Concepts from the Theory of the Firm:
K/S Chapter 2: Part 2.5, Imperfect Competition

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Imperfect Competition

• One or more “strategic player” firms can influence the market price through their actions

• Strategic player firms
  - Can influence the market price through their actions (price SETTERS rather than price takers)
  - Perceive and actively exploit potential demand for their output
  - Do not have “supply curves” in previously developed sense
  - Typically have a large market share of capacity/output/revenues

• A “competitive fringe” could still exist
  - Participants with a small market share
  - Take the market price as given

• Examples: Monopoly (single seller) models, Cournot and Bertrand multi-seller models of competition
Single-Price Monopoly Models

• A firm is a **monopoly** if it is the only supplier of a product \( y \) for which there is no close substitute.

• A monopoly that is limited to charging the same price \( \pi \) for each unit of its product \( y \) is called a **single-price monopoly**.

• This is the case treated in Kirschen/Strbac, Chapter 2, Section 2.5.3.
Single-Price Monopoly…Cont’d

- A single-price monopolist perceives it faces a downward sloping inverse demand curve $\pi(y) = D^{-1}(y)$.

- The monopolist exploits this knowledge by setting its price equal to $\pi(y) = D^{-1}(y) = (\text{maximum willingness of buyers to pay for } y)$, given it produces $y$.

- The monopolist sets its price with no worry this price can be undercut by existing rivals.

- However, new market entrants could be a concern, i.e., the market could be contestable.
Marginal Revenue for a Single-Price Monopolist

• **Total Revenue Function:** \( TR(y) = \pi(y) \cdot y \)

• **Marginal Revenue Function:**

\[
MR(y) = \frac{dTR(y)}{dy} \\
= \pi(y) + \frac{d\pi(y)}{dy} \cdot y \\
= \pi(y) + \frac{d\pi(y)}{dy} \cdot \frac{y}{\pi(y)} \cdot \pi(y) \\
= \pi(y) \left[ 1 - \frac{1}{|\varepsilon(y)|} \right]
\]

• **Note:** \( MR(y) < \pi(y) \) unless demand is perfectly elastic (i.e., \( \varepsilon(y) = -\infty \) )
Profix Maximization for a Single-Price Monopolist

• Basic Observation:

A small increase in output by a monopolist will *increase* its net earnings if MR > MC and *decrease* its net earnings if MR < MC.

• Single-Price Monopolist’s Short-Run Optimization Rule:

  ▪ Increase output if MR > MC;
  
  ▪ Decrease output if MR < MC;
  
  ▪ Choose output \( y^* \) where \( MR(y^*) = MC(y^*) \) and set price \( \pi^* \) equal to the consumer’s maximum willingness to pay at \( y^* \), i.e., set
    \[
    \pi^* = \pi(y^*) = D^{-1}(y^*)
    \]
  
  ▪ If revenue \( \pi^*y^* \) is at least as great as avoidable costs at \( y^* \), i.e., at least as great as \( \text{AvoidC}(y^*) = [C_v(y^*) + \text{AvoidFC}] = [\text{VariableCost}(y^*) + \text{Avoidable Fixed Cost}] \), then produce \( y^* \). Otherwise, shut down.
Single-Price Monopolist’s Profit Maximization:

\[ \text{Net Earnings} = |\text{Revenue} - \text{Avoidable Cost}| = \$390 \]

\[ y^* = 6 \]
\[ \frac{\text{AvoidC}(y^*)}{y^*} \]
\[ \pi^* = \$165 \]

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Natural Monopoly

- A production process is said to exhibit *economies of scale* if average avoidable costs of production decrease with increases in production.

- A production process is said to be a *natural monopoly* if it exhibits such extensive economies of scale that one monopolist firm can meet all demand at a lower average avoidable cost than two or more firms.

- **Key Issue for Power Industry:** What aspects of electric power production (if any) are properly considered to be “natural monopolies” and hence potential candidates for regulation?
Cournot Duopoly (Two-Firm) Model of Quantity Competition:

Let $Y = y_1 + y_2$, and let $\pi(Y) =$ market price given $Y$.

Problem for firm 1: $\max_{y_1} \left[ \pi \left( y_1 + y_2^e \right) y_1 - AvoidC(y_1) \right]$ 

$y_1 = f_1 \left( y_2^e \right) \quad (y_2^e =$ expected $y_2)$

Similar problem for firm 2:

$y_2 = f_2 \left( y_1^e \right) \quad (y_1^e =$ expected $y_1)$

Cournot-Nash Equilibrium: $\begin{cases} y_1^* = f_1 \left( y_2^* \right) \\ y_2^* = f_2 \left( y_1^* \right) \end{cases}$ \quad fulfilled firm expectations

Neither firm has any incentive to deviate from the equilibrium.
Cournot Oligopoly (Multi-Firm) Model:

Total industry output: \( Y = y_1 + \cdots + y_n \), \( s_i = y_i / Y \)

Firm i:
\[
\max_{y_i} \left\{ y_i \cdot \pi(Y) - \text{AvoidC}(y_i) \right\} = 0
\]

This equals 1 for perfect competition because \( d\pi / dy_i = 0 \)

This equals 1 – \( 1 / \varepsilon(Y) \) for a monopolist (i.e., when \( n = 1 \))
Cournot Oligopoly Model ... Continued

\[
\pi(Y) \left\{1 - \frac{s_i}{|\varepsilon(Y)|}\right\} = \frac{dc_i(y_i)}{dy_i} = MC(y_i)
\]

< 1 except for perfect competition case \(\varepsilon(Y) = -\infty\)

• Cournot firm i operates at a point where its marginal cost \(MC(y_i)\) is less than the market price \(\pi(Y)\)

• Ability of a Cournot firm i to advantageously deviate from “competitive” \(\pi(Y) = MC(y_i)\) point is a function of:

1) Market share \(s_i = y_i / Y\)

2) Inverse of price elasticity of demand \(1/\varepsilon(Y) = \frac{d\pi(Y)}{dy_i} \frac{Y}{\pi(Y)} < 0\)
Cournot Oligopoly Model…Continued

• We can now complete this model in the same way we did for the duopoly case (N=2)

• Consider the following equation:

\[
\pi(Y) \left\{1 - \frac{s_i}{|\varepsilon(Y)|}\right\} = \frac{dc_Y(y_i)}{dy_i}
\]

• Let Y be replaced by [ \( y_i + h(y_{e-i}) \) ] = Sum of \( y_i \) plus expected outputs for all firms other than firm i

• Then optimal \( y_i \) choice for each firm i can be expressed as a function \( y_i^* = f(y_{e-i}) \) of the expected output of other firms.

• Cournot-Nash equilibrium holds at \( y^* = (y_{1}, \ldots, y_{N}) \) if

\[
y_{i}^* = f(y_{e-i}^*) \text{ for each firm } i \text{ (i.e., all firms have correct expectations at } y^*)
\]
Bertrand Duopoly (Two-Firm) Model of Price Competition

- Two identical firms with constant marginal cost of production, $c$, are competing for demand

- Each firm offers a price with the promise to supply all quantity demanded from it at that price

**IF a firm was a monopolist, it would solve:**

$$\text{Max } [ \pi D^c(\pi) - cD^c(\pi) ] \Rightarrow \pi^*\left[1 - 1/|\varepsilon^*|\right] = c$$

(with $\varepsilon^* = \varepsilon(\pi^*) < 0$)

Market Demand Curve $Y = D^c(\pi)$ in ordinary form

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Bertrand Duopoly … Continued

- However, the firm that sets the lowest price captures the entire market, so firms have an incentive to undercut each other’s price offers.
- Neither firm will bid below its marginal cost of production, $c$, because it would sell at a loss.
- Thus the only Nash equilibrium is when each firm sells at the same price $\pi^*$, equal to the marginal cost of production $c$.
- Price=MC: Equivalent to competitive equilibrium!
- Note that the division between the firms of the output $Y^*$ resulting when $\pi^* = c$ is indeterminate.