Sets, Econ 500, Lecture 1

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A Review of Set Notation

- Definition of a Set: A set is any collection of objects.
  - The objects are called elements.
- If x is an element of the set S, we say that x belongs to S.
  - We write
$$x \in S$$
  - if y does not belong to S, we write
$$y \notin S$$
- Examples of sets:
$$A = \{1, 2, 4\}$$
$$C = \{x : x^2 + 2x - 3 = 0\}$$

Subsets

- Definition of a *Subset*: If all the elements of a set X are also elements of a set Y, then X is a subset of Y
  - We write
$$X \subseteq Y$$
- Definition of a *Proper Subset*: If X is a subset of Y, but not all elements of Y are in X, then X is a **proper** subset of Y.
  - We write
$$X \subset Y$$
- Definition of *Equality of Sets*: Two sets are equal if they contain exactly the same elements
  - We write
$$X = Y.$$

Examples of Sets

1. All corn farmers in Iowa.
2. All firms producing steel.
3. The set of all consumption bundles that a given consumer can afford,

$$B = \left[(x_1,\, x_2)\, \epsilon\, R^2 :\; x_1 \geq 0, x_2 \geq 0,\; p_1 x_1 + p_2 x_2 \leq I \right]$$

4. The people in Heady Hall who are taller than 5 feet.

Set Operations

- Intersections
  - The intersection, W, of two sets X and Y is the set of elements that are in both X and Y.
  - We write

$$W = X \cap Y = \{x : x \epsilon X \text{ and } x \epsilon Y\}$$

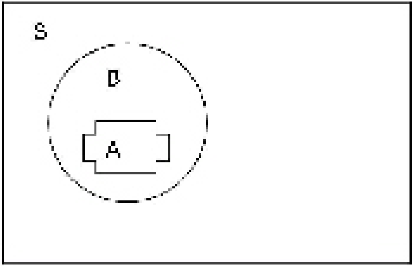
- Example: What is the intersection of students in the Masters program and people who are taller than 5 feet 5 inches?
- Empty or Null Sets
  - The empty set or the null set is the set with no elements.
  - Notation: $\emptyset$.
- Example: What is the intersection of students in the Masters program and people who are taller than 7 feet 5 inches?

Set Operations

- Disjoint Sets: If the sets A and B contain no common elements they are disjoint.
- Example: Students in the Masters program and people in Heady Hall who have a Ph.D.
- Unions: The union of two sets A and B is the set of all elements in one or the other of the sets.
    - We write
$$C = A \cup B = \{x : x \epsilon A \text{ or } x \epsilon B\}.$$
- Complements: The complement of a set X is the set of elements of the universal set U that are not elements of X.
    - We write X^C.
$$X^C = \{x : x \epsilon U \text{ and } x \notin X\}$$
- Example: Let U be the people in Heady Hall and X be the students in the Masters program. What is X^C?

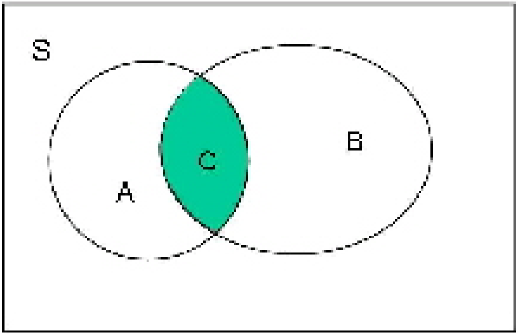
Venn Diagrams

- Venn diagram: Graphical representation of set relationships.
- Example: Let S be the universal set, and let A and B be two sets within the universal set.



- Here, $A \subset B$. Furthermore, $A \cap B = A$ and $A \cup B = B$

Venn Diagrams



What is:

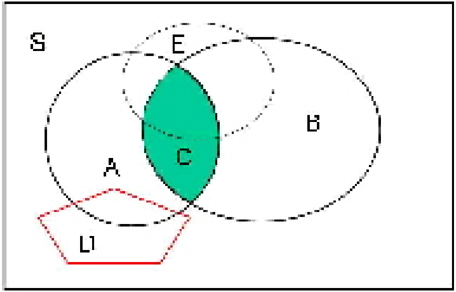
$A \cup B$?

$A \cap B$?

$(A \cap B)^C$?

$(A \cup B)^C$?

Venn Diagrams



- What is:
  - $B \cap D$?
  - $A \cap D$?
  - $E \cup (A \cup B)^C$

Manipulating Sets

Distributive Laws

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

DeMorgan's Laws

1. $(A \cup B)^C = A^C \cap B^C$
2. $(A \cap B)^C = A^C \cup B^C$

Proof of $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

To show that two sets are equal, e.g. $X = Y$ we need to show that:

(i) $\alpha \epsilon X \Rightarrow \alpha \epsilon Y$

(ii) $\alpha \epsilon Y \Rightarrow \alpha \epsilon X$

First prove (i). Suppose that $\alpha \epsilon A \cap (B \cup C)$. Then

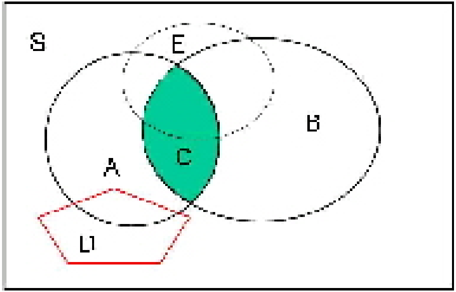
$$\begin{aligned}
\alpha &\in A \cap (B \cup C) \\
&\Rightarrow \alpha \in A \text{ and } \alpha \in (B \cup C) \\
&\Rightarrow \alpha \in A \text{ and } (\alpha \in B \text{ or } \alpha \in C) \\
&\Rightarrow (\alpha \in A \text{ and } \alpha \in B) \text{ or } (\alpha \in A \text{ and } \alpha \in C) \\
&\Rightarrow \alpha \in (A \cap B) \text{ or } \alpha \in (A \cap C) \\
&\Rightarrow \alpha \in (A \cap B) \cup (A \cap C)
\end{aligned}$$

Proof of $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Now prove (ii). Suppose that $\alpha \in (A \cap B) \cup (A \cap C)$. Then

$$\begin{aligned}
\alpha &\in (A \cap B) \cup (A \cap C) \\
&\Rightarrow \alpha \in (A \cap B) \text{ or } \alpha \in (A \cap C) \\
&\Rightarrow (\alpha \in A \text{ and } \alpha \in B) \text{ or } (\alpha \in A \text{ and } \alpha \in C) \\
&\Rightarrow \alpha \in A \text{ and } (\alpha \in B \text{ or } \alpha \in C) \\
&\Rightarrow \alpha \in A \text{ and } \alpha \in (B \cup C) \\
\alpha &\in A \cap (B \cup C)
\end{aligned}$$

Manipulating Sets
Examples



$$D \cap (A \cup B) = (D \cap A) \cup (D \cap B)$$
$$D \cup (A \cap E) = (D \cup A) \cap (D \cup E)$$

$$(A \cap D)^C = A^C \cup D^C$$
$$(A \cup E)^C = A^C \cap E^C$$

Power Set

- For a given set S, define power set of S,

$$\mathcal{P}_S = A : A \subset S = \text{set of all subsets of } S$$

- Example: Consider the set $S = \{1, 2, 3\}$

$$\mathcal{P}_S = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$