Solutions: Problem Set #9

(9.1) (a)

$$\begin{aligned}\Pr(Deny = 1|P/I = .35, Black = 1) &= \Phi(-2.26 + 2.74(.35) + .71)\\ &= \Phi(-.591)\\ &= .278\end{aligned}$$

(b)

$$\begin{aligned}\Pr(Deny = 1|P/I = .3, Black = 1) &= \Phi(-2.26 + 2.74(.3) + .71)\\ &= \Phi(-.728)\\ &= .233\end{aligned}$$

(c)

$$\begin{aligned}\Pr(Deny = 1|P/I = .35, Black = 0) &= \Phi(-2.26 + .959)\\ &= \Phi(-1.301)\\ &= .097\end{aligned}$$

$$\begin{aligned}\Pr(Deny = 1|P/I = .3, Black = 0) &= \Phi(-2.26 + .822)\\ &= \Phi(-1.438)\\ &= .075\end{aligned}$$

(d) Yes, the marginal effect does depend on race. Following class discussion, it follows that

$$\frac{\partial \Pr(Deny = 1|P/I, Black)}{\partial P/I} = \phi(\beta_0 + \beta_1 P/I + \beta_2 Black)\beta_1.$$

So, this marginal effect is clearly a function of the Black indicator variable.

9.2

We are now asked to repeat the analysis of 9.1a using results from the logit model. Based on equation (9.10), we have that

$$\begin{aligned}\Pr(\mathit{Deny} = 1 | P/I = .35, \mathit{Black} = 1) &= (1 + \exp[-(-4.13 + 5.37(.35) + 1.27)])^{-1} \\ &= (1 + \exp[.9805])^{-1} \\ &= .273\end{aligned}$$

$$\begin{aligned}\Pr(\mathit{Deny} = 1 | P/I = .3, \mathit{Black} = 1) &= (1 + \exp[-(-4.13 + 5.37(.3) + 1.27)])^{-1} \\ &= (1 + \exp[1.249])^{-1} \\ &= .223\end{aligned}$$

Note that the predicted probabilities from the logit are quite similar to those from the probit, even though the coefficient estimates from the logit are quite different from the probit coefficient estimates.

3 (a) The regression output is provided on the other portion of the problem set solutions. Note that the *linear probability model* implies:

$$\Pr(\widehat{\mathit{College} =} 1 | \mathit{Ability}) = .4264 + .251\mathit{Ability}.$$

Thus, a person at the mean of the ability distribution (Ability = 0) has (approximately) a 43 percent chance of attaining a college education. A person 3 standard deviations above the mean of the ability distribution would have a probability as follows:

$$\Pr(\mathit{College} \widehat{= 1} | \mathit{Ability} = 3) = .4264 + .251(3) \approx 1.18!!!$$

Of course, this is a non-sensical prediction. The probability must lie between 0 and 1. This is a shortcoming of the linear probability model.

(b) The probit coefficient and marginal effect estimates are provided in the attached file. The marginal effect estimates suggest that every unit increase in ability (which means an upward movement of one standard deviation in the ability distribution) increases the likelihood of college entry by about 34 percent.

(c) The logit estimates are provided in the attached file. If one were to calculate the marginal effects from the logit (though this was not required), one would get:

$$\frac{\partial \Pr(College = 1|Ability)}{\partial Ability} = \frac{\exp[-(-.522 + 1.5449 Ability)]}{(1 + \exp[-(-.522 + 1.5449 Ability)])^2}(1.5449)$$

Evaluating these at the mean value of Ability (Ability=0), we get

$$\begin{aligned}\frac{\partial \Pr(College = 1|Ability = 0)}{\partial Ability} &= \frac{\exp[.522]}{(1 + \exp[.522])^2}(1.5449) \\ &= \frac{2.604}{7.211} \\ &\approx .36\end{aligned}$$

So, again, the probit marginal effect of .34 and the logit marginal effect of .36 are quite close to one another.