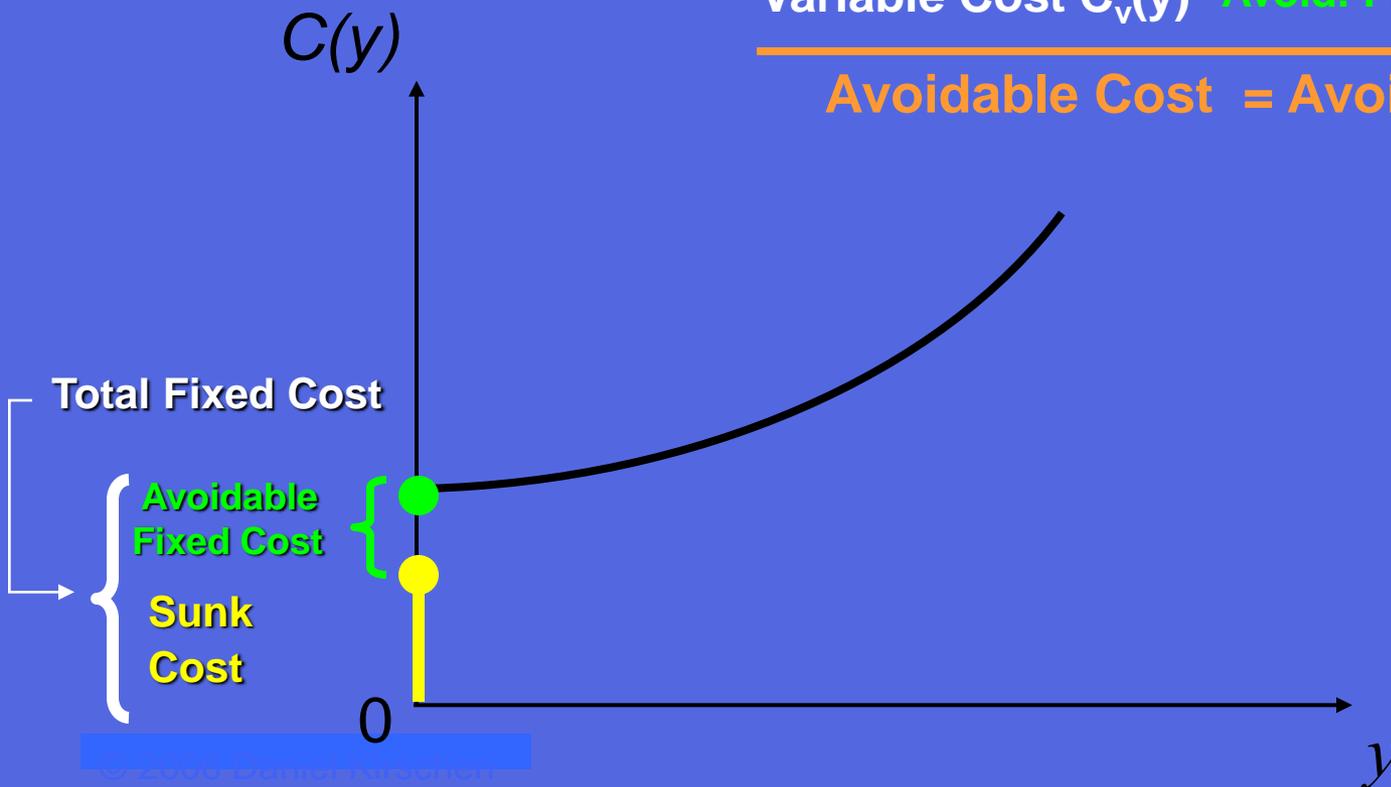
Total Cost Function: $c(y)$

For sunk input $\overline{\theta x_2}$, avoidable fixed input $[1-\theta]x_2$, given prices w_1, w_2 for inputs x_1, x_2 , and $x_1 = g(y) =$ input-output function for variable input x_1 , define:

$$c(y) = w_1 \cdot x_1 + w_2 \cdot x_2 = w_1 \cdot g(y) + w_2 \cdot [1-\theta]x_2 + w_2 \cdot \overline{\theta x_2}$$

Variable Cost $C_v(y)$ Avoid. Fixed Cost Sunk Cost SC

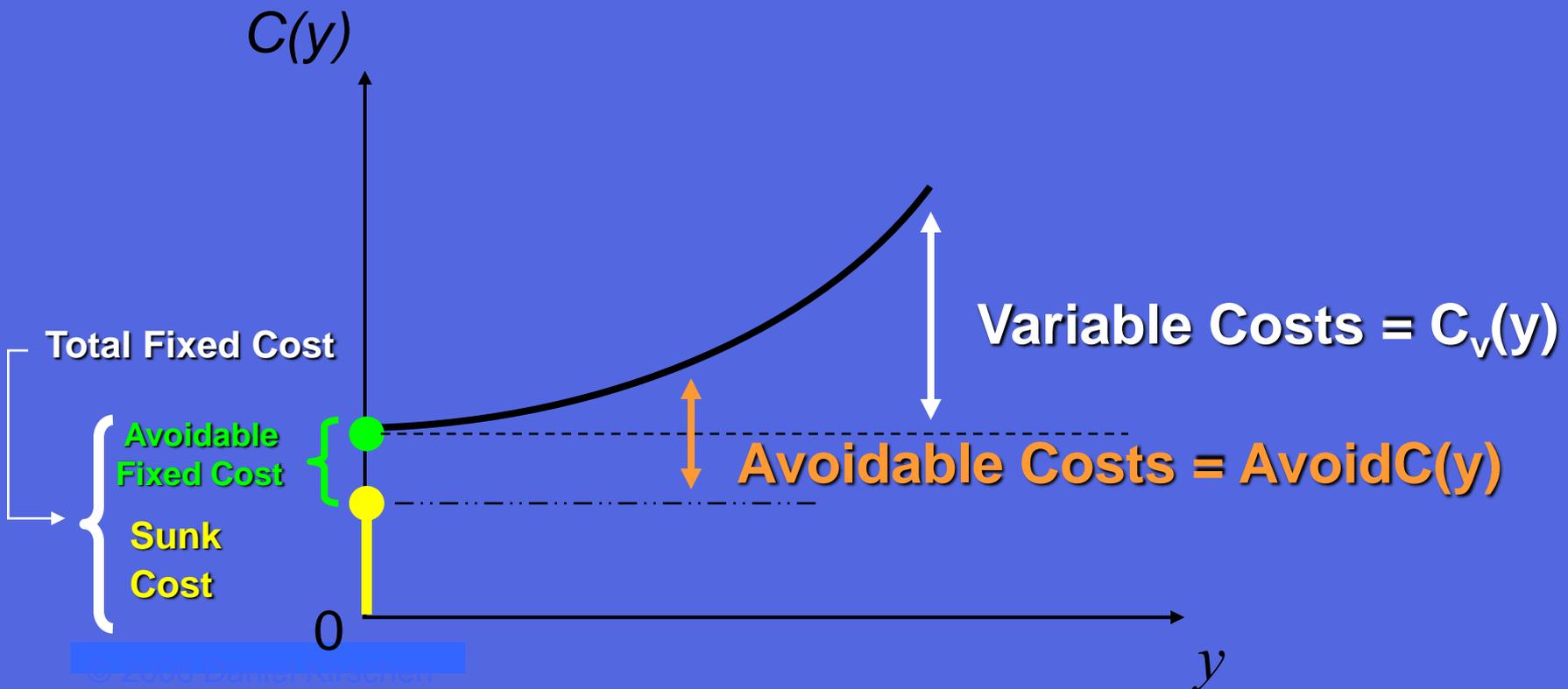
 Avoidable Cost = AvoidC(y)



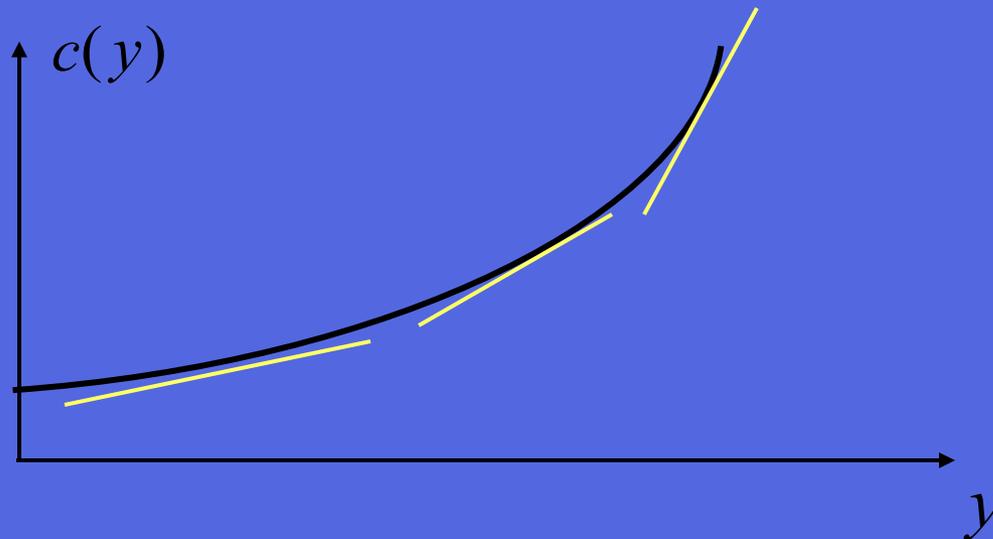
Total Cost Function: $c(y)$

For sunk input $\overline{\theta x_2}$, avoidable fixed input $[1-\theta]x_2$, given prices w_1, w_2 for inputs x_1, x_2 , and $x_1 = g(y) =$ input-output function for variable input x_1 , define:

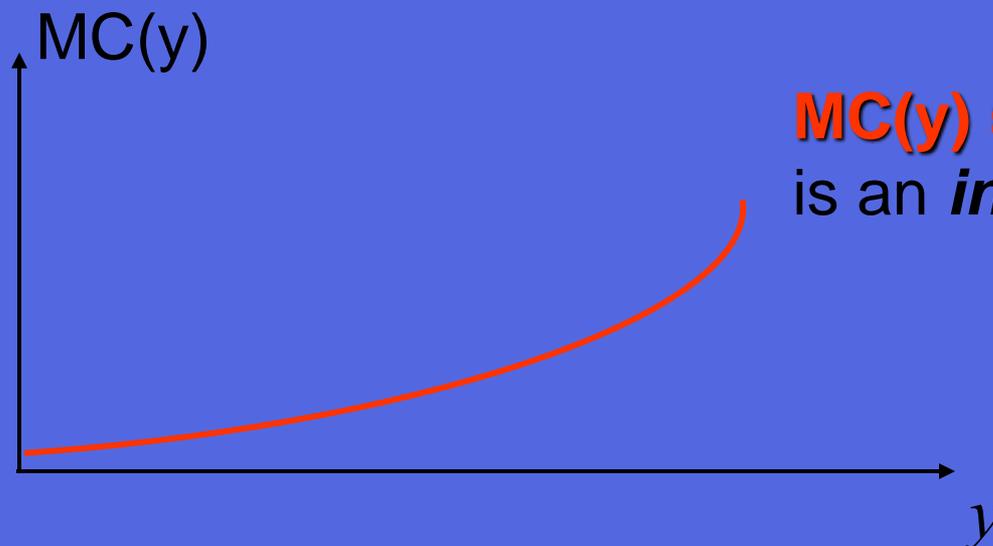
$$c(y) = w_1 \cdot x_1 + w_2 \cdot x_2 = \underbrace{w_1 \cdot g(y)}_{\text{Variable Costs}} + \overbrace{w_2 \cdot [1-\theta]x_2}^{\text{Avoidable Fixed Cost}} + \overbrace{w_2 \cdot \theta x_2}^{\text{Sunk Cost}}$$



Marginal Cost: $MC(y)$ for $y > 0$



If $c(y)$ is strictly convex due to the law of diminishing marginal product

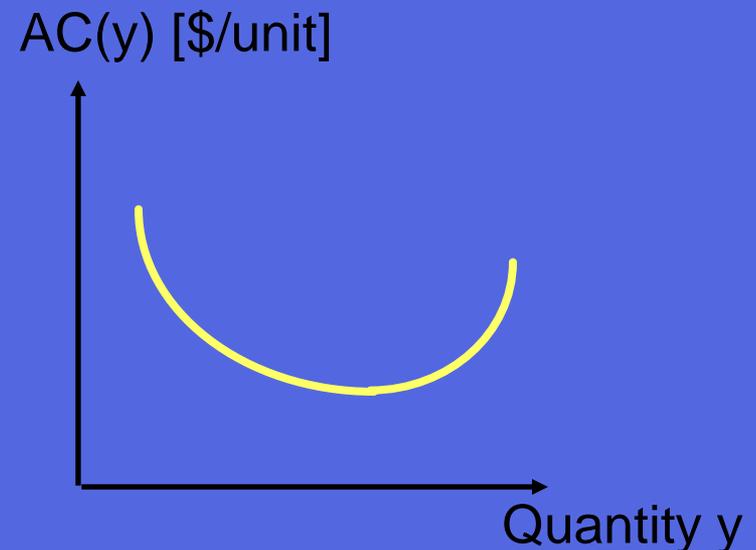
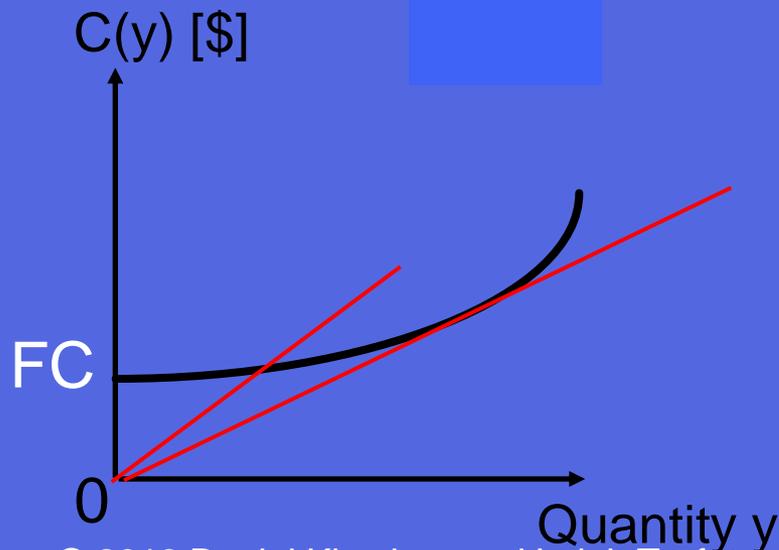


$MC(y) = dC(y)/dy = dC_v(y)/dy$ is an *increasing* function of y

Traditional Cost Curve Decompositions (cf. K/S): Average Cost $AC(y)$, Average Variable Cost $AC_v(y)$, and Average Fixed Cost AFC for $y > 0$

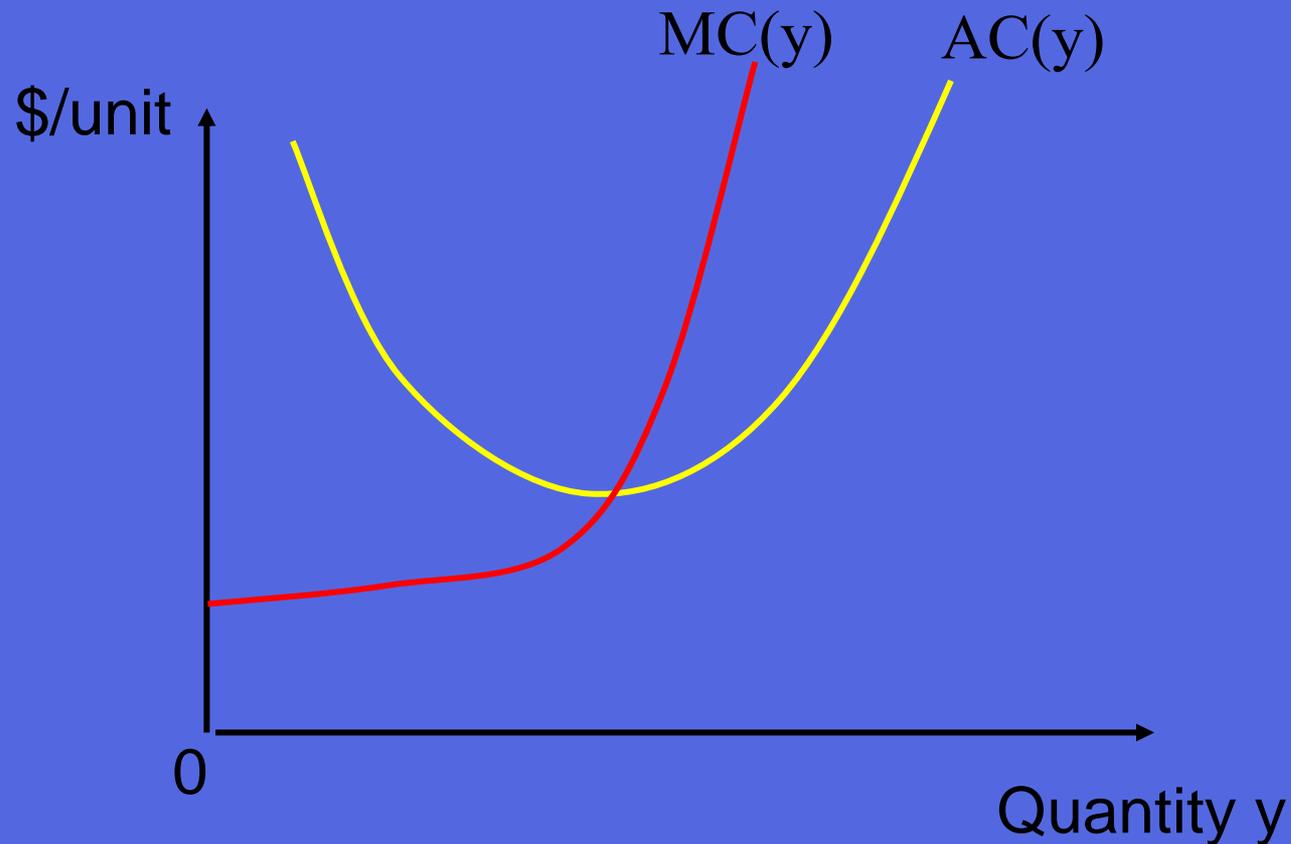
$$C(y) = c_v(y) + FC = \text{Variable Cost} + \text{Fixed Cost} \\ (\text{with } C_v(0) = 0)$$

$$AC(y) = \frac{C(y)}{y} = \frac{c_v(y)}{y} + \frac{FC}{y} = AC_v(y) + AFC$$

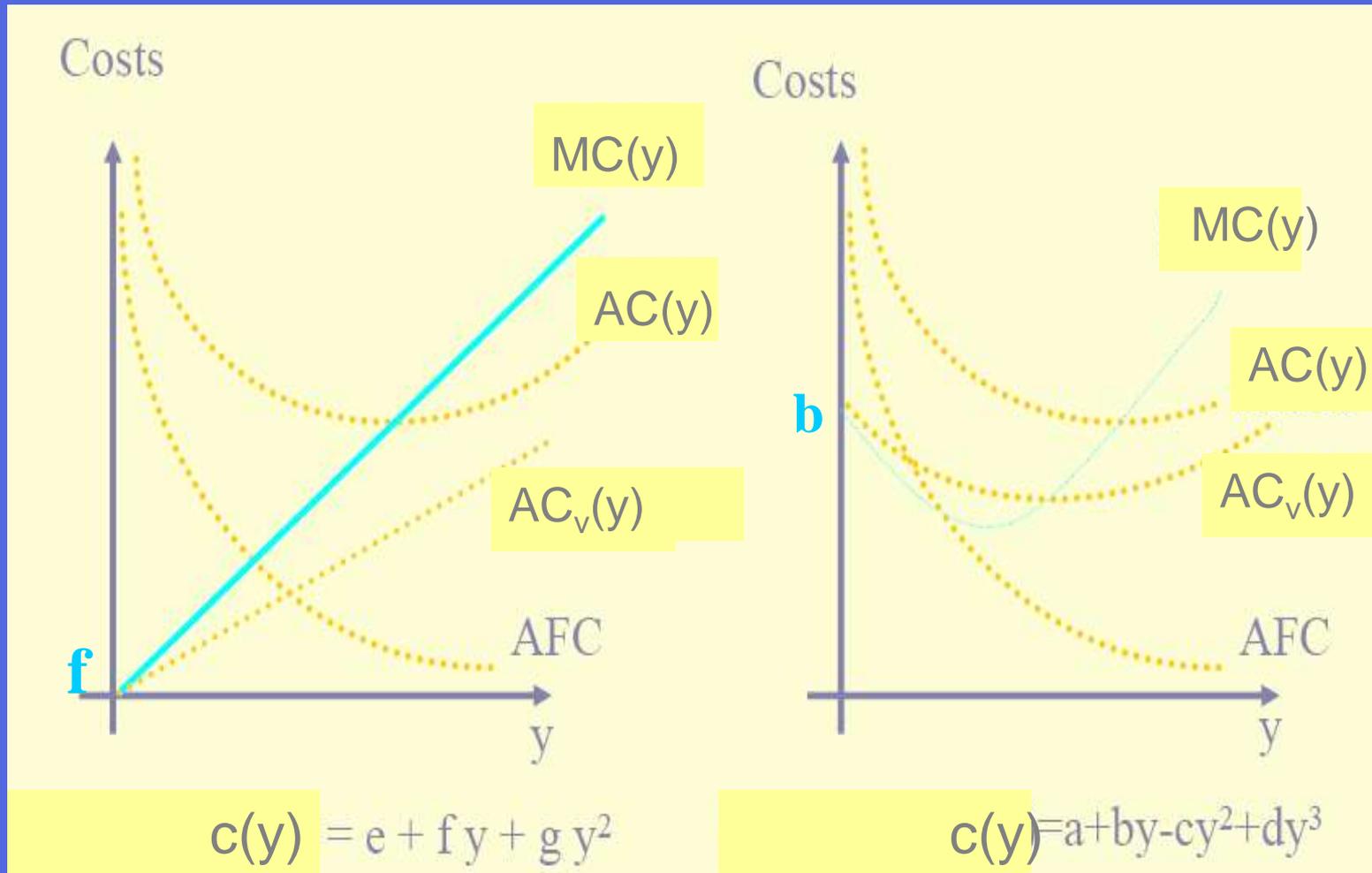


Relation between Marginal Cost and Average Cost

Note: $dAC(y)/dy = [MC(y) - AC(y)] / y$



A Fuller Depiction of Cost Relationships: Two Examples



Note: From the statement of these examples we do not know how much of the fixed costs e and a are avoidable versus sunk.

Consider again a competitive (price-taking) firm at time 1 trying to plan an optimal production level y^* for a future time $t^* > 1$:

π = Price of y at time t^*

$\text{NetEarnings}(y) = [\pi y - \text{AvoidC}(y)]$ for $y > 0$ (outcome at t^* if $y > 0$ at t^*)

$\text{NetEarnings}(0) = 0$ (outcome at time t^* given shut-down at time 1)

Time-1 Problem: Find $y^* \geq 0$ that maximizes $\text{NetEarnings}(y)$ at t^*

- **Step 1:**

Use the calculus at time 1 to find a production level $y' > 0$ for time t^* that maximizes $\text{NetEarnings}(y)$ over all $y > 0$

- **Step 2:**

If net earnings at y' are non-negative, plan to produce $y^* = y'$ at time t^* ;

If net earnings at y' are negative, **shut down** (i.e., plan $y^* = 0$ for time t^* and take action at time 1 to reduce all avoidable fixed cost to zero), so net earnings at time t^* are 0.

Additional Details About Step 1:

- If $y' > 0$ maximizes net earnings over the range $y > 0$, then y' must satisfy the following condition:

$$\max_y \{ \pi \cdot y - \text{AvoidC}(y) \}$$



$$\frac{d \{ \pi \cdot y - \text{AvoidC}(y) \}}{dy} = 0$$



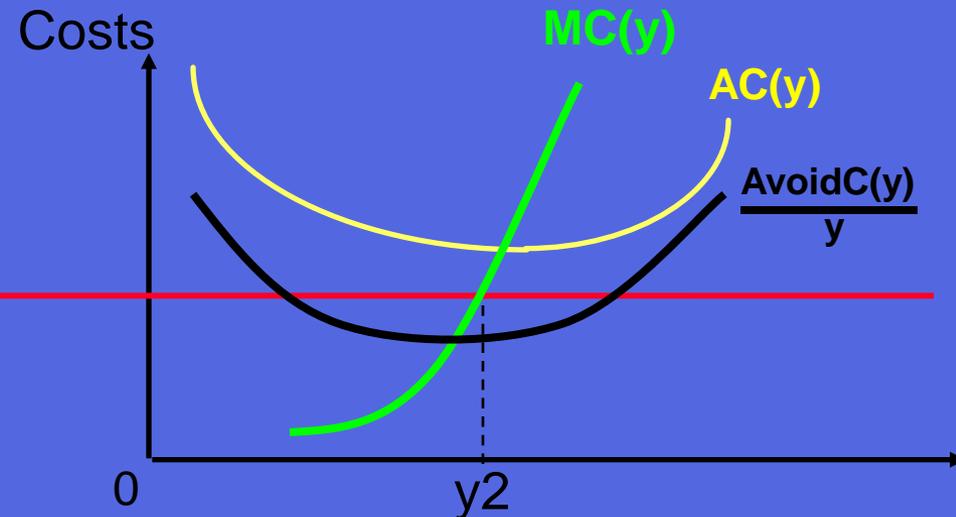
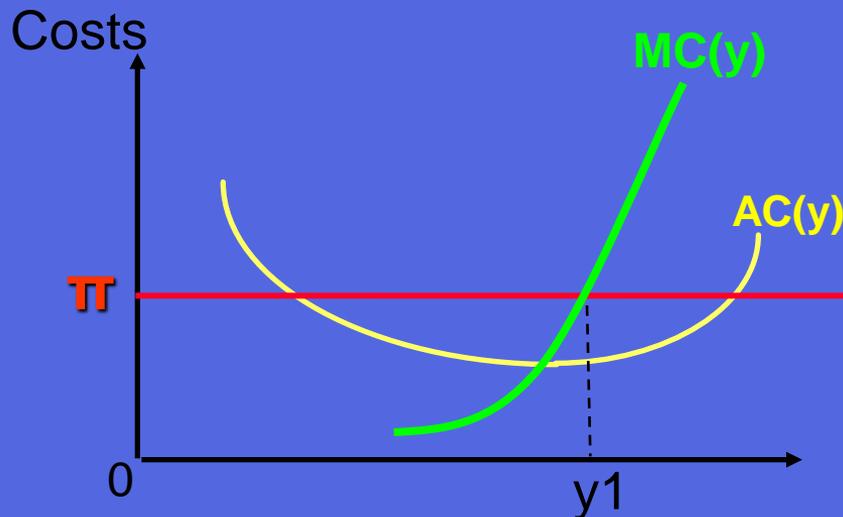
$$\pi = \frac{dC_v(y)}{dy} = MC(y)$$

Equality here only holds if firm perceives that changes in its output y have no effect on the market price π for y

↔ Competitive producer that perceives no way to affect market prices to its advantage through changes in its supply offer

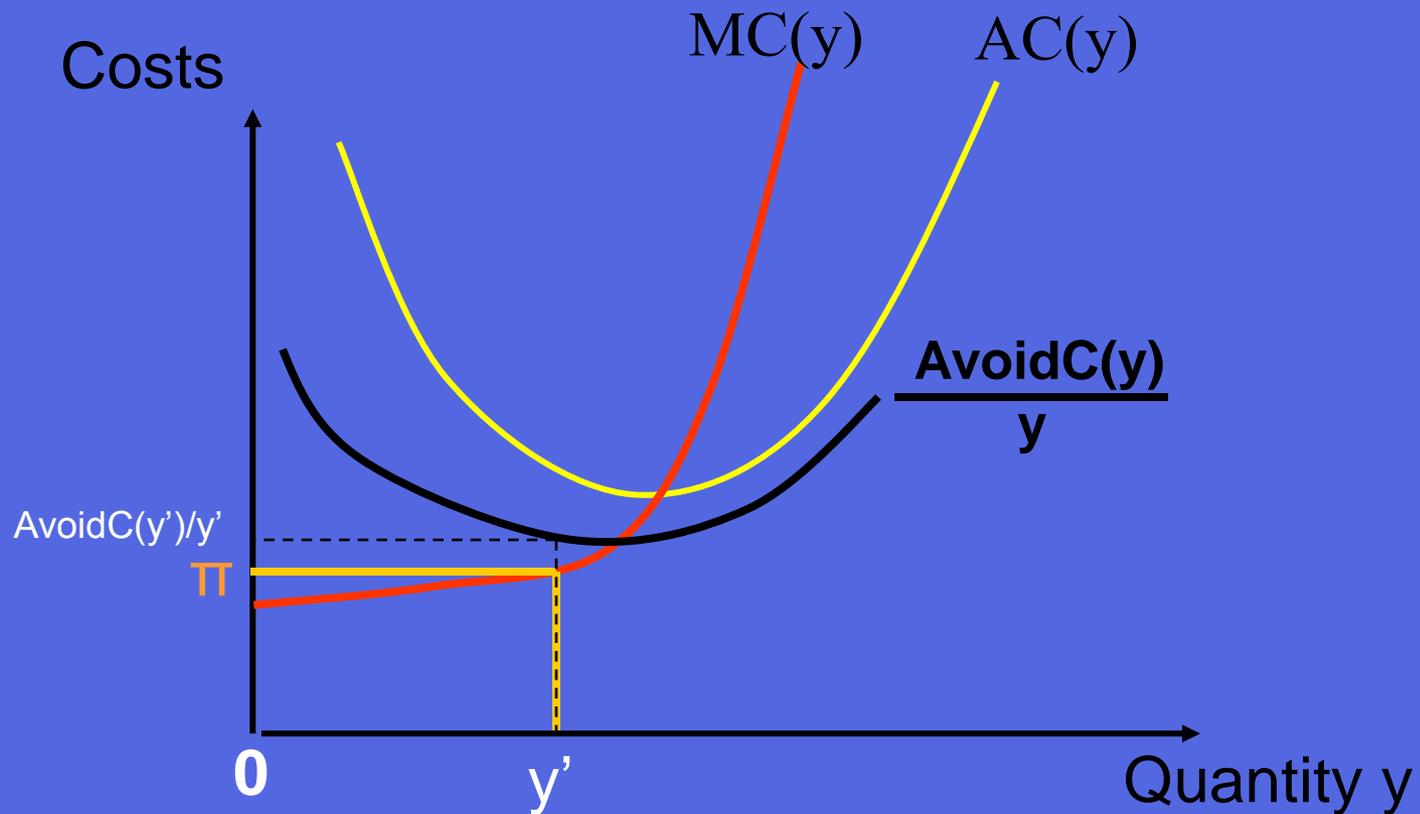
Additional details about Step 1...Continued

- $MC(y) = dC(y)/dy =$ Cost of producing 1 more unit
- If $MC(y) < \pi$, the next unit costs **less** than it returns.
- If $MC(y) > \pi$, the next unit costs **more** than it returns.
- Find $y' > 0$ where $MC(y')$ just equals π (cutting from below)
- If $NetEarnings(y') \geq 0$, produce $y^*=y'$. Otherwise, set $y^* = 0$ (shut down).

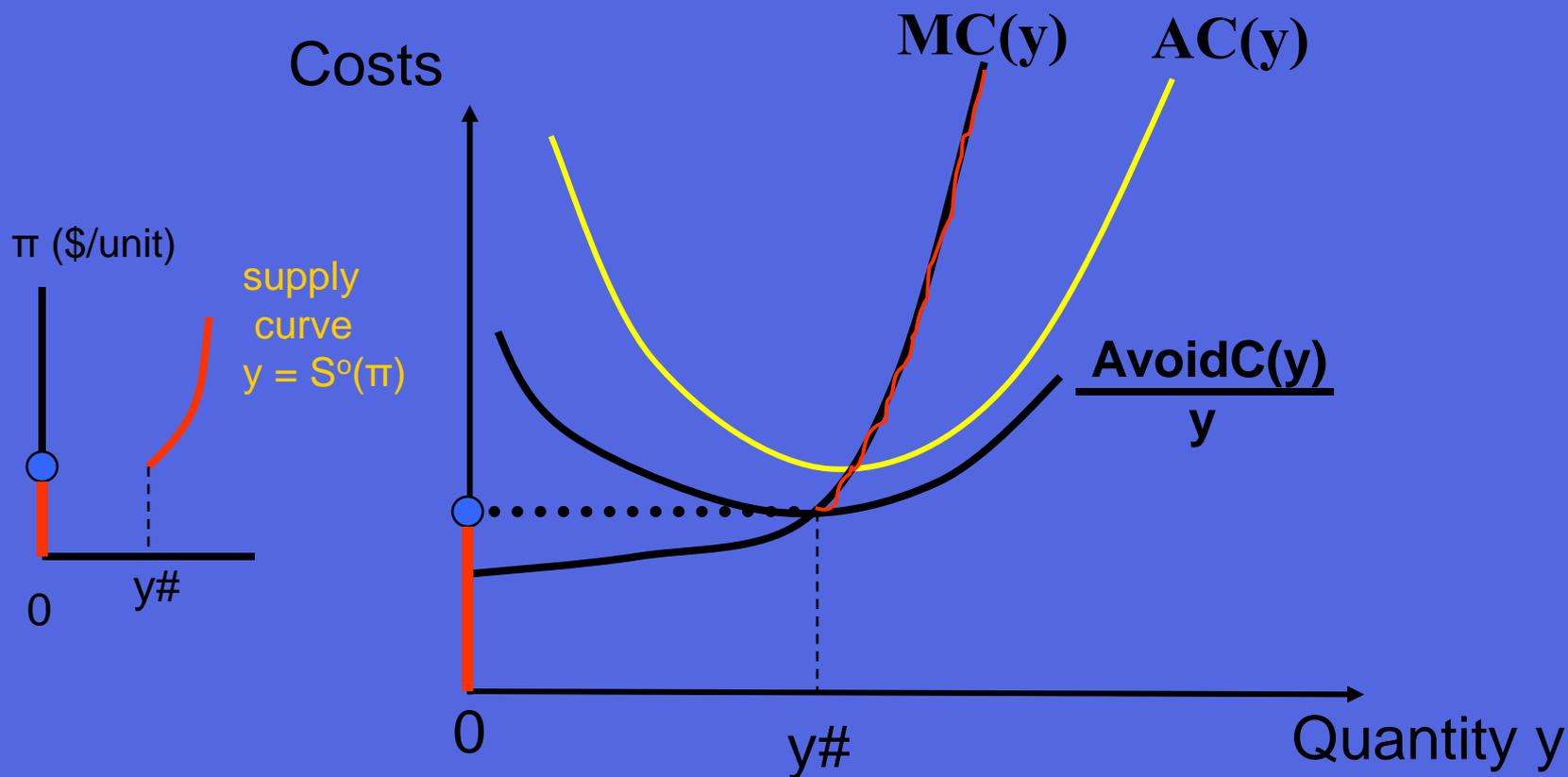


At optimal production level $y^*=y_1$, all costs are fully covered. At optimal production level $y^*=y_2$, all avoidable costs $AvoidC(y)$ are fully covered but some sunk costs are not.

Below, at y' where $MC(y) = \pi$, $NetEarnings(y') = [\pi \cdot y' - AvoidC(y')] < 0$, that is, avoidable cost $AvoidC(y')$ is not fully covered by revenues $\pi \cdot y'$. The firm should shut down and attain $NetEarnings(0) = 0$.

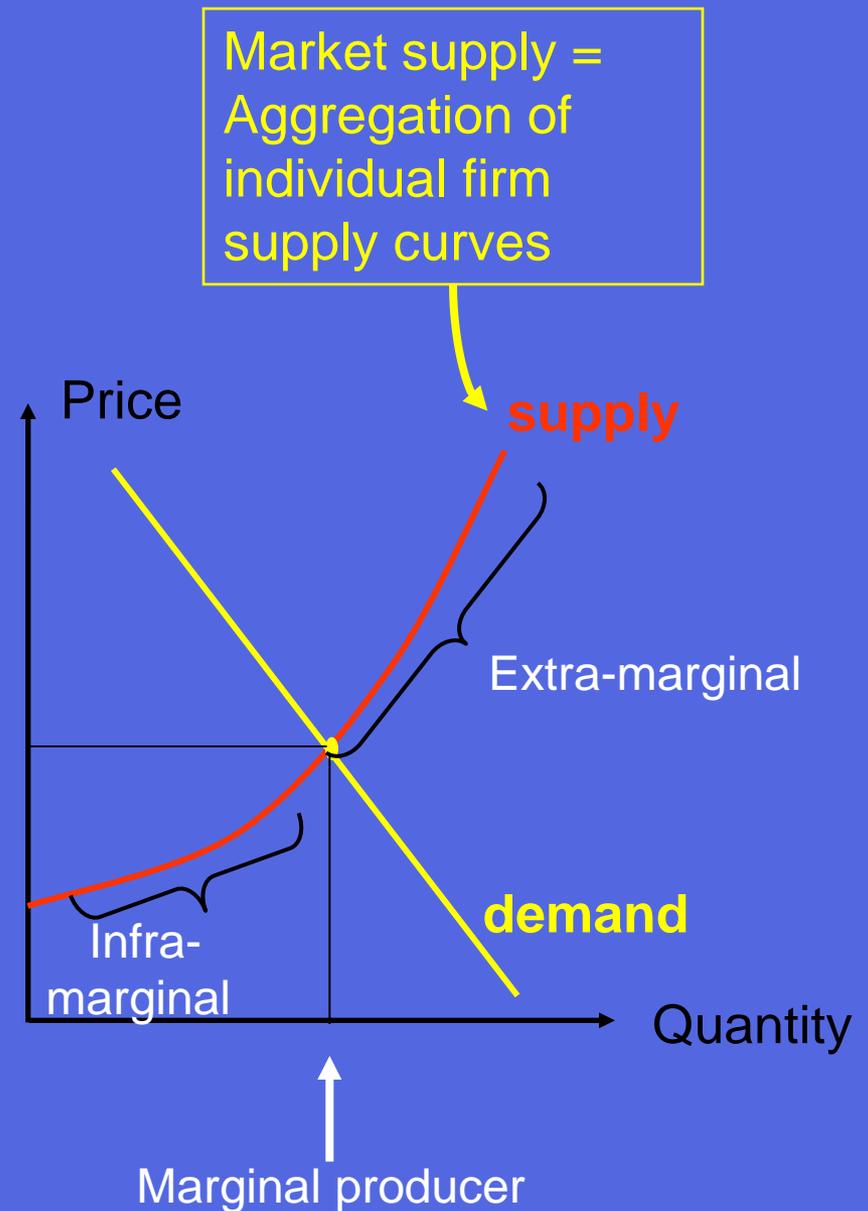


The supply curve for a competitive firm coincides with the part of its MC curve lying on or above its average avoidable cost curve, i.e., on or above the plot of $\text{AvoidC}(y)/y$ for $y > 0$.



Perfect Competition

- Perfect competition
 - The volume handled by each market participant is small compared to the overall market volume
 - No market participant can influence the market price by its own unilateral actions
 - All market participants act as price takers



Costs: Economist's Perspective

- *Opportunity cost:*
 - What is the **next best** use of the money that is currently being spent to produce something ?
 - Failure to exploit an opportunity to invest this money in a more advantageous use represents an “opportunity cost”
- **Examples:**
 - Growing apples or growing kiwis?
 - Using money to grow apples or instead depositing this money in a bank where it would earn interest?
- “Avoidable cost” includes income lost by not investing instead in the next best opportunity (i.e., opportunity cost)
- Thus, even if $\text{NetEarnings}(y^*) = 0$ at some production level $y^* > 0$, meaning y^* is sold “at cost” (i.e., at a break-even point), this does not necessarily mean zero dollar gain. It means the firm is just indifferent between current use and next best use of its productive resources.

Costs: Economist's Perspective ... Continued

- “Avoidable cost” includes income lost by not investing instead in next best opportunity (i.e., opportunity cost)
- Selling “at cost” (net earnings = 0) does not necessarily mean zero dollar gain. It means producer is just indifferent between current use and next best use.
- Suppose you have \$100 to invest for 1 year. If you buy Apple stock for \$100 on Jan 1 and resell it on Dec 31, you know you will receive a net return of \$3. Thus you should only buy IBM stock on Jan 1 if your revenues cover your “avoidable cost” = \$103, where
$$\$103 = \text{Investment Cost } \$100 + \text{Opportunity Cost } \$3.$$

Energy Plant Power Usage Over Time

- How to account for cost of each part?

-  = Start-up power usage
-  = No-load power usage
-  = Power injected into grid
-  = Shut-down power usage

