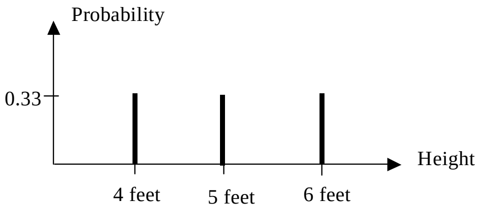
# Basic Probability, Econ 500, Lecture 2

Helle Bunzel

Fall 2006

# Relative Frequency as a Intuitive Measure of Probability

- Consider the height of people walking in a door.
  - Don't know what the height of the next person is.
  - Suppose people can only be 4,5 or 6 feet tall.
  - Imagine that 1/3 of people have each height.
    - This is a distribution of heights.
    - If random people walk in the door, there will be 1/3 probability of getting a person of each height.
    - This is a probability distribution that looks like:



# Relative Frequency as a Intuitive Measure of Probability

- Suppose we repeat the random experiment a number of times, $n$.
- If a certain outcome has occurred $f$ times in these $n$ trials; the number $f$ is called the **frequency** of the outcome.
- The ratio $f/n$ us called the **relative frequency** of the outcome.
- A relative frequencywill often stabilize about a number,$p$, as $n$ increases.
- $p$ is called the probability of the outcome.

Suppose a researcher randomly selected 100 households in Tulsa, Oklahoma. She then asks each household how many vehicles (truck and cars) they own. The data is:

**Table 1:** Data on number of vehicles per household

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 5 | 4 | 5 | 2 | 2 | 4 | 0 | 1 | 1 | 2 | 0 | 0 |
| 2 | 1 | 1 | 2 | 1 | 1 | 1 | 5 | 2 | 2 | 1 | 6 | 4 | 7 | 1 |
| 5 | 1 | 3 | 4 | 6 | 2 | 1 | 3 | 1 | 4 | 1 | 0 | 1 | 1 | 2 |
| 2 | 1 | 6 | 1 | 1 | 3 | 5 | 1 | 1 | 4 | 3 | 4 | 3 | 1 | 0 |
| 3 | 3 | 0 | 2 | 2 | 3 | 2 | 0 | 6 | 3 | 2 | 4 | 4 | 1 | 0 |
| 4 | 0 | 1 | 3 | 3 | 6 | 5 | 5 | 2 | 2 | 3 | 2 | 0 | 2 | 4 |
| 3 | 3 | 2 | 0 | 4 | 1 | 0 | 0 | 1 | 1 | | | | | |

We can construct a frequency table of the data by listing the all the answers possible (or received) and the number of times they occurred.

# Relative Frequency as a Intuitive Measure of Probability

## Example, continued

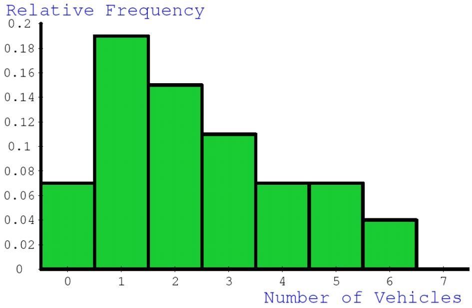
| Number of vehicles | Frequency | Relative Frequency |
|---|---|---|
| 0 | 13 | 0.13 |
| 1 | 28 | 0.28 |
| 2 | 20 | 0.20 |
| 3 | 14 | 0.14 |
| 4 | 12 | 0.12 |
| 5 | 7 | 0.07 |
| 6 | 5 | 0.05 |
| 7 or more | 1 | 0.01 |

- The probability of a household having 4 cars is around 12%.
- The probability of having less than six cars is 94%.

# Relative Frequency as a Intuitive Measure of Probability

## Example, continued

- Frequency Histogram.
  - Center a rectangle with a base of length one at each observed integer value and make the height equal to the relative frequency of the outcome. The total area of the relative frequency histogram is then one.




- The value that occurs most often is called the **mode**.

# Basic Concepts and Definitions of Probability

**Definition**

Experiment: The term **experiment** will denote doing something or observing something happen under certain conditions.

**Definition**

The performance of an experiment is called a **trial** of the experiment.

**Definition**

The observed result, on a trial of the experiment, is formally called an **outcome**.

**Example**

The **experiment** is to do 3 coinflips. Suppose we carry out three **trials**. The the **outcomes** are {H H H}, {H T T} and {T H T}

In economics we often have only one trial.

# Basic Concepts and Definitions of Probability

## Definition

Sample Space. The sample space of an experiment is the set of possible outcomes. We denote it by $\Omega$.

## Examples

If the experiment consists of tossing a coin, the sample space contains two outcomes, heads and tails

$$\Omega = \{H, \; T\}$$

If the experiment consists giving a quiz with a maximum score of 10 points to an economics student and only integer scores are recorded, the sample space is

$$\Omega = \{0, \, 1, \, 2, \, 3, \, \ldots \, , \, 10\}$$

- Sample spaces can be either countable or uncountable

# Basic Concepts and Definitions of Probability

**Definition**

Sample Point. A sample point is any member of $\Omega$ and is typically denoted $\omega$.

**Definition**

Event. A subset of $\Omega$ is called an event.

- We say an event $A$ occurs, if the outcome of the experiment is in the set $A$.

**Definition**

An event with only one member is called a simple event.

**Example**

If the experiment consists giving a quiz with a maximum score of 10 points an event $A$ (flunking) can be written $A = \{0, 1, 2\}$ and $A$ occurs if the students get any of the three scores.

# Probability Defined for a Discrete Sample Space

**Definition**

Discrete Sample Space. A discrete sample space is one that contains either a finite or countable number of distinct sample points.

**Axiom 1:** The probability of an event is a non-negative number $P(A) \geq 0$ for any subset $A$ of $\Omega$.
**Axiom 2:** $P(\Omega) = 1$.
**Axiom 3:** If $A_1, A_2, A_3, \ldots$ is a finite or infinite sequence of mutually exclusive events of $\Omega$, then

$$P(A_1 \cup A_2 \cup A_3 \cup ...) = P(A_1) + P(A_2) + P(A_3) + ...$$

# Probability Measure

- Let $\mathcal{F}$ denote a class of subsets of $\Omega$ to which we assign probabilities.
- A **probability measure** is a non-negative function $P$ on $\mathcal{F}$ having the following properties:

1. $P(\Omega) = 1$.
2. If $A_1, A_2, \ldots$ are pairwise disjoint sets in $\mathcal{F}$, then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

We often refer to the triple $(\Omega, \mathcal{F}, P)$ as a probability model. Only elements of $\Omega$ that are members of $\mathcal{F}$ are considered to be events.

# Elementary Properties of Probability

1. If $A \subset B$, then $P(B - A) = P(B) - P(A)$, where $-$ in the case of sets or events denotes the set theoretic difference or $B \cap A^c$.
2. $P(A^c) = 1 - P(A)$, $P(\varnothing) = 0$.
3. If $A \subset B$, $P(B) > P(A)$.
4. $0 \leq P(A) \leq 1$.
5. $P\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} P(A_i)$.
6. If $A_1 \subset A_2 \subset \ldots A_n, \ldots,$ then $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \lim_{n\to\infty} P(A_n)$.
7. $P\left(\bigcup_{i=1}^{k} A_i\right) \geq 1 - \sum_{i=1}^{k} P(A_i^c)$.

# Discrete Probability Models

**Definition**

A probability model is called discrete if $\Omega$ is finite or countably infinite and every subset of $\Omega$ is assigned a probability.

- Then we can write $\Omega = \{\omega_1, \omega_2, \ldots\}$ and $\mathcal{F}$ is the collection of subsets of $\Omega$.
- In this case

$$P(A) = \sum_{\omega_i \, \epsilon \, A} P(\{\omega_i\})$$

# Review of terms:

- Sample space: $\Omega$
- Class of subsets of $\Omega$ to which we assign probabilities: $\mathcal{F}$
  - If discrete: All possible subsets of $\Omega$.
- Probability measure: $P$, the function that assigns probabilities to each event.
- Probability model: $(\Omega, \mathcal{F}, P)$
- If the sample space is discrete, the probability model is discrete.

# Calculating the Probability of an Event Using the Sample Point Method

## Discrete Sample Space

Steps to Compute Probability:

1. Define the experiment and determine how to describe one simple event.
2. List the simple events associated with the experiment and test each one to ensure that it cannot be decomposed.
3. Assign reasonable probabilities to the sample points in $S$, making certain that $P(E_i) \geq 0$ and $\Sigma P(E_i) = 1$.
4. Define the event of interest, $A$, as a specific collection of sample points.
5. Find $P(A)$ by summing the probabilities of the sample points in $A$.

# Calculating the Probability of an Event Using the Sample Point Method

## Discrete Sample Space, Example

- A fair coin is tossed three times. What is the probability that exactly two of the three tosses results in heads?

Proceed with the steps as follows.

1. The experiment consists of observing what happens on each toss. A simple event is symbolized by a three letter sequence made up of $H$'s and $T$'s. The first letter in the sequence is the result of the first toss and so on.
2. The eight simple events in $S$ are:

$$\begin{array}{llll} E_1 : HHH & E_2 : HHT & E_3 : HTH & E_4 : HTT \\ E_5 : THH & E_6 : THT & E_7 : TTH & E_8 : TTT \end{array}$$

# Calculating the Probability of an Event Using the Sample Point Method

## Discrete Sample Space, Example

1. Given that the coin is fair, we expect that the probability of each event is equally likely; that is,

$$P\left(E_i\right) = \frac{1}{8}, \quad i = 1,\, 2,\, 3\, \ldots$$

2. The event of interest, $A$, is that exactly two of the tosses result in heads. This implies that

$$A = \left\{E_2\; E_3\; E_5\right\}$$

3. We find $P(A)$ by summing as follows

$$P(A) = P(E_2) + P(E_3) + P(E_5) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$